



# basic education

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Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**SENIOR CERTIFICATE EXAMINATIONS/  
SENIORSERTIFIKAAT-EKSAMEN  
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/  
NASIONALE SENIORSERTIFIKAAT-EKSAMEN**

**MATHEMATICS P2/  
WISKUNDE V2**

**MARKING GUIDELINES/NASIENRIGLYNE**

**2019**

**MARKS: 150  
PUNTE: 150**

**These marking guidelines consist of 20 pages.  
Hierdie nasienriglyne bestaan uit 20 bladsye..**

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**NOTA:**

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

<b>GEOMETRY • MEETKUNDE</b>	
<b>S</b>	<b>A mark for a correct statement (A statement mark is independent of a reason)</b>
	<i>'n Punt vir 'n korrekte bewering ( 'n Punt vir 'n bewering is onafhanklik van die rede)</i>
<b>R</b>	<b>A mark for the correct reason (A reason mark may only be awarded if the statement is correct)</b>
	<i>'n Punt vir 'n korrekte rede ( 'n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
<b>S/R</b>	<b>Award a mark if statement AND reason are both correct</b>
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

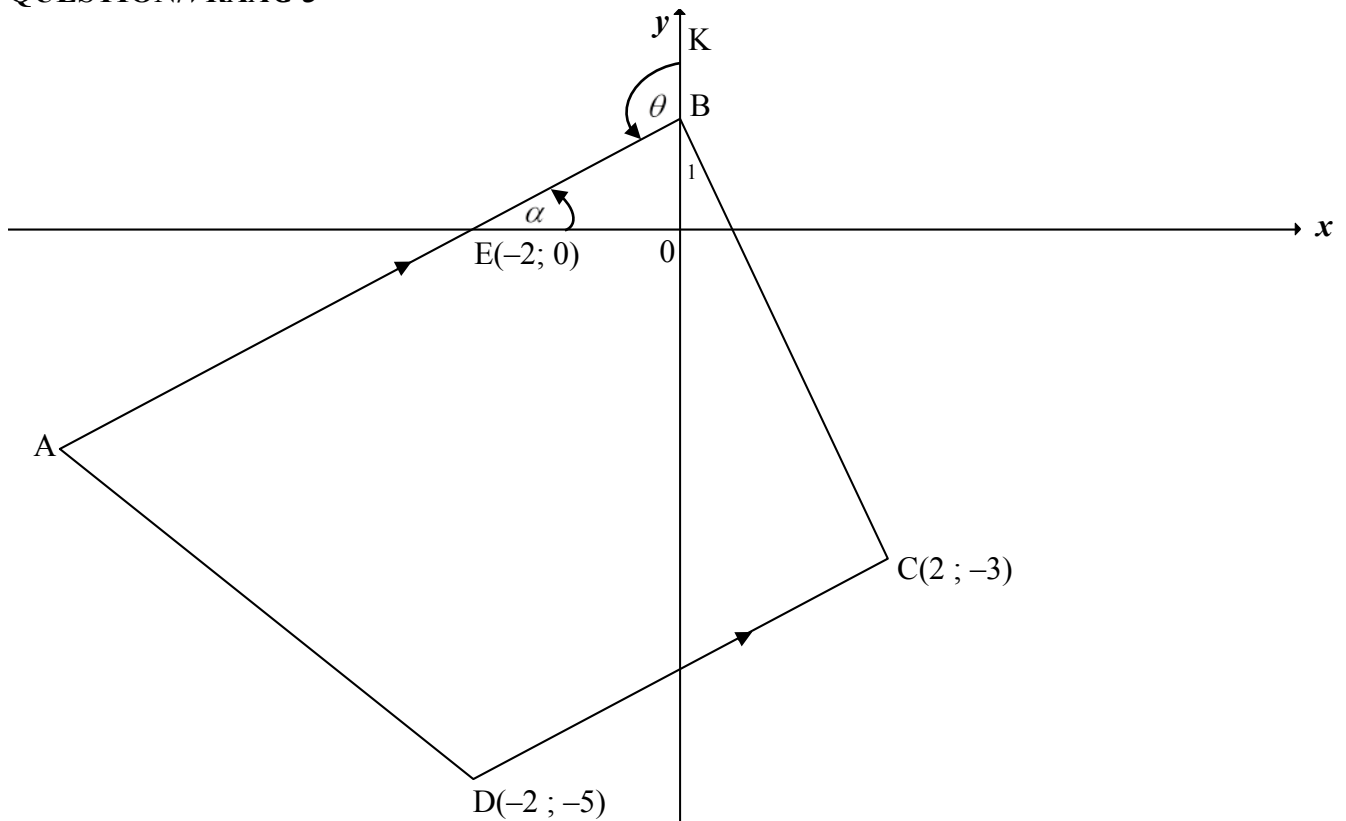
**QUESTION/VRAAG 1**

1.1	45 children	✓ answer (1)																								
1.2	$\bar{x} = \frac{\sum fx}{n} = \frac{(4 \times 2) + (8 \times 10) + (12 \times 9) + (16 \times 7) + (20 \times 8) + (24 \times 7) + (28 \times 2)}{45}$ $\bar{x} = \frac{692}{45} \text{ OR } \bar{x} = 15,38 \text{ minutes}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: full marks</div>	✓ 692 ✓ answer (2)																								
1.3	<table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th>Time taken (<i>t</i>) (in minutes)</th> <th>Number of children</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td><math>2 &lt; t \leq 6</math></td> <td>2</td> <td>2</td> </tr> <tr> <td><math>6 &lt; t \leq 10</math></td> <td>10</td> <td>12</td> </tr> <tr> <td><math>10 &lt; t \leq 14</math></td> <td>9</td> <td>21</td> </tr> <tr> <td><math>14 &lt; t \leq 18</math></td> <td>7</td> <td>28</td> </tr> <tr> <td><math>18 &lt; t \leq 22</math></td> <td>8</td> <td>36</td> </tr> <tr> <td><math>22 &lt; t \leq 26</math></td> <td>7</td> <td>43</td> </tr> <tr> <td><math>26 &lt; t \leq 30</math></td> <td>2</td> <td>45</td> </tr> </tbody> </table>	Time taken ( <i>t</i> ) (in minutes)	Number of children	Cumulative frequency	$2 < t \leq 6$	2	2	$6 < t \leq 10$	10	12	$10 < t \leq 14$	9	21	$14 < t \leq 18$	7	28	$18 < t \leq 22$	8	36	$22 < t \leq 26$	7	43	$26 < t \leq 30$	2	45	✓ first 4 cum freq correct ✓ last 3 cum freq correct (2)
Time taken ( <i>t</i> ) (in minutes)	Number of children	Cumulative frequency																								
$2 < t \leq 6$	2	2																								
$6 < t \leq 10$	10	12																								
$10 < t \leq 14$	9	21																								
$14 < t \leq 18$	7	28																								
$18 < t \leq 22$	8	36																								
$22 < t \leq 26$	7	43																								
$26 < t \leq 30$	2	45																								
1.4	<p style="text-align: center;"><b>CUMULATIVE FREQUENCY GRAPH (OGIVE)</b></p>	✓ plotting cum freq at upper limits correctly (all points) ✓ shape (smooth) ✓ grounding (2;0) (3)																								
1.5	On graph at the y-value of 22,5 or 23 Median = ± 15 minutes. <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: full marks</div>	✓ graph ✓ answer (2)																								
		<b>[10]</b>																								

**QUESTION/VRAAG 2**

2.1	$a = 12,44$ $b = 0,98$ $y = 12,44 + 0,98x$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ value of $a$ ✓ value of $b$ ✓ equation (3)
2.2.1	Percentage = $\frac{15}{50} \times 100$ $= 30\%$	✓ answer (1)
2.2.2	$\hat{y} = 12,44 + 0,98x$ $\hat{y} = 12,44 + 0,98(30)$ $\hat{y} = 41,84$ $= 42$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: full marks</div> <b>OR</b> $\hat{y} = 41,87$ (if using calculator) $\hat{y} = 42$  <b>OR</b> $\hat{y} = \frac{21}{50}$	✓ substitution of 30  ✓ answer as integer (2)  ✓ value of $y$ ✓ answer as integer (2)  ✓ ✓ answer (2)
2.3.1	standard deviation = 13,88	✓ ✓ answer (2)
2.3.2	$x = 50,67 - 45,67$ $= 5\%$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ $50,67 - 45,67$ ✓ answer (2)
		<b>[10]</b>

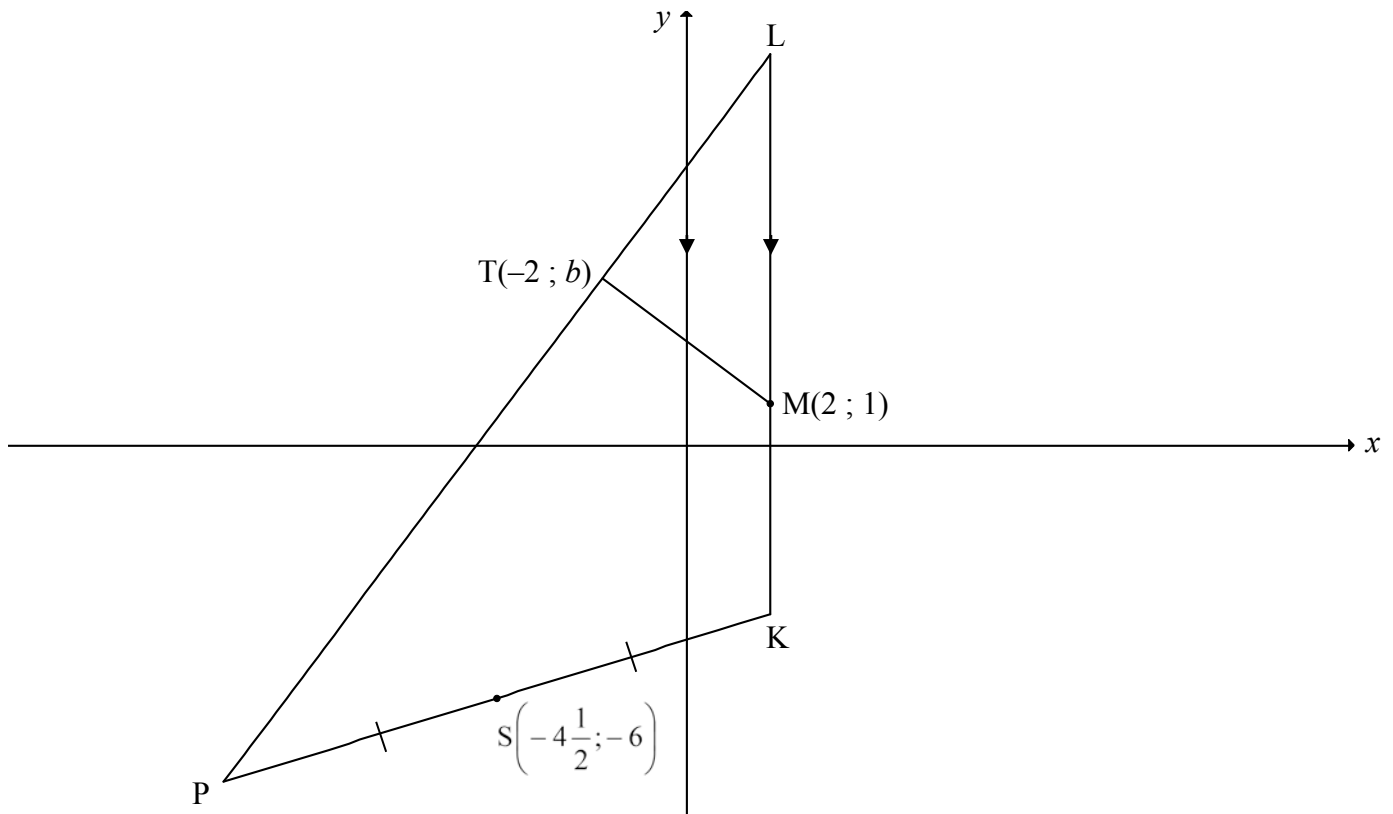
**QUESTION/VRAAG 3**



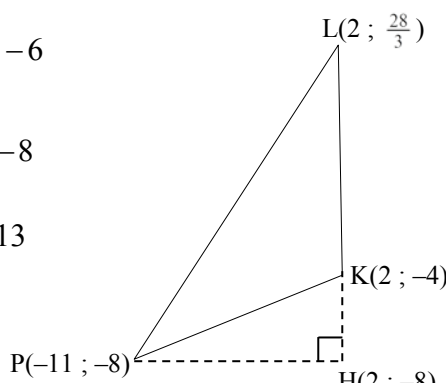
3.1.1	Midpoint of EC: $= \left( \frac{-2+2}{2} ; \frac{0+(-3)}{2} \right) = \left( 0 ; \frac{-3}{2} \right)$	✓ x value ✓ y value (2)
3.1.2	$m_{DC} = \frac{-3 - (-5)}{2 - (-2)} \text{ OR } \frac{-5 - (-3)}{-2 - 2}$ $= \frac{2}{4} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ substitution ✓ answer (2)
3.1.3	$m_{AB} = \frac{1}{2} \quad [AB \parallel DC]$ $y = \frac{1}{2}x + c$ $0 = \frac{1}{2}(-2) + c \quad \text{OR} \quad y - 0 = \frac{1}{2}(x - (-2))$ $c = 1$ $\therefore y = \frac{1}{2}x + 1$	✓ $m_{AB} = \frac{1}{2}$ ✓ substitution of (-2;0) ✓ equation (3)
3.1.4	$\tan \alpha = m_{AB} = \frac{1}{2}$ $\alpha = 26,57^\circ$ $\theta = 90^\circ + 26,57^\circ \quad [\text{ext } \angle \text{ of } \Delta]$ $= 116,57^\circ$	✓ $\tan \alpha = \frac{1}{2}$ ✓ value of $\alpha$ ✓ value of $\theta$ (3)

<p>3.2</p>	<p>B(0 ; 1)  <math>m_{BC} = \frac{1 - (-3)}{0 - 2}</math> <b>OR</b> <math>m_{BC} = \frac{(-3) - 1}{2 - 0}</math>  <math>= -2</math> <math>= -2</math>  <math>m_{AB} \times m_{BC} = \frac{1}{2} \times -2</math>  <math>= -1</math>  <math>\therefore AB \perp BC</math></p>	<p>✓ coordinates of B                  ✓ <math>m_{BC} = -2</math>                  ✓ product of gradients = -1                  (3)</p>
<p>3.3.1</p>	<p><math>\hat{A}BC = 90^\circ</math>  <math>\therefore EC</math> is diameter [converse: <math>\angle</math> in semi circle]  <math>\therefore</math> centre of circle = <math>\left(0 ; -\frac{3}{2}\right)</math></p>	<p>✓ answer                  (1)</p>
<p>3.3.2</p>	<p><math>(x - 0)^2 + \left(y + \frac{3}{2}\right)^2 = r^2</math>  <math>(-2 - 0)^2 + \left(0 + \frac{3}{2}\right)^2 = r^2</math> <b>OR</b> <math>(2 - 0)^2 + \left(-3 - \left(\frac{-3}{2}\right)\right)^2 = r^2</math>  <b>OR</b> <math>(0 - 0)^2 + \left(1 - \left(\frac{-3}{2}\right)\right)^2 = r^2</math>  <b>OR</b> <math>r = \frac{EC}{2} = \frac{\sqrt{(-2 - 2)^2 + (0 - (-3))^2}}{2}</math>  <b>OR</b> <math>r = 1 - \left(-\frac{3}{2}\right)</math>  <math>\therefore r^2 = \frac{25}{4}</math> or <math>r = \frac{5}{2}</math>  <math>x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}</math></p>	<p>✓ substitution of centre                  ✓ correct substitution of E(-1 ; 0), B(0 ; 1) or C(2 ; -3) to calculate <math>r^2</math> or <math>r</math>                  ✓ value of <math>r^2</math> or <math>r</math>                  ✓ equation                  (4)</p>
		<p>[18]</p>

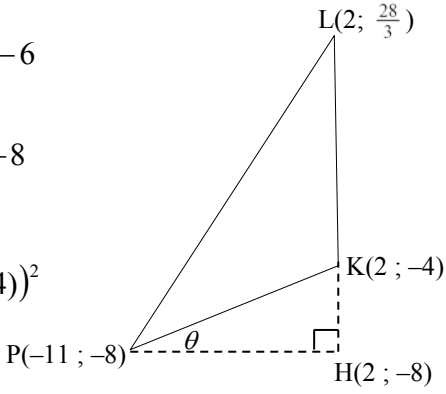
**QUESTION/VRAAG 4**



4.1	$(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $(b-1)^2 = 9$ <b>OF</b> $16 + b^2 - 2b + 1 = 25$ $b-1 = \pm 3$ $b^2 - 2b - 8 = 0$ $\therefore b = 4$ or $b \neq -2$ $\therefore b = 4$ or $b \neq -2$	✓ equation of the circle ✓ substitution of point T  ✓ simplification  ✓ answer  (4)
4.2.1	K(2 ; 1 - 5) $\therefore$ K(2 ; -4) <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>	✓ x value ✓ y value  (2)
4.2.2	$m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$ [radius $\perp$ tangent] $y = \frac{4}{3}x + c$ $4 = \frac{4}{3}(-2) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	✓ $m_{MT}$ ✓ $m_{PL} = \frac{4}{3}$  ✓ substitution of $m_{PL}$ and the point T  ✓ equation  (4)

	<p>OR</p> $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3} \quad [\text{radius} \perp \text{tangent}]$ $y - y_1 = \frac{4}{3}(x - x_1)$ $y - 4 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{20}{3}$ <p>OR</p> <p>P(-11 ; -8)</p> $m_{PL} = \frac{4 - (-8)}{-2 - (-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}(-11) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	<p>✓ <math>m_{MT}</math></p> <p>✓ <math>m_{PL} = \frac{4}{3}</math></p> <p>✓ substitution of <math>m_{PL}</math> and the point T</p> <p>✓ equation (4)</p> <p>✓ coordinates of P</p> <p>✓ <math>m_{PL} = \frac{4}{3}</math></p> <p>✓ substitution of <math>m_{PL}</math> and the point P or T</p> <p>✓ equation (4)</p>
<p>4.2.3</p>	$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ <p>L(2 ; <math>\frac{28}{3}</math>) and K(2 ; -4): <math>LK = \frac{28}{3} - (-4) = \frac{40}{3}</math></p> <p><u>Coordinates of P:</u></p> $\frac{x+2}{2} = -4 \frac{1}{2} \quad \text{and} \quad \frac{y-4}{2} = -6$ <p>∴ <math>x = -11</math>                      <math>y = -8</math></p> <p>∴ P(-11; -8)</p> <p>⊥ height (PH) = 2 - (-11) = 13</p> $\text{Area } \Delta PKL = \frac{1}{2} (LK)(PH)$ $= \frac{1}{2} \left( \frac{40}{3} \right) (13)$ $= \frac{260}{3} \quad \text{OR} \quad 86,67 \text{ square units}$ 	<p>✓ <math>y_L = \frac{28}{3}</math></p> <p>✓ length of LK</p> <p>✓ <math>x_p</math> ✓ <math>y_p</math></p> <p>✓ length of ⊥ height</p> <p>✓ substitution into the area formula</p> <p>✓ answer (7)</p>



<p>4.2.3</p>	<p><b>OR</b></p> $y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ $L\left(2; \frac{28}{3}\right) \text{ and } K(2; -4): \quad LK = \frac{28}{3} - (-4) = \frac{40}{3}$ <p><u>Coordinates of P:</u></p> $\frac{x+2}{2} = -4\frac{1}{2} \quad \text{and} \quad \frac{y-4}{2} = -6$ $\therefore x = -11 \qquad y = -8$ $\therefore P(-11; -8)$ $PK^2 = (-11-2)^2 + (-8-(-4))^2$ $PK = \sqrt{185} \text{ units}$ $m_{PK} = \frac{-8-(-4)}{-11-2} = \frac{4}{13}$ $\tan \theta = \frac{4}{13} \quad \therefore \theta = 17,1027\dots^\circ$ $\therefore \hat{PKL} = 90^\circ + 17,1027\dots^\circ = 107,1^\circ$ $\text{Area } \Delta PKL = \frac{1}{2}(PK)(LK) \cdot \sin \hat{PKL}$ $= \frac{1}{2}(\sqrt{185})\left(\frac{40}{3}\right) \sin 107,10^\circ$ $= 86,67 \text{ square units}$ 	<p>✓ <math>y_L = \frac{28}{3}</math></p> <p>✓ length of LK</p> <p>✓ <math>x_p</math> ✓ <math>y_p</math></p> <p>✓ <math>\hat{PKL}</math></p> <p>✓ substitution into the area rule</p> <p>✓ answer</p> <p style="text-align: right;">(7)</p>
<p>4.3</p>	<p>The centres of the two circles lie on the same vertical line  <math>x = 2</math>. and the sum of the radii = 10</p> $n-1 = 10 \qquad \text{or} \qquad 1-n = 10$ $n=11 \qquad \qquad \qquad n = -9$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: full marks</p> </div>	<p>✓ correct method</p> <p>✓ sum of radii = 10</p> <p>✓ <math>n=11</math> ✓ <math>n = -9</math></p> <p style="text-align: right;">(4)</p>
		<p><b>[21]</b></p>

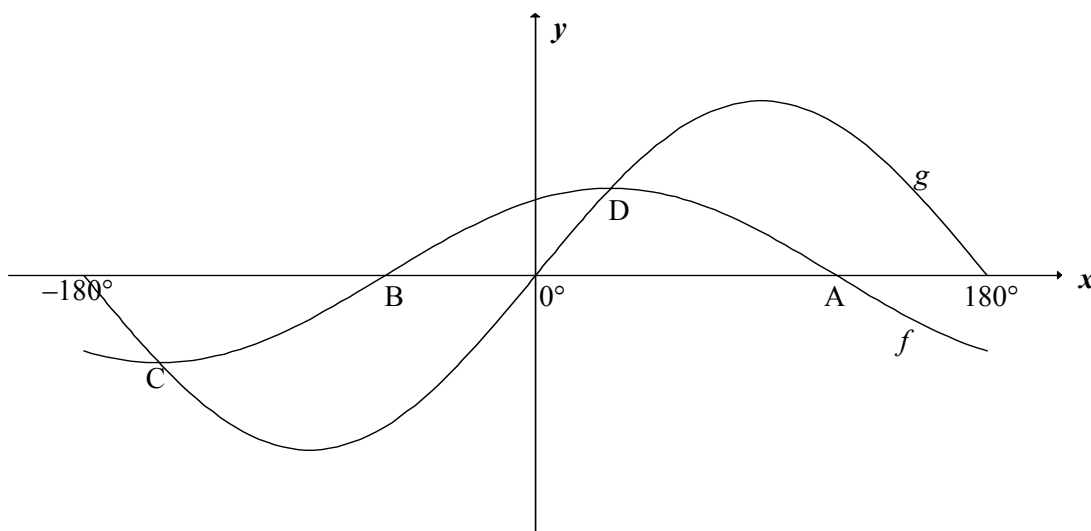
**QUESTION/VRAAG 5**

5.1.1	$\sin 191^\circ$ $= -\sin 11^\circ$	$\checkmark -\sin 11^\circ$ <p style="text-align: right;">(1)</p>
5.1.2	$\cos 22^\circ$ $= \cos(2 \times 11^\circ)$ $= 1 - 2\sin^2 11^\circ$	$\checkmark \text{ answer}$ <p style="text-align: right;">(1)</p>
5.2	$\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{1}{\sqrt{2}}\right) + \cos x \left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ <p><b>OR</b></p> $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{\sqrt{2}}{2}\right) + \cos x \left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	$\checkmark -\cos x \quad \checkmark \text{ expansion}$ $\checkmark \text{ special angle ratios}$ $\checkmark \text{ simplification of last 2 terms}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(5)</p> $\checkmark -\cos x \quad \checkmark \text{ expansion}$ $\checkmark \text{ special angle ratios}$ $\checkmark \text{ simplification of last 2 terms}$ $\checkmark \text{ answer}$ <p style="text-align: right;">(5)</p>
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2 \sin P \cos P + \cos^2 P = \frac{49}{25}$ $2 \sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	$\checkmark \sin Q = \cos P$ $\checkmark \text{ squaring}$ $\checkmark \text{ expansion}$ $\checkmark \sin^2 P + \cos^2 P = 1$ $\checkmark \text{ answer}$ <p style="text-align: right;">(5)</p>
		<b>[12]</b>

**QUESTION/VRAAG 6**

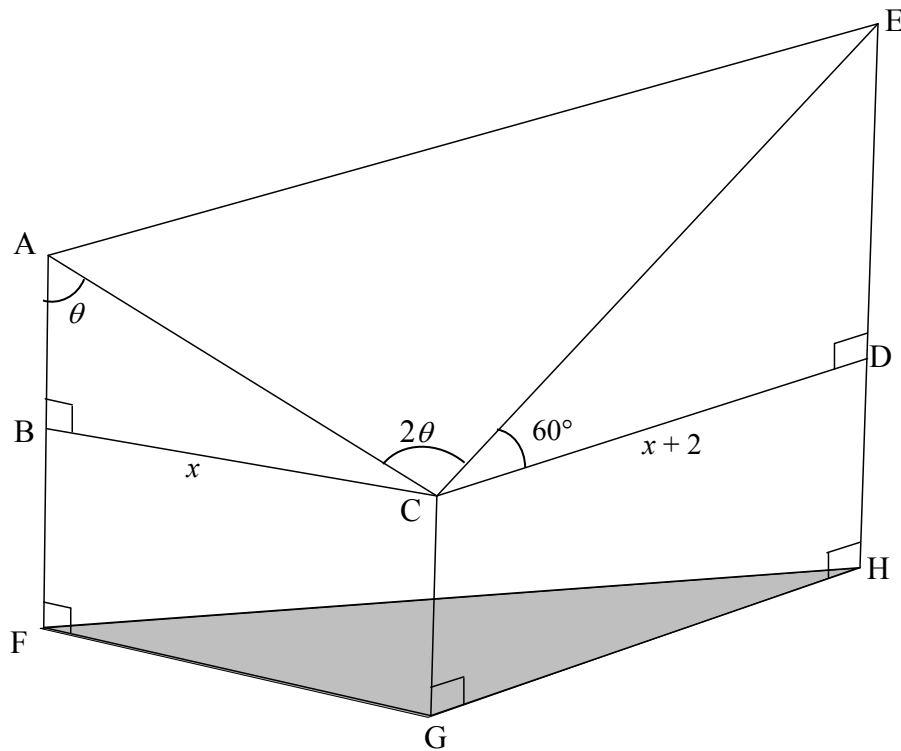
6.1	$\cos(x - 30^\circ) = 2 \sin x$ $\cos x \cos 30^\circ + \sin x \sin 30^\circ = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x = \frac{3}{2} \sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ + k \cdot 180^\circ; \quad k \in Z$ <p><b>OR</b></p> $x = 30^\circ + k \cdot 360^\circ \text{ or } x = 210^\circ + k \cdot 360^\circ; \quad k \in Z$	✓ expansion ✓ special $\angle$ s ✓ simplification ✓ equation in tan ✓ $30^\circ$ ✓ $k \cdot 180^\circ; k \in Z$ <b>OR</b> ✓ $30^\circ$ and $210^\circ$ ✓ $k \cdot 360^\circ; k \in Z$
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(6)



6.2.1(a)	A(120° ; 0)	✓ answer (1)
6.2.1(b)	C(-150° ; -1)	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90^\circ ; 30^\circ)$ OR $-90^\circ < x < 30^\circ$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160^\circ ; 20^\circ)$ OR $-160^\circ < x < 20^\circ$	✓ endpoints ✓ correct interval (2)
6.2.3	$y = 2^{2 \sin x + 3}$ Range of $y = 2 \sin x$ : $y \in [-2 ; 2]$ <b>OR</b> $-2 \leq y \leq 2$ Range of $y = 2 \sin x + 3$ : $y \in [1 ; 5]$ <b>OR</b> $1 \leq y \leq 5$ Range: $y = 2^{2 \sin x + 3}$ : $y \in [2 ; 32]$ <b>OR</b> $2 \leq y \leq 32$	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
<div style="border: 1px solid black; padding: 5px; display: inline-block;">Answer only: full marks</div>		
		<b>[18]</b>

**QUESTION/VRAAG 7**

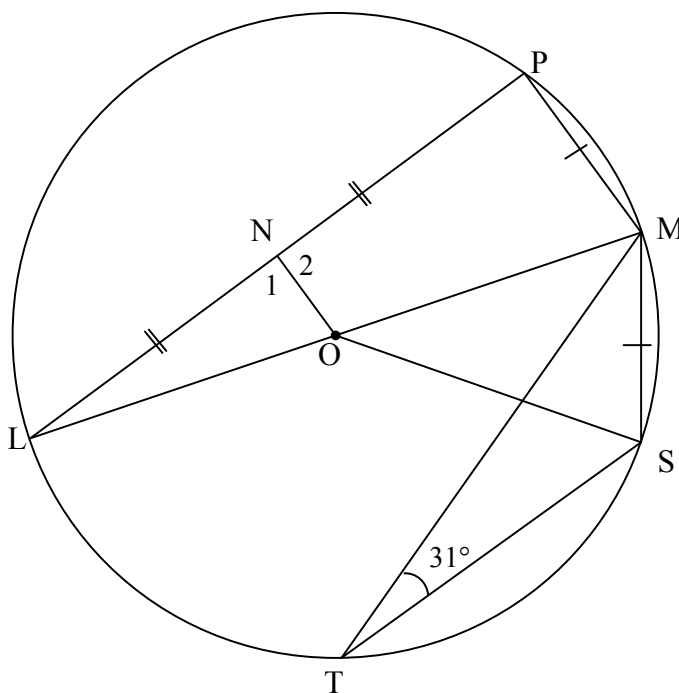


7.1.1	$\sin \theta = \frac{x}{AC} \quad \text{OR} \quad \frac{\sin \theta}{x} = \frac{\sin 90^\circ}{AC}$ $AC = \frac{x}{\sin \theta} \quad \text{OR} \quad AC = \frac{x}{\sin \theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60^\circ = \frac{x+2}{CE} \quad \text{OR} \quad \frac{\sin 30^\circ}{x+2} = \frac{\sin 90^\circ}{CE}$ $CE = \frac{x+2}{\cos 60^\circ} \quad \text{OR} \quad CE = \frac{x+2}{\sin 30^\circ}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2) \quad \text{OR} \quad = 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \triangle ACE = \frac{1}{2} AC \cdot EC \cdot \sin \hat{ACE}$ $= \frac{1}{2} \left( \frac{x}{\sin \theta} \right) (2(x+2)) \sin 2\theta$ $= \frac{x(x+2) \times 2 \sin \theta \cos \theta}{\sin \theta}$ $= 2x(x+2) \cos \theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin \theta} (2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

7.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos\hat{A}CE$ $= \left(\frac{12}{\sin 55^\circ}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55^\circ}\right)(28)\cos 110^\circ$ $AE = 35,77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		<b>[11]</b>

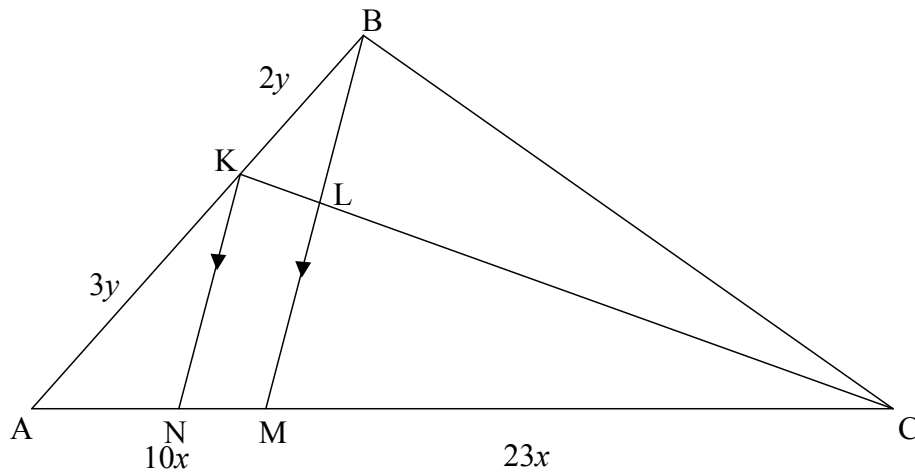
**QUESTION/VRAAG 8**

8.1



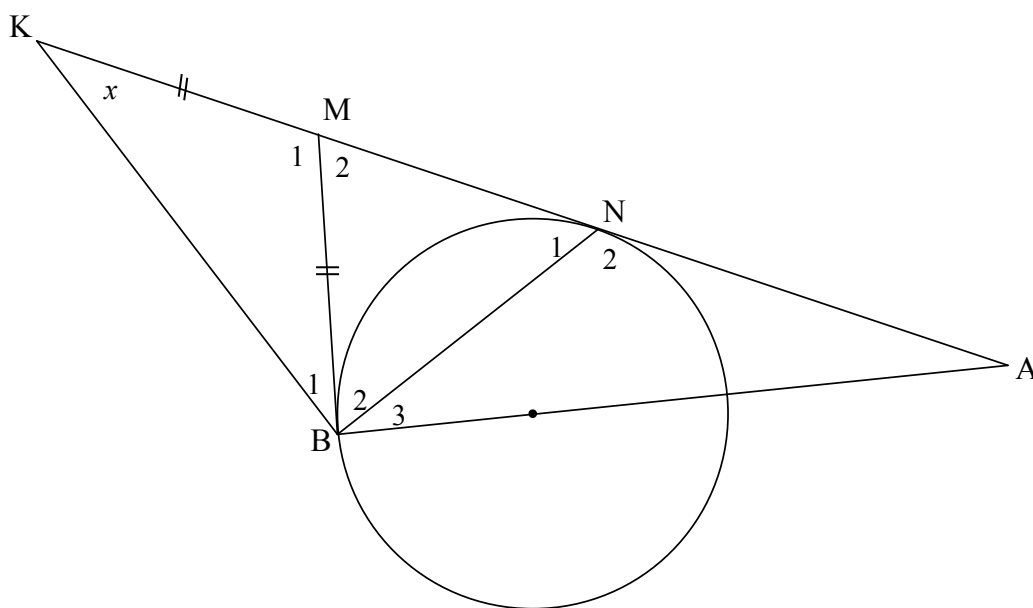
8.1.1(a)	$\hat{MÔS} = 62^\circ$ [ $\angle$ at centre = $2 \times \angle$ at circumf/ <i>middelpts</i> $\angle = 2$ omtreks $\angle$ ]	✓ S ✓ R (2)
8.1.1(b)	$\hat{L} = 31^\circ$ [equal chords; equal $\angle$ s / = <i>koorde</i> ; = $\angle$ e]	✓ S ✓ R (2)
8.1.2	<p>LN = NP and LO = OM</p> <p><math>\therefore ON = \frac{1}{2} PM</math> [midpoint theorem/<i>middelpuntstelling</i>]</p> <p><math>\therefore ON = \frac{1}{2} MS</math> [PM = MS]</p> <p><b>OR</b></p> <p><math>\hat{N}_1 = 90^\circ</math> [line from centre to midpt chord/<i>lyn v midpt na midpt kd</i>]</p> <p><math>\hat{P} = 90^\circ</math> [<math>\angle</math> in semi-circle/<math>\angle</math> in <i>halfsirkel</i>]</p> <p><math>\hat{L}</math> is common/<i>gemeen</i></p> <p><math>\therefore \triangle NLO \cong \triangle PLM</math> (<math>\angle \angle \angle</math>)</p> <p><math>\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}</math></p> <p><math>\therefore ON = \frac{1}{2} PM</math></p> <p><math>\therefore ON = \frac{1}{2} MS</math> [PM = MS]</p>	<p>✓ LO = OM</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>(4)</p> <p>✓ S R</p> <p>✓ S</p> <p>✓ S</p> <p>(4)</p>

8.2



<p>8.2.1</p>	$\frac{AN}{AM} = \frac{AK}{AB}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel BM</math>/ lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $\frac{AN}{AM} = \frac{3y}{5y} = \frac{3}{5}$	<p>✓ R</p> <p>✓ S</p> <p>(2)</p>
<p>8.2.2</p>	$\frac{AM}{MC} = \frac{10x}{23x}$ <p>[given]</p> $AM = 5y = 10x \quad \therefore y = 2x$ $\frac{LC}{KL} = \frac{MC}{NM}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel LM</math>/ lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$ <p><b>OR</b></p> $\frac{AM}{MC} = \frac{10x}{23x}$ <p>[given]</p> $\frac{AN}{MN} = \frac{3y}{2y} = \frac{6x}{4x}$ $\frac{LC}{KL} = \frac{MC}{NM}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel LM</math>/ lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$	<p>✓ S</p> <p>✓ R</p> <p>✓ S</p> <p>(3)</p> <p>✓ S</p> <p>✓ R</p> <p>✓ S</p> <p>(3)</p>
		<p>[13]</p>

**QUESTION/VRAAG 9**



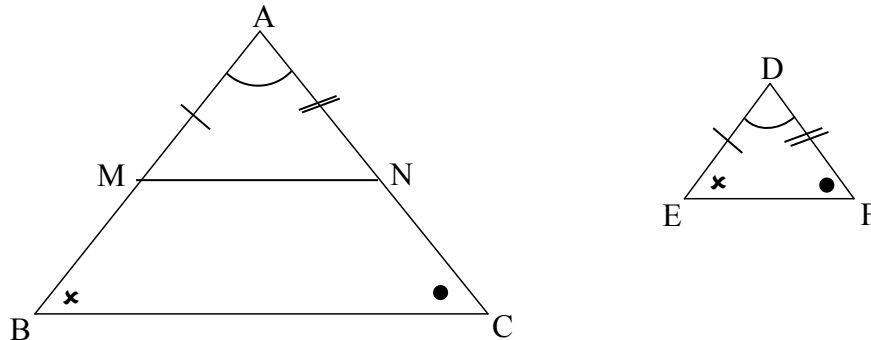
<p>9.1</p>	<p><math>\hat{B}_1 = x</math> [ <math>\angle</math>'s opp = sides/ <math>\angle</math>e teenoor = sye ]  <math>\hat{M}_2 = 2x</math> [ ext <math>\angle</math> of <math>\Delta</math> ] <b>OR</b> <math>\hat{M}_1 = 180^\circ - 2x</math> [ <math>\angle</math>s of <math>\Delta</math> ]  <math>BM = MN</math> [ 2 tans from a common point/raaklyne vanuit dieselfde punt ]  <math>\hat{N}_1 = \frac{180^\circ - 2x}{2} = 90^\circ - x</math> [ <math>\angle</math>'s opp = sides/ <math>\angle</math>e teenoor = sye ]  <b>OR</b>  <math>NM = BM</math> [ 2 tans from a common point/raaklyne vanuit dieselfde punt ]  <math>\hat{B}_2 = \hat{N}_1</math> [ <math>\angle</math>'s opp = sides/ <math>\angle</math>e teenoor = sye ]  <math>\hat{B}_1 = x</math> [ <math>\angle</math>'s opp = sides/ <math>\angle</math>e teenoor = sye ]                  In <math>\Delta KBN</math>:  <math>x + x + \hat{B}_2 + \hat{N}_1 = 180^\circ</math> [ sum of <math>\angle</math>'s of <math>\Delta</math> ]  <math>2x + 2\hat{N}_1 = 180^\circ</math>  <math>x + \hat{N}_1 = 90^\circ</math>  <math>\hat{N}_1 = 90^\circ - x</math></p>	<p>✓S                  ✓S ✓R                  ✓S ✓R                  ✓ answer                  (6)                  ✓S ✓R                  ✓S ✓R                  ✓S                  ✓ answer                  (6)</p>
<p>9.2</p>	<p><math>M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90^\circ</math> [ tangent <math>\perp</math> diameter/raaklyn <math>\perp</math> middellyn ]  <math>\hat{B}_3 = 90^\circ - \hat{B}_2</math>  <math>= 90^\circ - (90^\circ - x) = x</math>  <math>\hat{B}_3 = \hat{K} = x</math>  <math>\therefore AB</math> is a tangent/raaklyn converse tan-chord theorem/ omgekeerde raakl koordst ]]</p>	<p>✓S ✓R                  ✓S                  ✓S                  ✓R                  (5)</p>



	<p><b>OR</b></p> <p><math>\hat{B}_2 = \hat{N}_1</math></p> <p><math>\hat{B}_1 + \hat{B}_2 = x + (90^\circ - x) = 90^\circ</math></p> <p><math>\therefore</math> KN is diameter/<i>middel lyn</i> [converse <math>\angle</math> in semi-circle/ <i>omgekeerde <math>\angle</math> in halfsirkel</i>]</p> <p><math>\hat{MBA} = \hat{B}_2 + \hat{B}_3 = 90^\circ</math> [tangent <math>\perp</math> diameter]</p> <p><math>\therefore</math> AB is a tangent/<i>raaklyn</i> [converse tan-chord theorem/ <i>omgekeerde raakl koordst</i>]]</p>	<p>✓ S</p> <p>✓ R</p> <p>✓ S ✓ R</p> <p>✓ R</p> <p>(5)</p>
		<p><b>[11]</b></p>

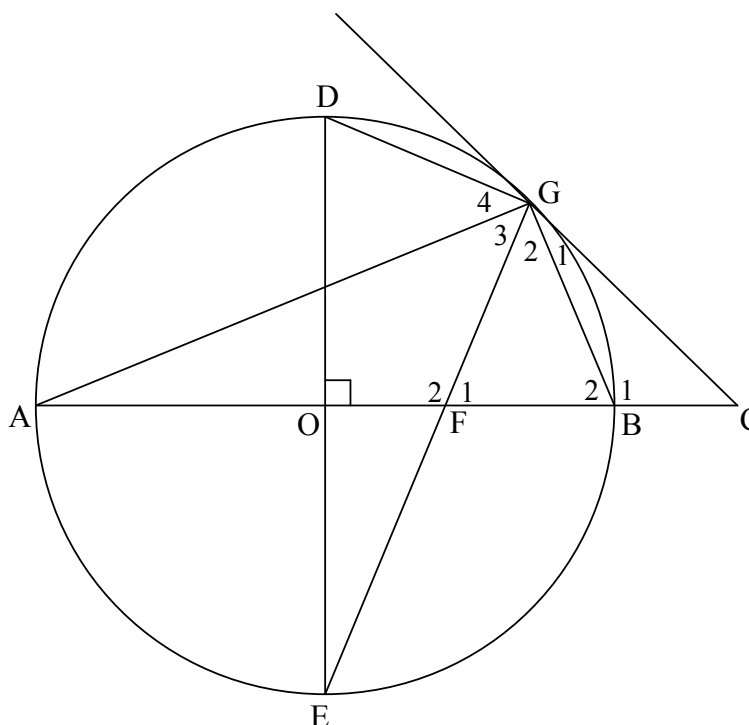
**QUESTION/VRAAG 10**

10.1



<p>10.1</p>	<p>Constr: Let M and N lie on AB and AC respectively such that <math>AM = DE</math> and <math>AN = DF</math>. Draw MN.  <i>Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat <math>AM = DE</math> en <math>AN = DF</math>. Verbind MN.</i></p> <p>Proof:                  In <math>\triangle AMN</math> and <math>\triangle DEF</math>  <math>AM = DE</math> [Constr]  <math>AN = DF</math> [Constr]  <math>\hat{A} = \hat{D}</math> [Given]  <math>\therefore \triangle AMN \cong \triangle DEF</math> (SAS)  <math>\therefore \hat{AMN} = \hat{E} = \hat{B}</math>  <math>MN \parallel BC</math> [corresp <math>\angle</math>'s are equal/ooreenkomstige <math>\angle e =</math>]  <math>\frac{AB}{AM} = \frac{AC}{AN}</math> [line <math>\parallel</math> one side of <math>\triangle</math> OR prop theorem; <math>MN \parallel BC</math>]  <math>\therefore \frac{AB}{DE} = \frac{AC}{DF}</math> [AM=DE and AN=DF]</p>	<p>✓ Constr / <i>Konstr</i></p> <p>✓ <math>\triangle AMN \cong \triangle DEF</math></p> <p>✓ SAS</p> <p>✓ <math>MN \parallel BC</math> and R</p> <p>✓ <math>\frac{AB}{AM} = \frac{AC}{AN}</math> ✓R</p> <p>(6)</p>
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10.2



<p>10.2.1(a)</p>	<p><math>\hat{D}\hat{O}\hat{B}=90^\circ</math>  <math>\hat{D}\hat{G}\hat{F}=\hat{G}_3+\hat{G}_4=90^\circ</math> [∠ in semi-circle/∠ in halfsirkel]  <math>\hat{D}\hat{O}\hat{B}+\hat{D}\hat{G}\hat{F}=180^\circ</math>  <math>\therefore</math> DGFO is a cyclic quad. [converse: opp ∠s of cyclic quad/  <i>omgekeerde teenoorst ∠e v koordevh</i>]                  OR  <math>\hat{\angle}</math>s of quad = <math>180^\circ</math>/∠e van koordevh = <math>180^\circ</math> ]  <b>OR</b>  <math>\hat{E}\hat{O}\hat{B}=90^\circ</math>  <math>\hat{D}\hat{G}\hat{F}=\hat{G}_3+\hat{G}_4=90^\circ</math> [∠ in semi-circle/∠ in halfsirkel]  <math>\hat{E}\hat{O}\hat{B} = \hat{D}\hat{G}\hat{F}</math>  <math>\therefore</math> DGFO is a cyclic quad. [converse: ext ∠ = opp int ∠/  <i>omgekeerde buite∠ = teenoorst ∠</i>]                  OR                  ext∠ of quad = opp int ∠/ <i>buite∠ v vh = teenoorst ∠</i> ]</p>	<p>✓ S ✓ R                  ✓ R                  (3)                  ✓ S ✓ R                  ✓ R                  (3)</p>
<p>10.2.1(b)</p>	<p><math>\hat{F}_1 = \hat{D}</math> [ext ∠ of cyclic quad/buite∠ v koordevh]  <math>\hat{G}_1 + \hat{G}_2 = \hat{D}</math> [tan-chord theorem/raakl koordst]  <math>\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2</math>  <math>\therefore GC = CF</math> [ sides opp equal ∠s/sye teenoor = ∠e ]</p>	<p>✓ S ✓ R                  ✓ S ✓ R                  ✓ R                  (5)</p>

10.2.2(a)	$AB = DE = 14$ [diameters/ <i>middel</i> lyne] $\therefore OB = 7$ units $\therefore BC = OC - OB = 11 - 7$ $= 4$ units <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">Answer only: full marks</div>	✓ S ✓ S ✓ S (3)
10.2.2(b)	In $\triangle CGB$ and $\triangle CAG$ $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/ <i>raakl koordst</i> ] $\hat{C} = \hat{C}$ [common] $\triangle CGB \parallel \triangle CAG$ [ $\angle, \angle, \angle$ ] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ or 8,49 units	✓ S/R ✓ S ✓ S ✓ CA = 18 ✓ answer (5)
10.2.2(c)	$OF = OC - FC$ $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11 - \sqrt{72}}{7} = 0,36$ $\hat{E} = 19,76^\circ$  <b>OR</b> $OF = OC - FC$ $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11 - \sqrt{72})^2$ $FE = 7,437.. = 7,44$ $\cos E = \frac{OE}{FE}$ OR $\sin E = \frac{OF}{FE}$ $= \frac{7}{7,44} = 0,94$ OR $= \frac{11 - \sqrt{72}}{7,44} = 0,338$ $\hat{E} = 19,76^\circ$ OR $\hat{E} = 19,76^\circ$	✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)  ✓ OF ✓ trig ratio ✓ substitution ✓ answer (4)
		<b>[26]</b>

	<b>TOTAL/TOTAAL: 150</b>
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