basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + x - 12 = 0$ (3)

1.1.2 $3x^2 - 2x = 6$ (answers correct to TWO decimal places) (4)

1.1.3 $\sqrt{2x+1} = x - 1$ (4)

1.1.4 $x^2 - 3 > 2x$ (4)

1.2 Solve for x and y simultaneously:
$x + 2 = 2y$ and $\frac{1}{x} + \frac{1}{y} = 1$ (5)

1.3 Given: $2^{m+1} + 2^m = 3^{n+2} - 3^n$ where m and n are integers.

Determine the value of $m + n$. (4)
[24]

QUESTION 2

2.1 Given the arithmetic series: $7 + 12 + 17 + \ldots$

2.1.1 Determine the value of T_{91} (3)

2.1.2 Calculate S_{91} (2)

2.1.3 Calculate the value of n for which $T_n = 517$ (3)

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)

[16]

QUESTION 3

3.1 Given the geometric series: $3 + 6 + 12 + \ldots$ to n terms.

3.1.1 Write down the general term of this series. (1)

3.1.2 Calculate the value of k such that: $\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98\,301$ (4)

3.2 A geometric sequence and an arithmetic sequence have the same first term.

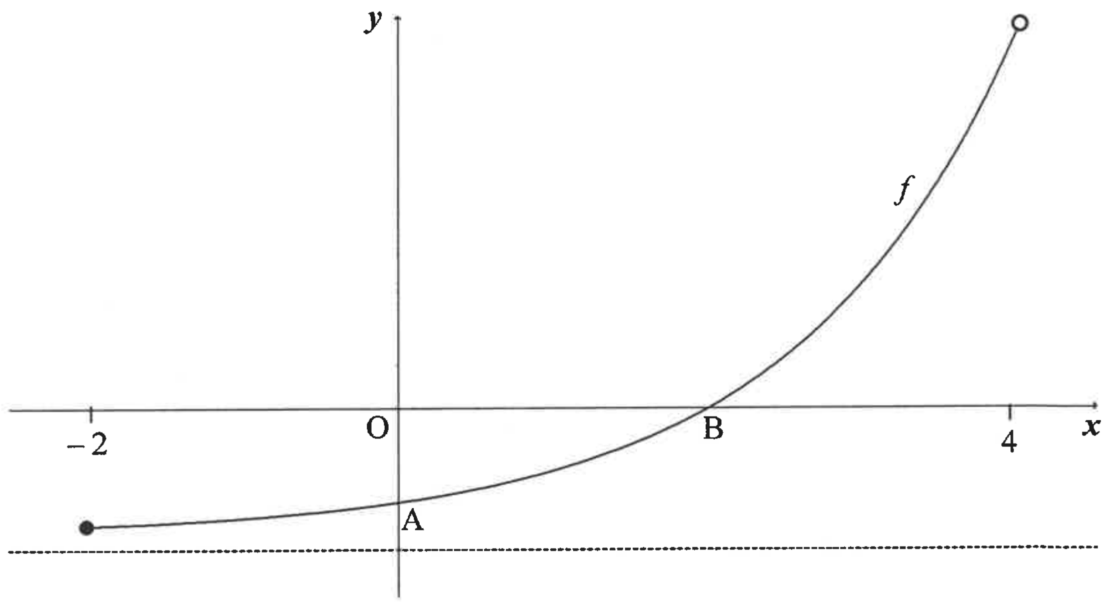
- The common ratio of the geometric sequence is $\frac{1}{3}$
- The common difference of the arithmetic sequence is 3
- The sum of 22 terms of the arithmetic sequence is 734 more than the sum to infinity of the geometric sequence.

Calculate the value of the first term. (5)

[10]

QUESTION 4

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2 ; 4)$.
A and B are respectively the y- and x-intercepts of f.



4.1 Write down the equation of the asymptote of f. (1)

4.2 Determine the coordinates of B. (2)

4.3 Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$ (3)

4.4 Calculate the vertical distance between k and f at $x = 1$ (3)

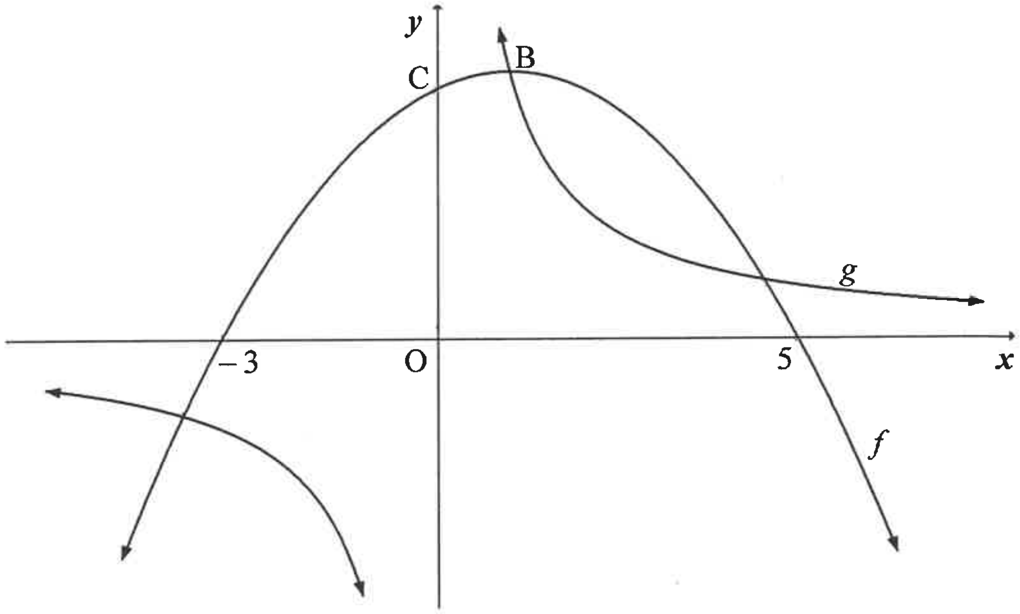
4.5 Write down the equation of g if it is given that $g(x) = f(x) + 4$ (1)

4.6 Write down the domain of g^{-1}. (2)

4.7 Write down the equation of g^{-1} in the form $y = \ldots$ (2)
[14]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f. The graph f has x-intercepts at $(-3\ ;\ 0)$ and $(5\ ;\ 0)$ and a y-intercept at C.



5.1	Write down the coordinates of the turning point of f.	(2)
5.2	Calculate the coordinates of C.	(2)
5.3	Calculate the value of d.	(1)
5.4	Write down the range of g.	(1)
5.5	For which values of x will $f(x).g(x) \leq 0$?	(3)
5.6	Calculate the values of k so that $h(x) = -2x + k$ will not intersect the graph of g.	(5)
5.7	h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y = f(x) + t$ intersects g at R.	(4)

[18]

QUESTION 6

6.1 Patrick deposited an amount of R18 500 into an account earning $r\%$ interest p.a., compounded monthly. After 6 months, his balance was R19 319,48.

6.1.1 Calculate the value of r. (3)

6.1.2 Calculate the effective interest rate. (2)

6.2 Kuda bought a laptop for R10 000 on 31 January 2019. He will replace it with a new one in 5 years' time on 31 January 2024.

6.2.1 The value of the old laptop depreciates annually at a rate of 20% p.a. according to the straight-line method. After how many years will the laptop have a value of R0? (2)

6.2.2 Kuda will buy a laptop that costs R20 000. In order to cover the cost price, he made his first monthly deposit into a savings account on 28 February 2019. He will make his 60^{th} monthly deposit on 31 January 2024. The savings account pays interest at 8,7% p.a., compounded monthly. Calculate Kuda's monthly deposit into this account. (4)

6.3 Tino wins a jackpot of R1 600 000. He invests all of his winnings in a fund that earns interest of 11,2% p.a., compounded monthly. He withdraws R20 000 from the fund at the end of each month. His first withdrawal is exactly 1 month after his initial investment. How many withdrawals of R20 000 will Tino be able to make from this fund? (5)

[16]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$ (5)

7.2 Determine:

7.2.1 $f'(x)$ if $f(x) = 2x^3 - 3x$ (2)

7.2.2 $D_x\left(7.\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

7.3 For which values of x will the tangent to $f(x) = -2x^3 + 8x$ have a positive gradient? (3)

[13]

QUESTION 8

Given: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Determine the coordinates of the turning points of f. (4)

8.2 Draw a sketch graph of f. Clearly label all the intercepts with the axes and any turning points. (4)

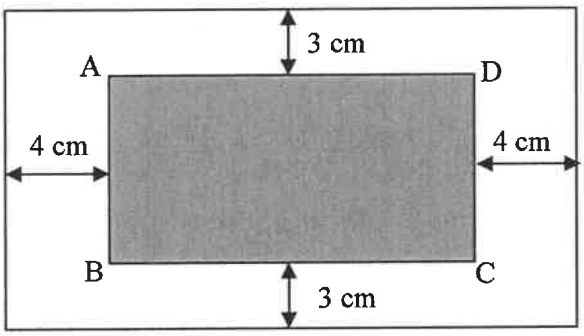
8.3 Use the graph to determine the value(s) of k for which $-x^3 + 6x^2 - 9x + 4 = k$ will have three real and unequal roots. (2)

8.4 The line $g(x) = ax + b$ is the tangent to f at the point of inflection of f. Determine the equation of g. (6)

8.5 Calculate the value of θ, the acute angle formed between g and the x-axis in the first quadrant. (2)
[18]

QUESTION 9

The diagram below represents a printed poster. Rectangle ABCD is the part on which the text is printed. This shaded area ABCD is 432 cm^2 and $\text{AD} = x$ cm. ABCD is 4 cm from the left and right edges of the page and 3 cm from the top and bottom of the page.



9.1 Show that the total area of the page is given by:
$$A(x) = \frac{3\,456}{x} + 6x + 480$$
(3)

9.2 Determine the value of x such that the total area of the page is a minimum. (3)
[6]

QUESTION 10

10.1 A and B are independent events. $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$

Determine:

10.1.1 P(A and B) (2)

10.1.2 P(at least ONE event occurs) (2)

10.2 The probability that it will snow on the Drakensberg Mountains in June is 5%.

- When it snows on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 72%.
- If it does not snow on the mountains, the probability that the minimum temperature in Central South Africa will drop below 0 °C is 35%.

10.2.1 Represent the given information on a tree diagram. Clearly indicate the probabilities associated with EACH branch. (3)

10.2.2 Calculate the probability that the temperature in Central South Africa will NOT drop below 0 °C in June 2024. (3)

10.3 Ten learners stand randomly in a line, one behind the other.

10.3.1 In how many different ways can the ten learners stand in the line? (1)

10.3.2 Calculate the probability that there will be 5 learners between the 2 youngest learners in the line. (4)
[15]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ ; r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$