NBT

NATIONAL BENCHMARK TEST

MATHS

MAT PREPARATION BOOKLET

PRACTICE

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National Benchmark Test (NBT) MAT Preparation booklet

Register for free online NBT practice: learn.olico.org.

This booklet can assist you in preparing for the NBT MAT.
It is recommended that you also prepare for the NBT AQL.
This is not an official publication of the NBT project so it is very important you get up to date information from http://nbt.ac.za

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Introduction

The National Benchmark Tests (NBTs) are a set of tests that measure an applicant's academic readiness for university. They complement and support, rather than replace or duplicate the National Senior Certificate.

A number of universities in South Africa use the NBTs to help interpret the National Senior Certificate (NSC) results. Universities use the NBT results in different ways:

- Some use them to help make decisions about an applicant's access to university. This means that the NBT results, in combination with the NSC results, are used to determine whether an applicant is ready for academic study.
- Some use them for placement within university. This means that the results are used to decide whether an applicant will need extra academic support after he/she has been admitted to university.
- Some use them to help develop curricula within their universities.

There are two NBT tests: the Academic and Quantitative Literacy (AQL), and the Mathematics Test (MAT).

THIS BOOKLET WILL HELP YOU PREPARE FOR THE MAT TEST ONLY.

Note that this is NOT an official publication from the NBT and the NBT test setters might choose to change the format and/or composition of the test over time. It is therefore advisable to check for up to date information on http://nbt.ac.za.

NSC and NBT

NBT MAT topics in the NSC context

In the NSC, Mathematics in the FET Phase covers various content areas. Each content area contributes towards the acquisition of specific skills. The main topics in the FET Phase are: Functions; Number patterns, sequences and series; Finance, growth and decay; Algebra; Differential Calculus; Probability; Euclidean Geometry and measurement; Analytical Geometry; Trigonometry; Statistics (Department of Basic Education CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS) FET BAND MATHEMATICS GRADES 10 – 12, p.12: www.thutong.doe.gov.za, accessed 24/04/2015).

Schools are provided with a pace setter guide (see p. 22 of the CAPS FET Band Mathematics Grades 10 – 12 document; reference above), which ensures that Grade 12 learners have sufficient time for revision before the final Grade 12 exam. There are a few topics which the pace setter guide suggests leaving until the third quarter of the Grade 12 year. Knowing that many applicants to universities need to write the NBTs as early as the end of May, the MAT tests exclude topics that are unlikely to have been taught by that time.

In some schools, especially those following curricula other than the NSC, Grade 12 learners are already exposed to more advanced mathematical topics, for example A levels, Advanced Programme, etc. However, it is assumed that studying advanced topics will not be possible unless learners already have a solid grounding in the topics that form part of the CAPS. Such learners should then be well prepared in CAPS topics.

The questions in the MAT tests are embedded in the concepts set out in the CAPS, but the tests are not constrained to testing everything covered by the CAPS. Whereas the Academic Literacy and Quantitative Literacy tests are intended as tests of generic skills in these domains, the MAT tests focus more on the specific knowledge and skills taught at school level, but are, as in the other domains as well, explicitly designed to measure the preparedness of candidates for Higher Education. The tests require writers to demonstrate sufficient understanding of concepts to enable them to apply those concepts in a variety of contexts. These higher order skills underlie success in Mathematics in Higher Education. These skills, developed deliberately in mathematical subjects such as Mathematics and Physical Science, are often implicitly expected by Higher Education institutions and are assumed in their curriculum design. It is important for teachers to focus on the intended curriculum and not be constrained by trends in the examined curriculum.

QUESTIONS IN THE MAT TESTS ARE SET IN SUCH A WAY THAT CALCULATORS ARE NOT NEEDED. CALCULATORS ARE THUS NOT PERMITTED IN THE TESTS.

Differences between the NSC Mathematics exam and the MAT tests

One difference between the MAT tests and the NSC Mathematics papers is that questions in the MAT tests do not cue the writers in any way. The practice of scaffolding questions does not take place.

For example, in an NSC paper the following might appear:

> Given a sketch, calculate the gradient of AC. Hence, determine the equation of BN (where BN is shown on the sketch to be perpendicular to AC).

In the MAT test the sketch would also be presented, but would then be followed by:

> The equation of BN is ... with four options to choose from.

Furthermore, in the MAT tests, no indication is given as to whether a question should be dealt with using geometrical or algebraic reasoning, by applying trigonometric principles, or by a combination of these. The fact that mathematics often requires learners to integrate many different skills and concepts in any given problem means that individual questions will assess across a range of mathematical competencies. For example, a question dealing with the graphical representation of a function may also assess spatial and algebraic competence. This means that writers must have a deep understanding of mathematics, and know what reasoning is appropriate in a given context; they will need these skills in Higher Education.

It may be assumed that multiple choice testing does not allow writers to obtain part marks for their reasoning in cases where they have reasoned correctly until the last step and then made a final careless mistake. This criticism is understandable, but the NBTP review process has, over a period of many years, made it possible to fine-tune the process of creating options for which this is unlikely. Firstly, if numerical reasoning is involved, the numbers are simple (sufficiently so to make calculators unnecessary); secondly the options given provide one correct answer and three others which are unlikely to have been reached by making careless mistakes. On the whole test writers must know what to do, in which case they find the correct option; or guess, in which case they choose one of the incorrect options. In some cases misconceptions are deliberately probed, so that one of the incorrect options will be a popular but incorrect answer. This practice can 'trap' students into selecting the incorrect option in a pressurised testing environment, so it is only occasionally used, since testing should create opportunities for writers to demonstrate what they know.

What can we expect from the MAT tests?

MAT test topics

The topics from which test questions can be drawn are the following.

1. PROBLEM SOLVING AND MODELLING

   1.1. Algebraic processes.

   - Pattern recognition, sequences and series, use of sigma notation
   - Operations involving relationships such as ratios and percentages
   - Modelling situations by making use of mathematical process skills (translation from language to algebra, solution of problems)
   - Operations involving surds, logarithms and exponents, including solution of exponential equations

- Financial calculations (compound interest, appreciation, future value, etc.)
- Number sense – manipulations/simple calculations involving integers, rational and irrational numbers
- Algebraic manipulation (includes expressions, equations, inequalities, simplification, factorisation, completing the square)

1.2. Functions represented by graphs and equations; 'functions' to include linear, quadratic, hyperbola, cubic, exponential and logarithmic. Other graphs such as circles are also included.

- Comprehension of function notation, substitution, domain, range
- Function representation (algebraic and graphic); properties of functions and graphs (such as intercepts, turning points, asymptotes); relationship between graphs and their equations; interpretation of graphical information
- Transformations of graphs of the functions noted above; solution of related problems; inverses of functions
- Applications of principles of differential calculus and related problems involving simple linear, non-linear functions (i.e. critical points, increasing/decreasing functions, tangents); interpretation of behaviour of function from derivative and vice versa

2. BASIC TRIGONOMETRY, INCLUDING GRAPHS OF TRIGONOMETRIC FUNCTIONS, PROBLEMS REQUIRING SOLUTIONS OF TRIGONOMETRIC EQUATIONS AND APPPLICATION OF TRIGONOMETRIC CONCEPTS

- Definitions of trigonometric ratios (sine, cosine, tangent)
- Characteristics and interpretations of trigonometric functions and their graphs (e.g. domain, range, period, amplitude), including transformations of trigonometric functions
- Solving of trigonometric equations and using identities; simplification of trigonometric expressions using identities and reduction formulae where necessary; special angles; compound and double angles
- Application of area, sine and cosine rules
- Application of trigonometric concepts in solving problems, including two- and three-dimensional problems

3. SPATIAL PERCEPTION INCLUDING ANGLES, SYMMETRIES, MEASUREMENTS, REPRESENTATIONS AND INTERPRETATION OF TWO-DIMENSIONAL AND THREE-DIMENSIONAL SHAPES

3.1. Geometric objects

- Properties of 2D figures and 3D objects (such as the circle, rectangle, trapezium, sphere, cone, pyramid)
- Scale factor
- Perimeter, area, volume (also of composite figures and objects)

3.2. Analytic geometry (linking geometric and algebraic properties in the Cartesian plane)

3.3. Circle Geometry

- Cyclic quadrilaterals
- Relationships between tangents, and chords, and angles in a circle

4. DATA HANDLING and PROBABILITY

- Measurement (and related interpretations)
- Representation (such as histograms, line graphs, pie charts, ogives, box-and-whisker plots)

and related interpretations)
- Probability

4.2. COMPETENT USE OF LOGICAL SKILLS IN MAKING DEDUCTIONS AND DETERMINING THE VALIDITY OF GIVEN ASSERTIONS

What should teachers do?

Principles for proactive teaching

Dealing with multiple choice questions

Unless multiple choice questions are already being used in the classroom, it might be helpful to give learners some guidelines regarding how to deal with tests in this format. It would be helpful if teachers go through the following points, perhaps with some examples to make the principles clear.

- Read the question very carefully without looking at any of the possible options.
- Try to work out the question before looking at any of the possible options.
- Look at the options and see whether one of these corresponds to the answer that has been obtained, in which case select that option. **But** be critical of the reasoning involved, in case the answers reflects a specific misconception, as in the following example:

  For $x > 0$, $\sqrt{9x^2 + 16x^2}$ is equal to:

  (A) $5x$ (B) $7x$ (C) $\pm 5x$ (D) $\pm 7x$

  Working out the question before looking at the answers, and being aware of the misconceptions that (a) the square root of a sum is not equal to the sum of the square roots, and (b) 'square root' is by definition positive, should help writers make the correct choice.

- **Pace yourself!** If none of the given options corresponds to the answer you have found, start the question over, and try once more. If none of the options is then found, leave the question for later and move on. All questions have one correct option - this has been checked beforehand, and writers need not worry that there may be a mistake in the question.
- Questions in which it is possible to eliminate options by substitution are deliberately avoided. So, for example, there will not be questions asking for a specific solution to an equation, because it is easy to substitute each of the given options and find the correct one by elimination. For example, if we were to ask the following: "The solution of $3x + 4 = -8$ is

  (A) -4 (B) $-\frac{4}{3}$ (C) 4 (D) $\frac{4}{3}$"

  you can easily substitute -4 and see that (A) is the correct option.

Helping learners prepare for the MAT tests

The suggestions below are an attempt to guide teachers who want their learners to develop competence and skill in mathematics. The greater their competence, the better they will score in the NBTs.

- Ensure active engagement in class where learners are encouraged to ask questions (this pre-supposes solid teacher knowledge and understanding).
- Affirm learners - very few questions are stupid questions; all questions are opportunities for deeper and broader engagement.
- Develop learners' conceptual understanding by asking them to explain their reasoning at all times.

- Make explicit the academic literacy skills needed in mathematics: it is east to assume learners understand the nuances of the language of mathematics, but this is not necessarily the case. For example, do they understand the difference between 'but and 'and', between 'twice as much as' and 'two more than'; do they understand the language related to inequalities, such as 'at least three units' or 'not more than 5' etc.?
- Make explicit the quantitative skills required in mathematics. Since ratio, percentage, numerical manipulation, etc., are not specific skills required by the Grade 12 curriculum (although they are presupposed by the fact that they have been taught in the earlier grades), learners have often forgotten (or perhaps didn't ever understand) these quantitative concepts. In the MAT tests they can't use calculators, and must now demonstrate understanding of the relevant concepts. Undue dependence on calculators also programmes learners to lose arithmetic skill, and to lose their understanding of numbers, their relative size and position on a number line.
- Wherever possible consider alternative approaches to problem solving: could a geometric problem (area, volume) be approached from a trigonometric perspective, or could a trigonometric equation be solved using a trigonometric graph?
- Wherever possible, depend on mathematical concepts rather than calculators to solve problems. It is possible for a calculator to solve an equation, but does this show that the learner has understood the concepts required in solving equations? They may for instance not realise that the equation $\frac{x^2(x+1)}{x} = 0$ has only one solution.
- **Most important: do learners understand?**

The information above was taken from: "The National Benchmark Tests: Preparing your learners for the Mathematics (MAT) test"

Dr. Carol Bohlmann; NBTP Mathematics research lead, Centre for Education testing for access and placment (CETAP), Centre for Higher Education Development (CHED), University of Cape Town, April 2015. We recommend you visit http://www.nbt.ac.za/content/preparing-your-learners-0 to read the full text for further information.

IMPORTANT: HOW TO USE THIS BOOKLET FOR GRADES 10 - 12

Given the level of conceptual work that needs to be done with learners in order to prepare them for the MAT tests, it is advisable to include NBT style questions in your grade 10 and 11 classes while working through these topics. In order to assist you to do this, this booklet has been divided into two sections.

SECTION A contains NBT style questions that have been divided into topic areas with an indication of the grade they are appropriate for.

SECTION B consists of 2 full mock NBT papers to prepare learners for the MAT tests.

The nature of the NBT tests is such that they change from year to year. As the learners work through the practice tests, focus on their approach in terms of **understanding and problem solving**. It is not possible to predict what questions will come up, so having developed a solid approach to problem solving is the ultimate goal of their preparation.

PRACTISE NBT ONLINE (WITH IMMEDIATE FEEDBACK): LEARN.OLICO.ORG

NOTE: NO CALCULATORS ARE PERMITTED IN THE NBT.

Section A

Algebra and exponents

Grades 10, 11 and 12

1. Simplify the expression:

$$\frac{1+2\ \div 3}{\frac{2}{3}-\frac{1}{4}}$$

A) 4 B) $\frac{5}{6}$ C) $\frac{12}{5}$ D) $\frac{25}{36}$

2. Find the value of r in the sequence $8; 5; 3; 2; 1; 1; 0; 1;\ -1; 2; q; r$.

A) 4 B) 3 C) 8 D) 5

3. Evaluate $2^{1001}-2^{1000}-2\times 2^{999}$

A) 0 B) 2 C) $3(2^{1001})$ D) 2^{1002}

4. Find an expression for r, if

$$\frac{s-2}{t}=\frac{1}{p}+\frac{1}{r}.$$

A) $r=\frac{tp}{ps-2p-t}$ B) $r=\frac{s-2-p}{t}$ C) $r=\frac{p}{ps-2p-1}$ D) $r=\frac{t}{s-2}-p$

5. Find $\left(\sqrt{x}\right)^6$ if $x=4\times\sqrt[3]{25}$.

A) 640 B) $2^{12}\times 25$ C) 1 600 D) $2^8\times 10^4$

6. Renata drives to work at an average speed of y km/h, and she drives the same route home at an average speed of x km/h. Find an expression for the average speed of her total trip.

A) $\frac{2(x+y)}{xy}$ B) $\frac{2xy}{x+y}$ C) $\frac{2}{x+y}$ D) $\frac{x+y}{2}$

7. Two integers have a sum of m and a product of n. Find an expression for the sum of the squares of the two integers.

A) m^2+2n B) m^2-2n C) m^2-n^2 D) $(m+n)^2$

8. Find the value of $x - y$ if $2^{x+2} + 2^x = 5^{y+1} - 5^y$.

A) 0 B) 1 C) 2 D) 3

9. For how many integer values of n is $\frac{n+3}{n-1}$ an integer?

A) 1 B) 2 C) 3 D) 6

10. Four men can lay 3 200 bricks in 6 hours. How long will three men take to lay 800 bricks?

A) 1½ hours B) 2 hours C) 3 hours D) 4½ hours

11. Marla is given money to buy supplies for a class party. She spends 20% of the money on sweets and cooldrinks and 10% on paper plates. Then she spends 40% of the money leftover at this stage on sandwiches. She then has R168 left over. How much money did she spend on paper plates?

A) R22 B) R28 C) R36 D) R40

12. Find the value of n for which $4(9^n) - 9^n = 3^{801}$.

A) 199 B) 798 C) 267 D) 400

13. If $a \otimes b = a - 2 \times b$, find the value of $7 \otimes (5 \otimes 2)$.

A) 5 B) 4 C) 2 D) –7

14. Find the sum of b and c if the equation $x^3 + bx = c$ has solutions 2 and –3.

A) –1 B) –13 C) 5 D) –6

15. Find the value of $1005^2 - 1004 \times 1006$

A) 1 B) 2 C) 3 D) 4

16. Sipho and Maria share an amount of money, in the ratio 3 : 4. After they each spend R100, the ratios of the amounts they have left over is 1 : 2. What is the amount with which they started?

A) R250 B) R300 C) R350 D) R400

17. A point C lies on a line AB so that AB : AC = AC : CB. If CB = 1, find the length of AC.

A) $\frac{1+\sqrt{5}}{2}$ B) $\frac{1+\sqrt{3}}{2}$ C) $\frac{\sqrt{5}}{3}$ D) $\frac{\sqrt{3}}{2}$

18. Solve for x if 199 + 195 + 191 + ... + 7 + 3 = x + 1 + 5 + ... 189 + 193 + 197.

A) 2 B) 25 C) 50 D) 100

19. Find the value of k, if the following three lines all pass through the same point.

$$y - x - 2 = 0 \quad ...(1)$$
$$y - 3x + 2 = 0 \quad ...(2)$$
$$3y - kx - 5 = 0 \quad ...(3)$$

A) 5,6 B) 3,5 C) –1,4 D) –3,2

20. How many prime numbers n exist so that $n + 1$ is the square of a whole number?

A) none B) one C) two D) four

Grades 11 and 12

21. Solve for x if $4x^2 < 8x$.

A) $0 < x < \frac{1}{2}$ B) $x < 0$ or $x > 2$ C) $0 < x < 2$ D) $x < 2$

22. Given the expression $\sqrt{4x^2 - 16x + 20}$, which statement is true?

A) The expression has a maximum value of 6.
B) The expression has a maximum value of 1.
C) The expression has a minimum value of 4.
D) The expression has a minimum value of 2.

23. Solve the inequality $(3 - x)^2 \left(\frac{-4}{x}\right) < 0$.

A) $x > -4$ B) $x < 0$ or $x > 3$ C) $x > 0$ D) $0 < x < 3$

24. Simplify the expression:

$$\frac{3\sqrt{12}-\sqrt{27}}{\sqrt{24}}$$

A) $\frac{3}{2\sqrt{2}}$ B) $\frac{9}{4\sqrt{2}}$ C) $\frac{3}{2\sqrt{6}}$ D) $\frac{3-3\sqrt{3}}{\sqrt{2}}$

25. Find the product of the roots of the equation $x^2 - 4x + 1 = 0$.

A) 1 B) 4 C) $4-\sqrt{3}$ D) $8+4\sqrt{3}$

26. Find a positive solution to the equation $\sqrt{x}\times\sqrt{x}\times\sqrt{x}\times\sqrt{x} = \sqrt{x}+\sqrt{x}+\sqrt{x}$.

A) $3^{3/2}$ B) $\sqrt[3]{3}$ C) $\sqrt[3]{9}$ D) $\sqrt[3]{2}$

27. Solve the inequality $\frac{x.(x^2-4)(-2-(x-3)^2)}{3^x} \leq 0$

A) $0 \leq x \leq 3$ B) $0 \leq x \leq 2$ or $x \geq 3$
C) $-2 \leq x \leq 0$ D) $-2 \leq x \leq 0$ or $x \geq 2$

Grade 12

28. Find:

$$\sum_{n=1}^{p}(2n-1)$$

A) p^2 B) p^2+1 C) p^2-1 D) $(p+1)^2$

29. When $f(x) = 2x^4 - kx^2 + bx$ is divided by $x + 1$ the remainder is –3, and when $f(x)$ is divided by $x-1$ the remainder is 5. Determine the value of k.

A) –1 B) 1 C) 3 D) 4

30. Find the values of x for which $\log_y(x^4-3) = \log_x y^0$.

A) $\pm\sqrt{2}$ B) ± 2 C) $\pm\sqrt{3}$ D) $\sqrt{2}$

31. Find the value of ab if $2^a = 3$ and $9^b = 16$.

A) 4 B) 2 C) $\frac{4}{3}$ D) $\frac{3}{4}$

32. Evaluate the sum $(52^2 - 51^2) + (50^2 - 49^2) + \cdots + (2^2 - 1^2)$.

A) $26^2 \times 53^2$ B) $(26)(53)$ C) $51^2(26)$ D) $(52)(53)$

33. Two boys are born in January of different years. In the January when the boys turn 7 and 11, their father invests R12 000 at 7% compounded monthly. When each son turns 21, they receive R8 000 from the investment. How much will be left in the account immediately after the second son receives his money?

A) $\left[\left(12\,000\left(1+\frac{0{,}07}{12}\right)^{120} - 8\,000\right)\left(1+\frac{0{,}07}{12}\right)^{48}\right] - 8\,000$

B) $\left[\left(12\,000\left(1+\frac{7}{1200}\right)^{10} - 8\,000\right)\right] + \left[12\,000\left(1+\frac{7}{1200}\right)^{4} - 8\,000\right]$

C) $\left[\left(12\,000\left(1+\frac{7}{12}\right)^{120}\right)\right] \times \left[\left(1+\frac{7}{12}\right)^{48} - 16\,000\right]$

D) $\left[12\,000\left(1+\frac{0{,}07}{12}\right)^{10}\right]\left[\left(1+\frac{0{,}07}{12}\right)^{14} - 16\,000\right]$

34. Find an expression for x if $a = b^{x^2}$.

A) $\frac{\log a - \log 2}{\log b}$ B) $\pm\sqrt{\log b - \log a}$ C) $\pm\sqrt{\frac{\log a}{\log b}}$ D) $\frac{\log a}{2\log b}$

35. Find the value of p if

$$\sum_{n=1}^{p} 2n = 420$$

A) 22 B) 21 C) 20 D) 19

36. If $\log_2 x = \log_4(x + 12)$, find the value of x.

A) 1 B) 2 C) 3 D) 4

37. Find T_9 if

$$\sum_{n=1}^{k} T_n = 3k^2 - 1.$$

A) 51 B) 57 C) 191 D) 242

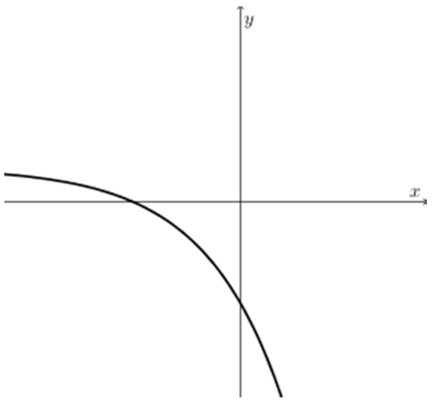
Functions and graphs

Grades 10, 11 and 12

38. One of the x-intercepts of the graph of the quadratic $y = f(x)$ is (–6 ; 0). The maximum value of the function is given by $f(p)$. Find the other x-intercept of the graph, in terms of p.

A) $(2p - 6; 0)$ B) $(2p + 6; 0)$ C) $(p + 6; 0)$ D) $(p - 6; 0)$

39. The graph of $y = a(3^x) + b$ is shown below. Choose the correct descriptions of a and b.

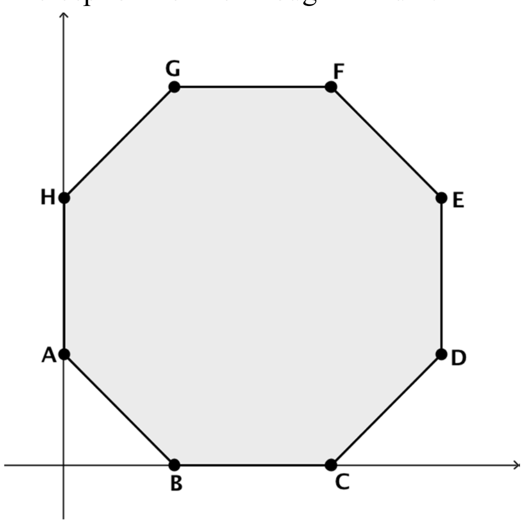


A) $0 < a < 1$ and $b < 0$
B) $a > 0$ and $-1 < b < 0$
C) $a < 0$ and $b < 0$
D) $a < 0$ and $b > 0$

40. If $f(x) = 1 - \frac{1}{x}$, find $f(f(x))$.

A) $\frac{1}{1-x}$ B) $\frac{x-1}{x^2}$ C) $\frac{1}{x-1}$ D) $\frac{2-x}{x-1}$

41. A regular octagon is formed as shown (not drawn to scale), with A(0; 4) and B(4; 0). Find the y-intercept of the line through E and F.



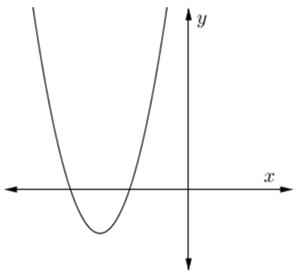
A) $12\sqrt{2}$ B) $8\sqrt{2}$ C) $12 + 8\sqrt{2}$ D) $6 + 4\sqrt{2}$

42. A function f satisfies the equation $yf(xy) = f(x)$ and $f(30) = 20$. Find $f(40)$.

A) 10 B) 60 C) 15 D) 35

Grades 11 and 12

43. The graph of $y = ax^2 + bx + c$ is shown below. Which one of the following statements is correct?



A) $a > 0, b > 0, b^2 - 4ac < 0$
B) $a < 0, b < 0, b^2 - 4ac > 0$
C) $a > 0, b > 0, b^2 - 4ac > 0$
D) $a > 0, b < 0, b^2 - 4ac > 0$

44. The graph of $3y - 2x + 4 = 0$ is reflected in the x-axis and shifted three units to the left. The equation of the resulting graph is:

A) $3y - 2x + 1 = 0$
B) $3y + 2x + 7 = 0$
C) $-3y - 2x + 1 = 0$
D) $3y + 2x + 2 = 0$

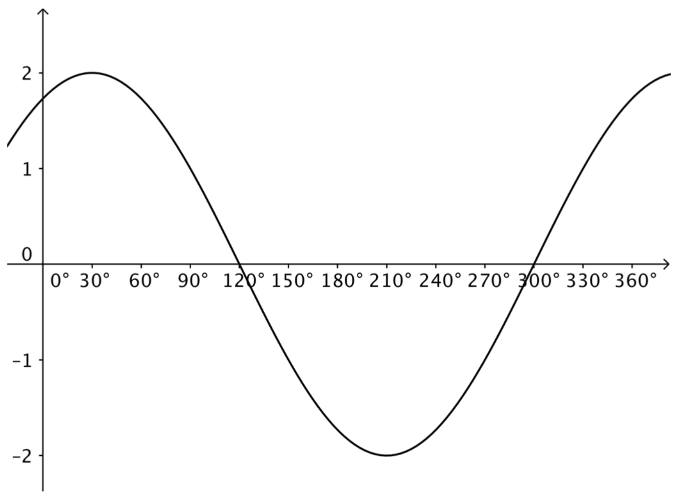
45. Find the range of $f(x) = 2\sin(3x) - 1$.

A) $y \in [-2; 2]$ B) $y \in [-3; 1]$ C) $y \in [-1; 3]$ D) $y \in [0; 4]$

46. For which value(s) of x will the function $f(x) = 4px^2 - 6px - 6x + 3$ achieve its maximum value (where $p < 0$)?

A) $\frac{-6p-1\pm\sqrt{(6p+1)^2-48}}{8}$
B) $\frac{3p+3}{4p}$
C) $\frac{-3p-3}{4}$
D) $\frac{6p+1\pm\sqrt{(6p+1)^2-48p}}{8p}$

47. A possible equation for the graph shown below is



A) $y = \cos(2x + 60°)$ B) $y = 2\cos(x - 30°)$
C) $y = -2\sin(2x + 60°)$ D) $y = \sin(x - 30°) + 2$

48. Find the range of the function $-x^2 + 6x - 11$.

A) $y \in (-\infty;\ -2]$ B) $y \in (-\infty;\ 3]$ C) $y \in [-2;\ \infty)$ D) $y \in [-1;\ \infty)$

49. A graph is translated 3 units to the left and reflected in the x-axis. If the resulting graph has the equation $y = \frac{2}{x-1}$, find the equation of the graph before these transformations.

A) $y = \frac{2}{4-x}$ B) $y = \frac{2}{-x+2}$ C) $y = \frac{-2}{x+2}$ D) $y = \frac{-2}{4-x}$

50. Find the range of $f(x) = \frac{3}{\cos 2x}$.

A) $y \in \left[-\frac{3}{2};\ \frac{3}{2}\right]$ B) $y \in (-\infty; 6]$ or $[6;\ \infty)$
C) $y \in \left(-\infty; -\frac{1}{3}\right]$ or $\left[\frac{1}{3};\ \infty\right)$ D) $y \in (-\infty; -3]$ or $[3;\ \infty)$

Grade 12

51. Find $f'(9)$ if $f(x) = \frac{3}{2\sqrt{x}}$.

A) $-\frac{1}{12}$ B) $-\frac{1}{36}$ C) $\frac{1}{9}$ D) $\frac{9}{2}$

52. Find $f^{-1}(x)$ if $f(x) = \frac{2x}{1-3x}$

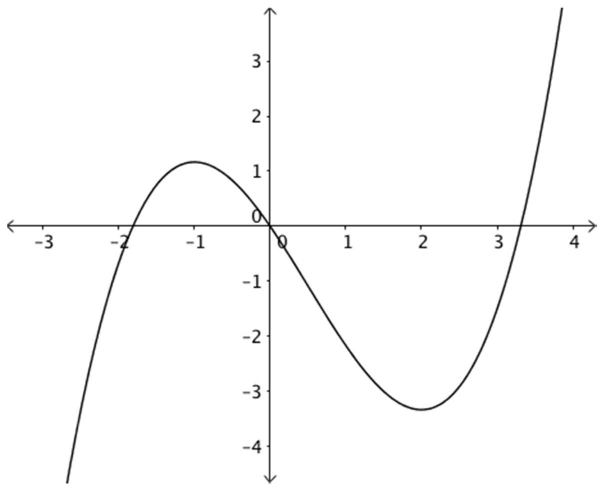
A) $f^{-1}(x) = \frac{x}{3x+2}$

B) $f^{-1}(x) = \frac{x}{2x+3}$

C) $f^{-1}(x) = 2\left(\frac{1}{x} - 3\right)$

D) $f^{-1}(x) = \frac{1}{2}\left(x - \frac{1}{3}\right)$

53. The graph of a polynomial, $f(x)$, is shown, with turning points at $x = -1$ and $x = 2$. Which statement about $f'(x)$ is true?



A) $f'(0) = 0$

B) $f'(-1) > f'(-2)$

C) $f'(2) < 0$

D) $(x-2)$ is a factor of $f'(x)$

54. The line $y = 7x + p$ is a tangent to the graph of $f(x) = kx^3 + x$ at the point where $x = -1$. Find the value of p.

A) 2

B) 4

C) 8

D) 10

55. For a linear function $f(x) = mx + c$, $f^{-1}(4) = 0$ and $f(f(0)) = 2$. Find $f(6)$.

A) 0

B) 1

C) 2

D) 3

56. Find the domain of the function $f(x) = 1 + \log(4x - 6)$.

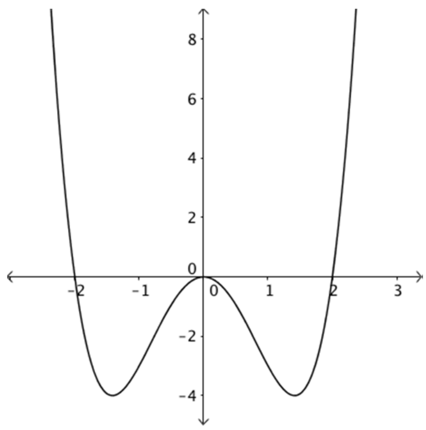
A) $x \in (\frac{2}{3};\ 1)$

B) $x \in (\frac{2}{3};\ \infty)$

C) $x \in (1{,}5;\ \infty)$

D) $x \in [1{,}5;\ \infty)$

57. The graph of $f(x)$ is shown here. For how many values of k will $f'(k) = 0$?



A) 1 B) 2 C) 3 D) 5

58. If $f'(x) = (x^2 + 2x + 2)(2x - 4)(x + 4)^2$, for which value(s) of x will the graph of $f(x)$ have a turning point?

A) $x = 2$ or $x = -4$
B) $x = 2$ or $x = -1$
C) $x = 2$
D) $x = \frac{1}{2}$

59. Find the x-coordinate of the point of intersection in the fourth quadrant of the graphs of $xy = -4$ and $y = 5 - x^2$.

A) $\frac{-1+\sqrt{17}}{2}$ B) $\frac{-1+\sqrt{15}}{2}$ C) $\frac{1+\sqrt{15}}{2}$ D) $\frac{1+\sqrt{17}}{2}$

60. A circle centered at the origin touches each branch of the hyperbola $y = \frac{8}{x}$, at exactly one point. Find the equation of the circle.

A) $x^2 + y^2 = 64$
B) $x^2 + (y - 2)^2 = 16$
C) $x^2 + y^2 = 16$
D) $x^2 + y^2 = 8$

61. Find the values of x for which the function $f(x) = 2x^4 - 8x$ decreases.

A) $x < 1$ B) $x > 1$ C) $0 < x < 2$ D) $-2 < x < 0$

62. If $(x; y) = (\log(t - 1); 3t)$, with $t > 1$, then

A) $y = 3(10^x + 1)$
B) $y = 3(10^x) + 1$
C) $y = \frac{\log(x+1)}{3}$
D) $y = \frac{1}{3}(10^{x+1})$

Trigonometry

Grades 11 and 12

63. Simplify $\cos^2 20° + \sin 70°.\cos 200°$.

A) 0 B) $2\sin^2 20°$ C) $\sin 20°$ D) $2\cos^2 70°$

64. In triangle ABC, AB = AC = $\sqrt{3}$ and ∠ABC = 30°. Find the area of triangle ABC.

A) $\frac{3\sqrt{3}}{4}$ B) $\frac{3\sqrt{3}}{2}$ C) $\frac{3}{4}$ D) $\frac{\sqrt{3}}{4}$

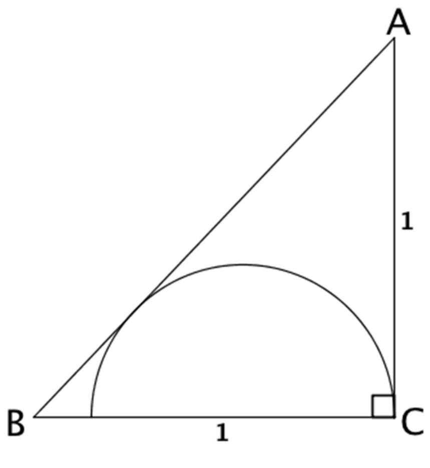
65. A regular pentagon is drawn inside a circle, with all vertices on the circumference. Find an expression for the radius of the circle if the pentagon has a perimeter of 50 cm.

A) $10 \sin 72°$ B) $5 \cos 36°$ C) $\frac{10}{\cos 72°}$ D) $\frac{5}{\sin 36°}$

66. Find the value of $\cos\theta$ if $\frac{1}{\cos\theta} + \tan\theta = 2$.

A) $\frac{2}{3}$ B) $\frac{4}{7}$ C) $\frac{4}{5}$ D) $\frac{5}{4}$

67. Triangle ABC is a right-angled isosceles triangle with AC = BC = 1. A semicircle is drawn with its diameter on BC, so that AB and AC are tangents to the semicircle. Find the radius of the semicircle.



A) $1 - \frac{1}{\sqrt{2}}$ B) $\frac{2}{\sqrt{2}-1}$ C) $\frac{\sqrt{2}+1}{2}$ D) $\sqrt{2} - 1$

68. Express $\cos 110°$ in terms of p, if $\sin 20° = p$.

A) $-p$ B) $\frac{1}{p}$ C) $-\frac{1}{p}$ D) $\sqrt{1-p^2}$

69. Evaluate $\frac{\sin 300°}{\tan 240°}$.

A) $-\frac{1}{2\sqrt{3}}$ B) $\frac{1}{2\sqrt{3}}$ C) $-\frac{1}{2}$ D) $\frac{1}{2}$

70. Find the interval(s) in which $\tan 2x < 0$, for $x \in [0°; 180°]$.

A) (0°; 45°) or (90°; 135°) B) (45°; 90°)
C) (0°; 90°) or (135°; 180°) D) (45°; 90°) or (135°; 180°)

71. Simplify the expression $\tan(180° + x) - \frac{\cos x}{\sin(180°+x)}$.

A) $\frac{2}{\sin 2x}$ B) $\frac{2\cos 2x}{\sin 2x}$ C) $\frac{1}{2\sin 2x}$ D) $\frac{\cos 2x}{2\sin 2x}$

Grade 12

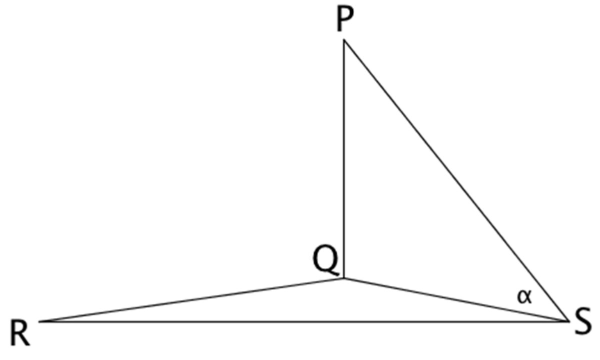
72. Find $\cos 2\theta$ in terms of m if $\sin\theta = \frac{1}{m}$.

A) $\frac{2m^2}{\sqrt{m^2-1}}$ B) $\frac{2m^2-1}{m-1}$ C) $1 - \frac{2}{m^2}$ D) $\frac{2\sqrt{m^2-1}}{m}$

73. Find the general solution to the equation $\cos 2x \cos x = \sin 2x \sin x$.

A) $x = 60° + k.120°$ B) $x = 60° + k.360°$
C) $x = 30° + k.60°$ D) $x = 45° + k.180°$

74. A vertical flagpole PQ of height h metres is supported by a stay wire PS, which makes an angle of α with the ground. A point R lies on the same horizontal plane as S so that RQ = RS = x and angle QSR = θ. Find an expression for height h.



A) $\sqrt{2x\cos\theta\sin\theta}\sin\alpha$ B) $x\sqrt{2(1+\cos 2\theta)}\tan\alpha$
C) $x\sqrt{2(1-\cos 2\theta)}\tan\alpha$ D) $2\sqrt{x+\sin 2\alpha}\cos\theta$

75. Find the values of x for which $\frac{\cos 2x}{\cos x} \leq 0$, for $x \in [0°; 180°]$.

A) $x \in [0°; 45°]$ or $x \in [90°; 135°]$
B) $x \in [45°; 135°]$
C) $x \in [90°; 180°]$
D) $x \in [45°; 90°]$ or $x \in [135°; 180°]$

76. Express $\sin(2x) - 2\sin^3 x \cos x$ as a product.

A) $2 \sin x(1 - \cos^3 x)$
B) $2 \sin x \cos^3 x$
C) $2 \sin x(1 - \sin^2 x. \cos x)$
D) $2 \cos x \sin^3 x$

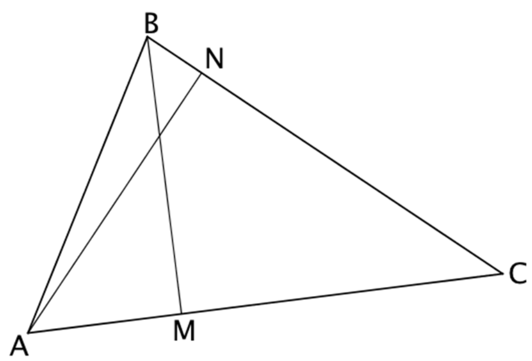
77. Find the value of $\cos 2\alpha$ if $\alpha + \beta = 90°$ and $\tan \beta = 0{,}3$.

A) $\frac{91}{109}$
B) $-\frac{91}{109}$
C) $\frac{73}{91}$
D) $-\frac{73}{91}$

Geometry and Measurement

Grades 10, 11 and 12

78. In triangle BAC, AN ⊥ BC and BM ⊥ AC. Find the length of BM if AC = 12 cm, BC = 10 cm and AN = 8 cm.

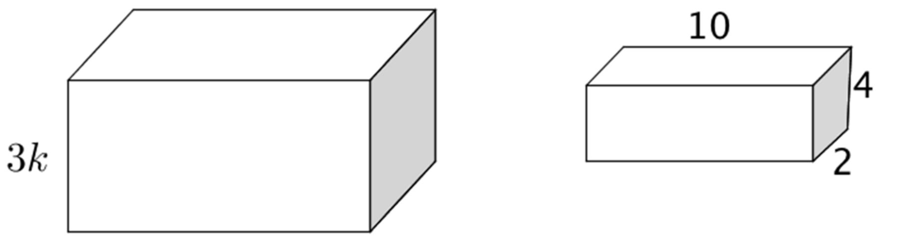


A) 6 cm
B) 9 cm
C) $9\frac{3}{5}$ cm
D) $6\frac{2}{3}$ cm

79. A water sprinkler is placed in the centre of a square field, and rotates to water a circular area. If the sprayed water just reaches the four sides of the field, what fraction of the field is not watered?

A) $\frac{4-\pi}{\pi}$
B) $r^2(4 - \pi)$
C) $1 - \pi$
D) $\frac{4-\pi}{4}$

80. The smaller cuboid (rectangular box) shown below is a scale model of the larger one.
Find the volume of the larger cuboid in terms of *k*.

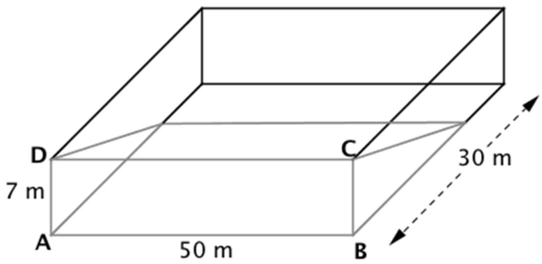


A) $120k^3$ B) $270k^3$ C) $\frac{k^3}{40}$ D) $\frac{135k^3}{4}$

81. A square lawn has a 1 metre wide path all around it. Find the area of the lawn if the area of the path is 40 m^2.

A) 64 m^2 B) 144 m^2 C) 81 m^2 D) 121 m^2

82. A rectangular water storage tank is 30 m wide, 50 m long and 7 m deep and partly filled with water. When the tank is tipped forward just far enough to make sure that the water completely covers side ABCD, two thirds of the base is covered by the water. How deep was the water in the tank when the base was still horizontal?

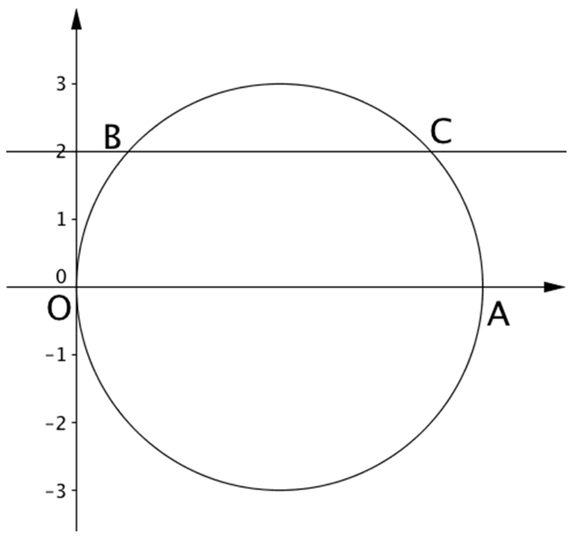


A) $\frac{14}{3}$ m   B) $\frac{7}{3}$ m   C) $\frac{7}{6}$ m   D) $\frac{14}{9}$ m

83. A cube is constructed using 64 equal smaller cubes, and the resulting solid is painted. How many of the smaller cubes will be painted on exactly two faces?

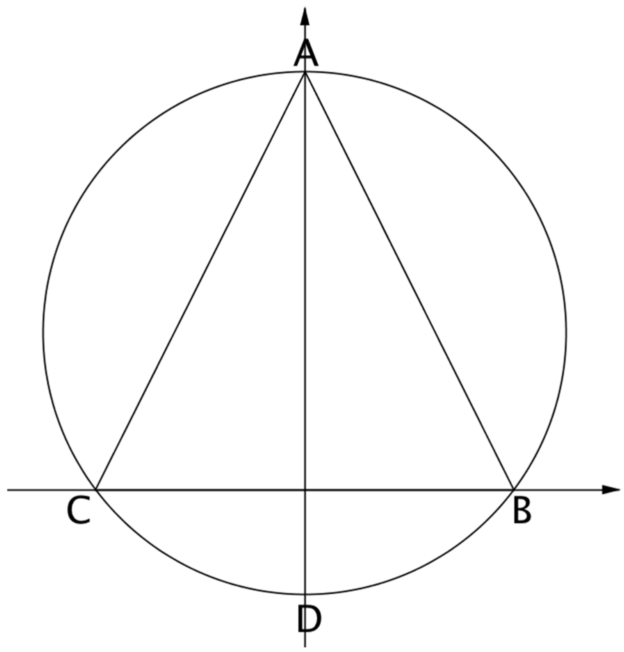
A) 12   B) 24   C) 36   D) 48

84. The diagram shows a circle in the Cartesian plane with diameter OA, where O is the origin and A is the point (6; 0). The horizontal line through the point (0; 2) intersects the circle at B and C. Find the x-coordinate of point C.



A) $5 - \sqrt{3}$ B) $6 - 2\sqrt{2}$ C) $3 + 2\sqrt{2}$ D) $3 + \sqrt{5}$

85. A circle passes through the points A(0; 8) and D(0; –2), and has x-intercepts at B and C. If AD is a diameter of the circle, find the length of AC.

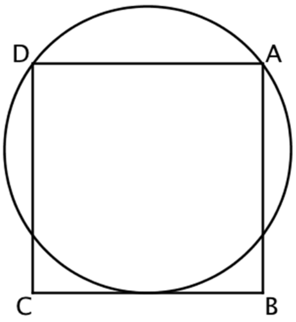


A) $8\sqrt{3}$ B) $4\sqrt{3}$ C) $3 + \sqrt{5}$ D) $4\sqrt{5}$

86. How many different shaped cuboids (rectangular boxes) can be formed from 12 equal cubes?

A) 3 B) 4 C) 6 D) 8

87. A circle passes through vertices A and D of a square, and is tangential to the side BC. If the square has side length 2, find the radius of the circle.



A) $\frac{5}{4}$ B) $\frac{4}{5}$ C) 1 D) $\frac{5}{2}$

88. Find the volume of the largest possible cylinder that can fit into a cube of side *x* cm.

A) $\frac{(\pi x)^3}{8}$ B) $\pi\frac{x^3}{2}$ C) $\pi\frac{x^3}{8}$ D) $\pi\frac{x^3}{4}$

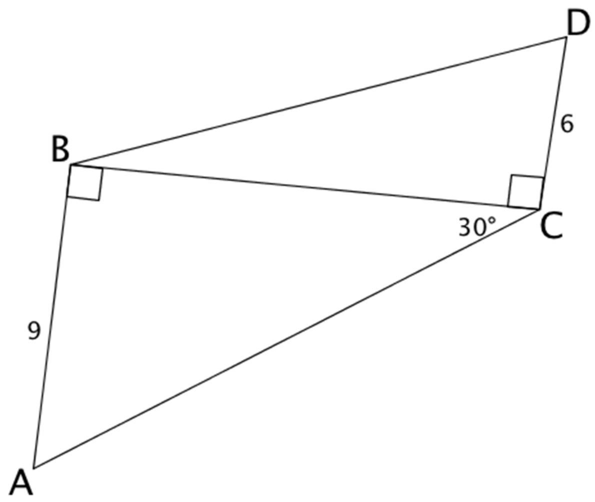
89. Rectangle ABCD has AB = 6 and AD = 2. Find the area of circle with its centre at A, passing through C.

A) $4\sqrt{2}\pi$ B) $\sqrt{40}\pi^2$ C) 40π D) 20π

90. The sum of the interior angles of a polygon is 2340°. How many sides does the polygon have?

A) 13 sides B) 14 sides C) 15 sides D) 16 sides

91. Find the length of BD in the given diagram.



A) $3\sqrt{23}$ B) $3\sqrt{31}$ C) $3\sqrt{7}$ D) $3\sqrt{10}$

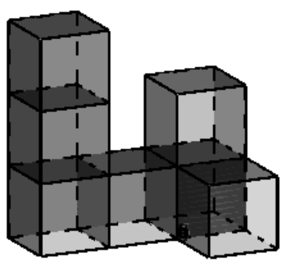
92. A circle has the same area as a square with side *s* metres. Find an expression for the radius of the circle in terms of *s*.

A) $\frac{s}{\sqrt{\pi}}$ B) $\sqrt{\frac{s}{\pi}}$ C) $s\sqrt{\pi}$ D) $\frac{\sqrt{\pi}}{s}$

93. In triangle ABC, ∠A = 80° and the bisectors of angles B and C meet at point P inside the triangle. Find the size of ∠BPC.

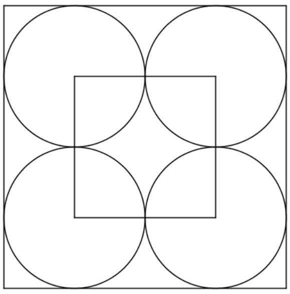
A) 40° B) 100° C) 130° D) 160°

94. The solid shown here is constructed using equal cubes, and its total volume is 189 cm^2. Find its surface area.



A) 270 cm^2 B) 290 cm^2 C) 145 cm^2 D) 170 cm^2

95. Four equal circles are drawn to fit exactly inside a square. A smaller square is drawn with its vertices at the centres of the four circles. Find the ratio of the area of the smaller square to the area of the larger square.

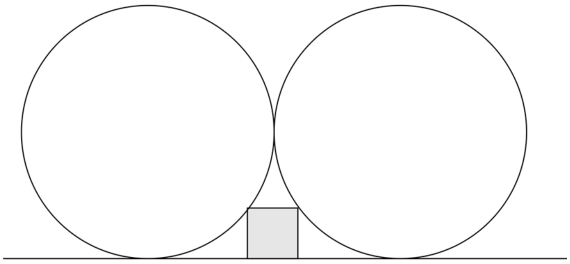


A) 1 : 2 B) 1 : 4 C) 2 : 5 D) 3 : 5

96. One diagonal of a rhombus is 8 cm shorter than the other. If the area of the rhombus is 24 cm2, find the sum of the lengths of the diagonals.

A) 16 B) 12 C) 24 D) 36

97. The diagram shows two circles of radius 1, with a square drawn to just touch both circles. Find the length of the side of the square.



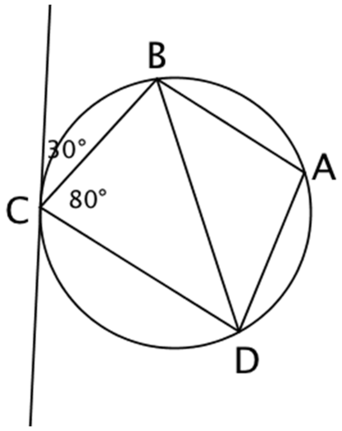
A) $\frac{\sqrt{2}}{3}$ B) $\frac{2}{5}$ C) $\frac{3}{8}$ D) $\frac{1}{2\sqrt{2}}$

Grades 11 and 12

98. The diagonals of square ABCD intersect at O. If the area of triangle ABO is x cm2, find an expression for diagonal AC in terms of x.

A) $2\sqrt{x}$ B) $2\sqrt{2x}$ C) $\sqrt{2x}$ D) $2x\sqrt{2}$

99. The diagram shows a tangent to the circle at C, making an angle of 30° with chord BC. If chord BA is parallel to chord CD and ∠BCD = 80°, find the size of ∠ADB.

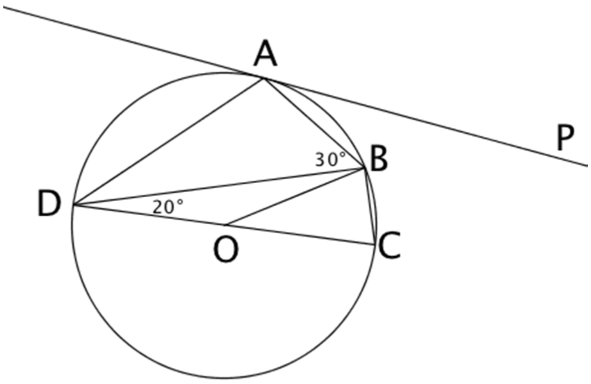


A) 30° B) 40° C) 45° D) 50°

100. Triangle ABC has sides of length 9 cm, 12 cm and 15 cm. Find the radius of the circle that is drawn inside the triangle to touch all three sides.

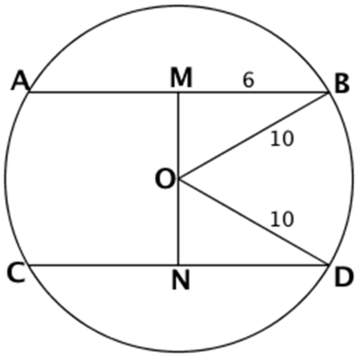
A) 2 cm B) 3 cm C) 4 cm D) 5 cm

101. In the given diagram, DOC is a diameter of the circle, and a tangent line AP is drawn at A. Find the size of ∠PAB if ∠ODB = 20° and ∠ABD = 30°.



A) 20° B) 30° C) 35° D) 40°

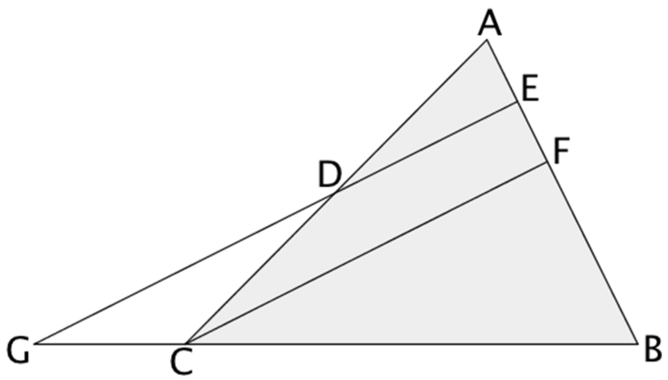
102. Two parallel chords AB and CD lie 14 cm apart on opposite sides of the centre of a circle of radius 10 cm. If AB is 12 cm long, find the length of CD.



A) 8 cm B) 12 cm C) 13 cm D) 16 cm

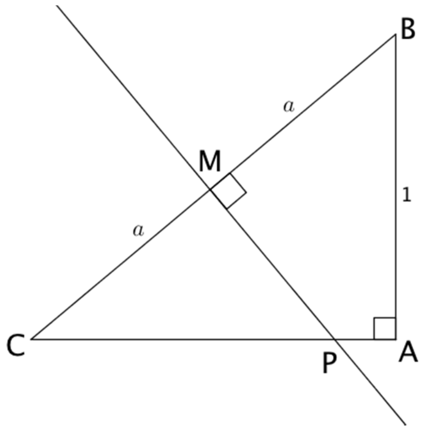
Grade 12

103. In the given figure, AE = DC = 3, AB = 10 and AD = 4,5. If line GDE is parallel to CF, find FC : EG.



A) 5 : 7 B 3 : 5 C) 6 : 7 D) 5 : 6

104. In the diagram, MP is the perpendicular bisector of CB so that CM = MB = a. If AB = 1, find the length of MP.

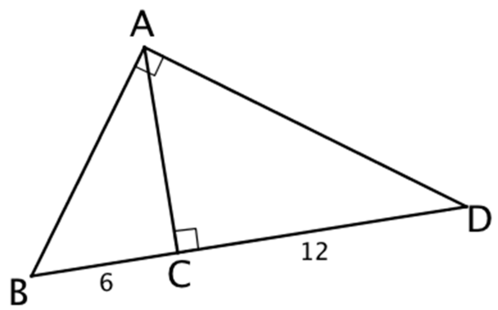


A) $\frac{a}{\sqrt{4a^2+1}}$ B) $\frac{a}{\sqrt{2a^2-1}}$ C) $\frac{a}{\sqrt{4a^2-1}}$ D) $a\sqrt{4a^2+1}$

105. A circle of radius 13 has its centre at the point O(4; –2). If the point P(–1; b) lies on the circumference of the circle, find the sum of the possible values of b.

A) 11 B) 9 C) 4 D) –4

106. In the given diagram, angles BAD and ACD are right angles, BC = 6 and CD = 12. Find the length of AC.

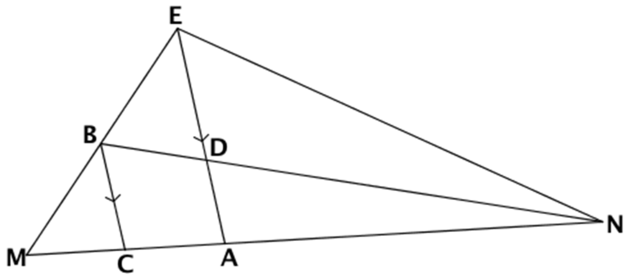


A) 8 B) 10 C) $6\sqrt{2}$ D) $4\sqrt{3}$

107. Rectangle ABCD is drawn in the Cartesian plane with sides parallel to the x- and y-axes and one diagonal through the points B(12; 12) and D(8 ; 6). Find the equation of the circle with diameter AC.

A) $(x-8)^2+(y-6)^2=25$
B) $(x-12)^2+(y-2)^2=5$
C) $(x-10)^2+(y-9)^2=13$
D) $(x-9)^2+(y-10)^2=\sqrt{13}$

108. In the diagram (not drawn to scale), BC is parallel to EA, MA = $\frac{3}{8}$ MN and 2MB = BE. Find the value of BD : DN.



A) 1 : 2 B) 2 : 3 C) 1 : 3 D) 2 : 5

109. The line $y = mx + c$, with $m < 0$, makes an angle of 30° with the x-axis. The line is a tangent to the circle defined by $(x-2)^2 + y^2 = 4$, with the point of contact in the first quadrant. Find the y-intercept of the tangent line.

A) $\frac{3\sqrt{3}}{4}$ B) $4\sqrt{3}$ C) $3\sqrt{3}$ D) $2\sqrt{3}$

Data and Probability

Grades 10, 11 and 12

110. Find the median value of the 29 data values given in the stem and leaf diagram.

Stem	Leaf
1	1 2 3 3 5 7 7 7 9
2	1 3 5 5 6 7 8
3	0 1 3 4
4	1 4
5	2 3 7 8
6	0 1
7	2

Key: 6 | 2 = 62

A) 7 B) 17 C) 26 D) 27

111. Newspapers A and B are sold in a suburb. In a survey of 30 people, 8 people read both newspapers and 4 people read neither of these newspapers. The number of people reading only newspaper B is twice as large as the number of people who read only newspaper A. How many people read newspaper A?

A) 14 B) 18 C) 12 D) 16

112. A player draws three cards from a standard deck of 52 playing cards. In order to win, the third card must be a number between the first and second cards. If the player draws a 6 and then a 10, what is the probability that he will win the round?

A) $\frac{6}{25}$ B) $\frac{3}{50}$ C) $\frac{3}{13}$ D) $\frac{4}{13}$

113. In a group of boys and girls, the average height of all the children in the group is 165 cm, the average height of the boys is 172 cm and the average height of the girls is 160 cm. Find the ratio of boys to girls in the group.

A) 7 : 5 B) 5 : 7 C) 5 : 8 D) 4 : 7

114. All of the 12 values in a data set are increased by 2. Which one of the following statements is true?

A) The mode will not change.
B) The median will increase by a value of 2.
C) The mean will not change.
D) The mean will increase by a value of 24.

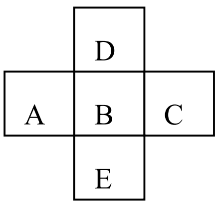
Logic

Grades 10, 11 and 12

115. Two positive natural numbers have their difference, sum and product in the ratio 1 : 3 : 4. Find the smaller of the two numbers.

A) 2 B) 3 C) 4 D) 6

116. In the diagram, each letter represents a different number from 1 to 5. Find the number of different ways the letters can be arranged so that A + B + C = D + B + E.



A) 12 B) 18 C) 24 D) 36

117. Find the last digit in the expansion of 7^{77}.

A) 7 B) 3 C) 1 D) 9

118. If $3 \leq x \leq 6$ and $7 \leq y \leq 9$, find the sum of the largest and smallest values of $\frac{x}{y}$.

A) $\frac{69}{63}$ B) $\frac{75}{63}$ C) $\frac{75}{18}$ D) $\frac{23}{9}$

Section B

NBT Prep. Test 1 3 hours No calculators

1. $3^{n+1} - 3^n + 3^{n-1}$ is equal to:

A) 3^n B) 2×3^n C) 9^n D) $7 \times 3^{n-1}$

2. The area of triangle ABC = 18 cm^2, and BC = AC = m. Find sin C in terms of m.

A) $\frac{6}{m}$ B) $\frac{36}{m^2}$ C) $\frac{m^2}{36}$ D) $\frac{m}{6}$

3. If m% of r cars are sold, how many cars are left unsold?

A) $\frac{mr}{100}$ B) $\frac{100r-mr}{100}$ C) $\frac{100r-m}{100}$ D) $\frac{r-m}{100}$

4. The mean of the data values $x^2 + x; 3x - 5; 5x$ and $6 - 7x$ is 1. Find the median of the data if x is a negative number.

A) –4 B) –3 C) 2,5 D) 5,5

5. Given $f'(x) = (x^2 + 1)(x - 3)(x + 2)^2$, find the x-coordinates of any turning point(s) of the graph of $f(x)$.

A) $x = 3$ or ± 1 B) $x = 3$ or -2 C) $x = 3$ D) $x = 3, \pm 1$ or -2

6. Four horses are running in a race. In how many different ways can the first three positions be filled?

A) 8 B) 12 C) 24 D) 36

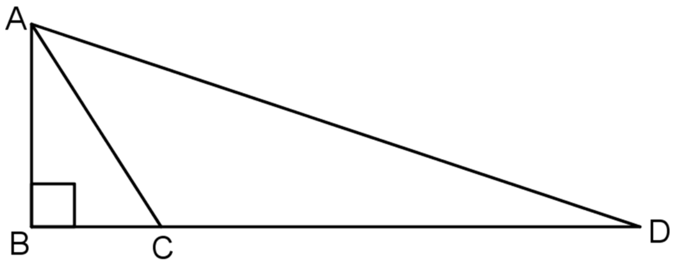
7. Simplify the expression $\cos^2 x + \cos^2 x \tan^2 x$.

A) $\tan^2 x$ B) $\cos^4 x$ C) $\cot^2 x$ D) 1

8. If p is an odd number and q is an even number which of the following expressions represents an even number?

A) $p^2 - q^2$ B) $p^2 + q^2$ C) $(qp + q)^{q+1}$ D) $(p - q)^2$

9. Find the length of CD, if AB = 16 and angle ACB = 60° and angle D = 30°.



A) $\frac{16}{\sqrt{3}}$ B) $32\sqrt{2}$ C) $\frac{32}{\sqrt{3}}$ D) $\frac{32}{\sqrt{2}}$

10. An investment grows by 12% per annum compounded monthly. How many years will it take for the investment to double?

A) $(\log_{1,01} 2) \div 12$ B) $1{,}01^{12}$ C) $\log_{1,01} 2$ D) $(\log_{1,12} 2) \div 12$

11. A cube has a surface area of A cm^2. Find an expression for the length s of each side of the cube, in terms of A.

A) $s = \sqrt[3]{A}$ B) $s = \sqrt[3]{\frac{A}{6}}$ C) $s = \frac{\sqrt{A}}{6}$ D) $s = \sqrt{\frac{A}{6}}$

12. Find the value of k if $(p + 2)$ is a factor of the polynomial $2p^3 - p^2 + 12 - k$.

A) 0 B) –8 C) 20 D) 24

13. Find x such that $x.\log_2 3 = \log_{10} 3$.

A) $x = \log_{10} 2$ B) $x = \log_5 1$ C) $x = 3\log_2 10$ D) $x = \log_2 10$

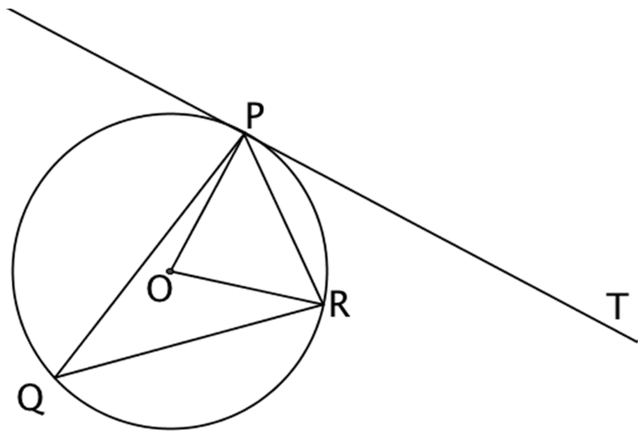
14. Simplify $(a + b)^{-1}(a^{-1} + b^{-1})$.

A) $\frac{1}{(a+b)^2}$ B) 2 C) $\frac{1}{ab}$ D) $\frac{1}{a+b}$

15. The graph of $f(x) = x^3 + bx - 12$ has a tangent line at the point where $x = 3$. Find the value of b if this tangent line is parallel to the line defined by $2y - 6x = 4$.

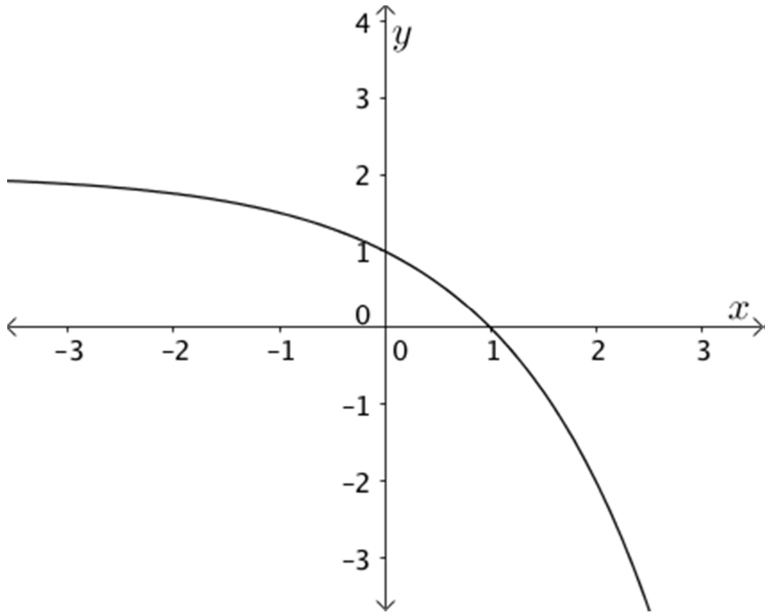
A) –4 B) –24 C) –33 D) –21

16. The diagram shows circle centre O with a tangent line PT drawn at P. If ∠TPR = 40°, find the size of ∠OPR.



A) 35° B) 40° C) 45° D) 50°

17. Which of the equations below could be the equation of the given graph?



A) $y = -(2^x) + 2$ B) $y = (2^{-x}) + 1$ C) $y = (3^x) + 1$ D) $y = -(3^x) + 2$

18. The circumference of a circle is doubled. Find the ratio $\frac{new\ area}{original\ area}$.

A) 2π B) $\frac{\pi}{2}$ C) 2 D) 4

19. If $x = 4$, find the third term of the series defined by

$$\sum_{k=3}^{12}(10-2x)^k.$$

A) 8 B) 16 C) 32 D) 64

20. Find all real numbers x for which $-2(x+3)(5-x) < 0$.

A) $-3 < x < 5$ B) $x < -3$ or $x > 5$

C) $x < 5$ or $x > -3$ D) $-3 > x > 5$

21. Find the sum of the roots of $x + 1 = x^2$.

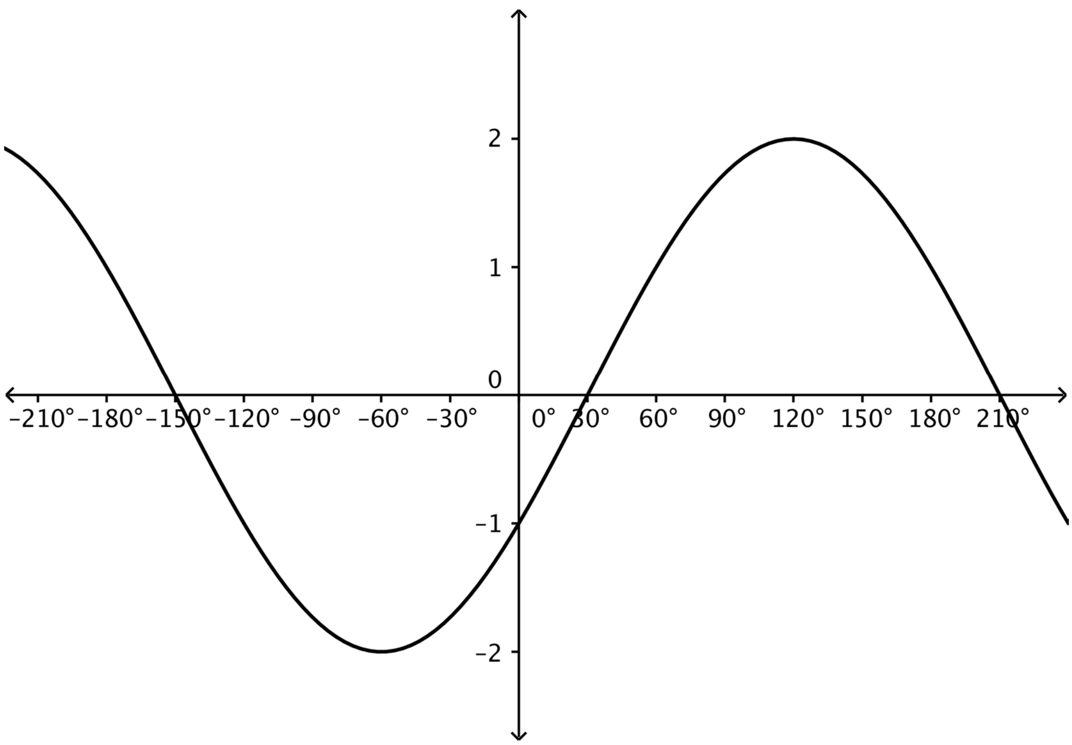
A) $1 + \sqrt{5}$ B) $\sqrt{5}$

C) $2\sqrt{5}$ D) 1

22. Find the domain of the function $f(x) = 3 - 5\log(2x+4)$.

A) $[-2;\ \infty)$ B) $(-2;\ \infty)$ C) $[-2; 3]$ D) $(-2; 5)$

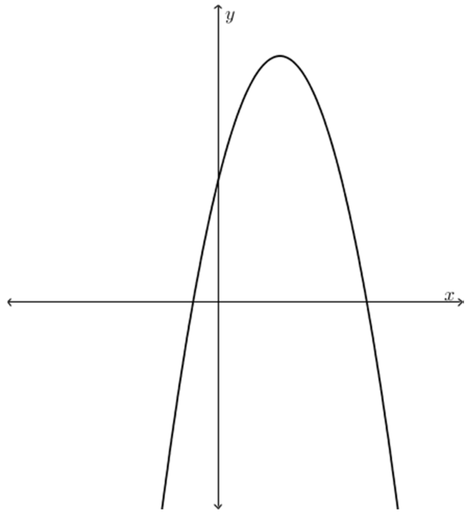
23. If $f(x) = \sin x$, what is the equation of the graph below?



A) $g(x) = f(2x - 30°)$ B) $g(x) = f(2x) + 30°$

C) $g(x) = 2f(x - 30°)$ D) $g(x) = 2f(x + 30°)$

24. The graph of the function $f(x) = ax^2 + bx + c$ is shown below. Choose the correct information about the values of *a, b* and *c*.



A) $a > 0, b > 0, c > 0$
B) $a < 0, b > 0, c > 0$
C) $a < 0, b < 0, c > 0$
D) $a > 0, b < 0, c < 0$

25. A rhombus has a side length of 6 cm and one diagonal of length 10 cm. Find the length of the other diagonal.

A) $2\sqrt{11}$ cm B) $\sqrt{11}$ cm C) $\sqrt{61}$ cm D) 8 cm

26. If $x > 0$, which one of the following statements is true?

A) $\frac{x}{x+1} > 1$ B) $0 \leq \frac{x}{x+1} \leq 1$ C) $0 < \frac{x}{x+1} < 1$ D) $0 \leq \frac{x}{x+1} < 1$

27. One solution to the equation $\cos(x - 20°) = k$ is $x = 80°$. Which of the following will also be a solution to the equation?

A) 100° B) 260° C) –80° D) –40°

28. Find the 2 000th letter in the sequence

ABCDCBAABCDCBA ABCDCBABC ...

A) *A* B) *B* C) *C* D) *D*

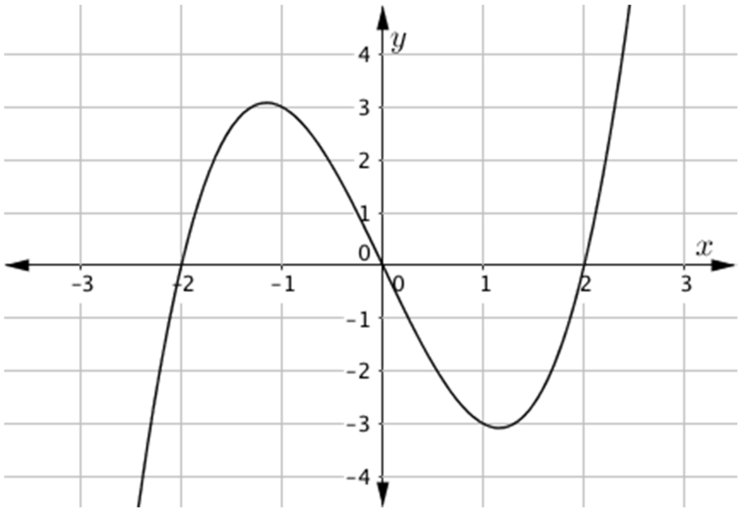
29. The *x*-axis is a tangent line to a circle with centre A($\sqrt{17}$; 9). Find the *y*-coordinates of the points where the circle cuts the *y*-axis.

A) 8 and $\sqrt{98}$ B) 1 and 17 C) ± 8 D) 8 and 6

30. The graph of $f(x) = ax^2 + bx + c$ has its turning point at (–2; 4). Which one of the following functions will be equal to 0 for exactly one value of x?

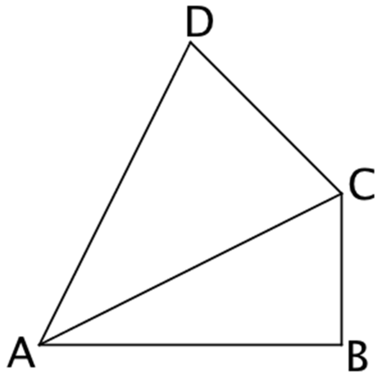
A) $y = f(x + 2)$ B) $y = f(-x) + 2$ C) $y = f(x - 4)$ D) $y = f(x) - 4$

31. Choose the *incorrect* statement about the graph of $y = f(x)$.



A) $f'(0) < 0$ B) $f(-2) = f(0) = f(2)$
C) $f'(-2) > 0$ D) $f(-1) < f(2)$

32. In the figure, ∠B and ∠ACD are right-angles, AD = $\sqrt{29}$, DC = 2, CB = 3 and AB = 4. Find sin(∠BAD).

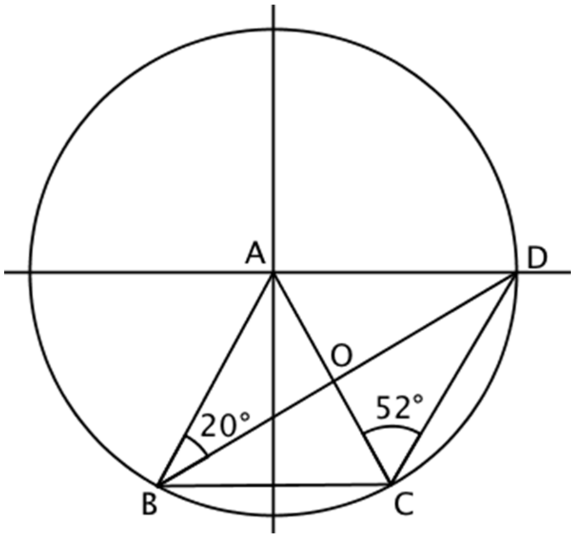


A) $\frac{7}{5\sqrt{29}}$ B) $\frac{23}{5\sqrt{29}}$ C) $\frac{3\sqrt{29}+10}{5\sqrt{29}}$ D) $\frac{1}{\sqrt{29}}$

33. If $\sin 6x = \frac{2}{3}$, find the value of $(\sin 3x + \cos 3x)^2$.

A) $\frac{5}{3}$ B) $\frac{4}{9}$ C) $\frac{4}{3}$ D) $\frac{1}{9}$
x

34. In the given diagram the centre of the circle is A, angle ABD equals 20° and angle DCA equals 52°. Find angle BOC.



A) 64° B) 40 ° C) 52° D) 84°

35. If $p\otimes q = p^2 - q$, find the value of $3\otimes(5\otimes 2)$.

A) -14 B) 14 C) 8 D) -8

36. A large rabbit is 18 m behind a small rabbit. The large rabbit jumps 4 m every time the small rabbit jumps 2 m. In how many leaps does the large rabbit overtake the small rabbit?

A) 6 B) 9 C) 18 D) 24

37. If $(x+2)(y+2) = 18$ and $x + y = 7$, find the value of xy.

A) 11 B) 14 C) 25 D) 0

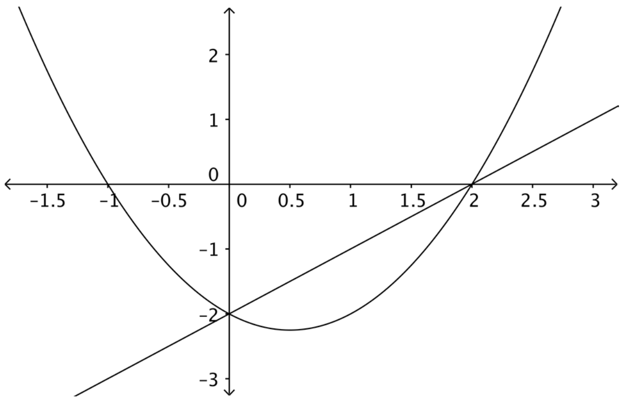
38. A rectangular solid has side, front and bottom faces with areas of $2x$, $\frac{y}{2}$ and xy cm^2 respectively. Find the volume of the solid in cubic centimetres.

A) x^2y^2 B) $8x^3 + \frac{y^3}{8} + x^3y^3$ C) $4x^2 + \frac{y^2}{4} + x^2y^2$ D) xy

39. A circle centre P has radius r_1 and a circle centre Q has radius r_2. The two circles will **not** intersect each other if:

A) $r_1 + r_2 < PQ$ B) $r_1 - r_2 > PQ$ C) $r_1 + r_2 = PQ$ D) $r_1 + r_2 > PQ$

40. The graphs of the linear function $f(x)$ and the quadratic function $g(x)$ are given below. For which values of x is $f(x).g(x) < 0$?



A) $x < -1$ B) $0 < x < 2$ C) $x < -1$ or $x > 2$ D) $-2 < x < 2$

41. What is the minimum value of $6 \sin x \cos x + 4$?

A) –10   B) 2   C) 1   D) –2

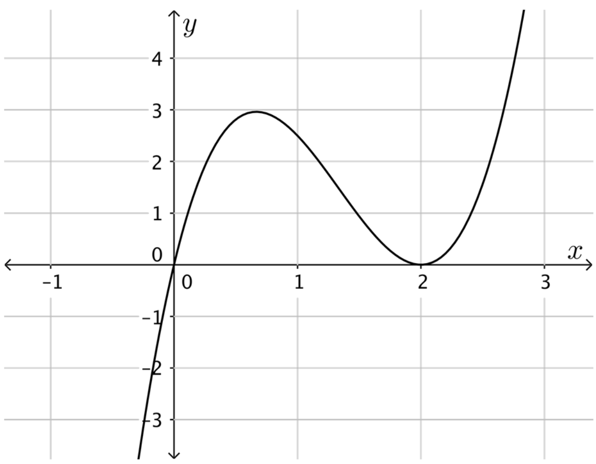
42. If n is a natural number, we write $n!$ for the product of the first n natural numbers, so that $n! = 1 \times 2 \times 3 \times ... \times (n-1) \times n$. Write $\log_{10} \frac{100!}{99!}$ in its simplest form.

A) $\log_{10} \frac{1}{99}$ B) 2   C) $\log_{10} 99$ D) -2

43. A and B are complementary angles such that $\sin B = 0{,}6$. Find the value of $\tan(180° - A)$.

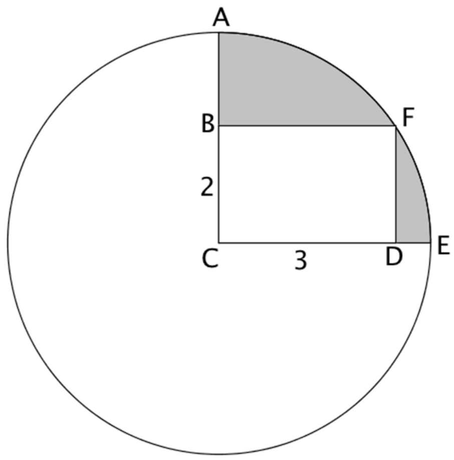
A) $-\frac{3}{4}$ B) $-\frac{5}{3}$ C) $\frac{4}{3}$ D) $-\frac{4}{3}$

44. The graph of $y = f(x)$ is shown below. For what values of k will the equation $f(x) = k$ have three distinct roots?



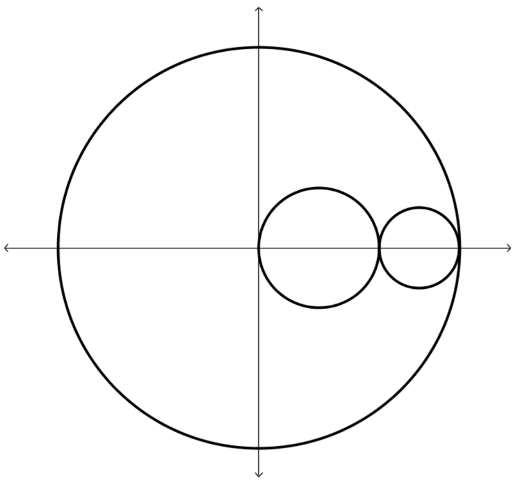
A) $0 < k < 3$ B) $k > 3$ C) $k < 3$ D) $0 < k < 2$

45. C is the centre of the circle and F is a point on the circle such that BCDF is a 2cm by 3cm rectangle. What is the area of the shaded region? (in cm^2).



A) $\frac{\sqrt{5}\pi}{4} + 1{,}5$ B) $\sqrt{5}\pi - 6$ C) $\frac{13\pi}{4} - 6$ D) $13\pi - 6$

46. The diagram shows two tangential circles with their centres on the *x*-axis. These circles fit within the larger circle defined by $x^2 + y^2 = 144$, as shown. If the radius of the smallest circle is half the radius of the middle-sized circle, find the equation of the smallest circle.

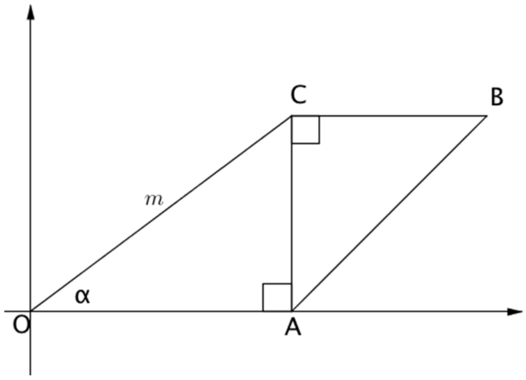


A) $(x-6)^2 + y^2 = 36$ B) $x^2 + (y-10)^2 = 4$
C) $(x-5)^2 + y^2 = 36$ D $(x-10)^2 + y^2 = 4$

47. A pyramid has a slant height of 5 m (along the sloping sides) and a square base of length 4 m. Find the vertical height of the pyramid.

A) $\sqrt{17}$ B) $3\sqrt{2}$ C) $2\sqrt{3}$ D) $\sqrt{15}$

48. Find an expression for the x-coordinate of B in the Cartesian plane, if ∠COA = α, OC = m and AC = CB.



A) $\frac{\sin A+\cos A}{m}$ B) $\frac{\sin A-\tan A}{m}$ C) $m(\cos\alpha-\sin\alpha)$ D) $m(\cos\alpha+\sin\alpha)$

49. Find the area enclosed by the graphs of $y=2x$, $x+y=6$ and $y=0$.

A) 6 B) 12 C) 18 D) 24

50. Find the minimum value of

$$\frac{2}{\sqrt{-2x^2+4x+2}}$$

A) ½ B) 1 C) 2 D) $\sqrt{2}$

51. Point C is drawn on line BD so that BC = 9 cm and CD = 4 cm. Line CA is drawn perpendicular to BD, so that angle BAD is a right angle. Find the length of AC.

A) 5 cm B) $\sqrt{97}$ cm C) 6 cm D) $\sqrt{65}$ cm

52. If x, y and z represent positive numbers and $x-y=z$, which of the following must equal 2?

A) $\frac{x-y}{2z}$ B) $\frac{2y+2z}{x+z}$ C) $\frac{2x+y}{y+z}$ D) $\frac{2x}{z+y}$

53. Find the values of a and b such that $\sum_{r=5}^{18} 2(2^{r-1}) = \sum_{p=1}^{18} 2^p - \sum_{p=a}^{b} \left(\frac{1}{2}\right)^{-p}$.

A) a = 1 and b = 18 B) a = 1 and b = 4 C) a = 4 and b = 18 D) a = 1 and b = 5

54. A function $f(x)$ satisfies the equation $f(1-x)+2f(x)=3x$. Find the value of $f(0)$.

A) –2 B) –1 C) 0 D) 1

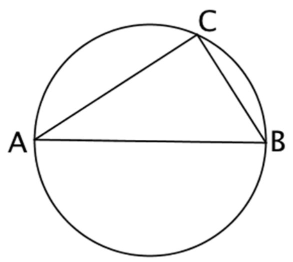
55. After the parabola $y = x^2 - 8$ is shifted and rotated, the original x-intercepts become the points A(2; 4) and B(4; 2). Find the equation of the axis of symmetry of the transformed parabola.

A) $y = 2x$ B) $y = \frac{1}{2}x$ C) $y = x$ D) $y = x - 2$

56. The lines $y = 6x + 3$ and $y = mx$ intersect below the line $y = -9$. For which values of m is this possible?

A) $-6 < m < 12$ B) $4\frac{1}{2} < m < 6$ C) $0 < m < 6$ D) $m < 6$

57. AB is a diameter of the circle, with AB = 10 cm and the area of ΔABC = 11 cm^2. Find the perimeter of triangle ABC. Hint: Expand $(AC + CB)^2$.



A) $10 + \sqrt{11}$ B) $10 + \frac{\sqrt{11}}{5}$ C) 22 cm D) 21 cm

58. There are red, green and yellow marbles in a bag. The probability of choosing a red one at random is $\frac{1}{3}$ and the probability of choosing a yellow one is $\frac{1}{5}$. Find the minimum number of green marbles that can be in the bag.

A) 3 B) 5 C) 7 D) 8

59. Annie and Basil both save all the money they have left after their expenses are paid. Annie has an income which is five eighths that of Basil. Annie's expenses are one-half those of Basil and Annie saved 40% of her income. What percentage of his income does Basil save?

A) 25% B) 75% C) 12,5% D) 50%

60. The points A, B, C and D lie in a plane, so that B is the midpoint of AC, BC = BD = 13 and CD = 24. Find the length of AD.

A) 9 B) 10 C) 11 D) 12

NBT Prep. Test 2 3 hours No calculators

1. If $\log_{10} 3 = a$, then $\log_{10} 0{,}009 =$

A) $a^2 - 3$ B) $\frac{a^3}{3}$ C) $3a - 2$ D) $2a - 3$

2. The length of a rectangle is decreased by 20% and its breadth is increased by 20%. Find the percentage change in the area of the rectangle.

A) No change in area B) decrease of 10% C) increase of 4% D) decrease of 4%

3. Find the value of $1 - 2\cos^2 15°$.

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) $1 - \frac{\sqrt{3}}{2}$ D) $\frac{1}{2}$

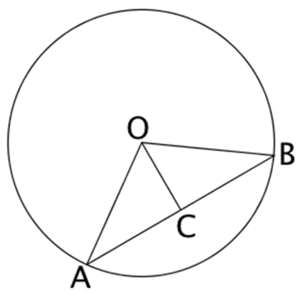
4. For which real values is $x(x^2 + 9)(x^3 + 1) = 0$?

A) $x = 0$ or $x = -1$ B) $x = \pm 3$ or $x = 0$ C) $x = \pm 1$ or $x = 0$ D) $x = 0$

5. Find the 231st term of the sequence 0; 2; 0; 4; 0; 6; 0; 8; …

A) 0 B) 2231 C) 2115 D) 2116

6. Two right-angled triangles AOC and BOC share a common apex at the centre of the same circle as shown. Find an expression for AB, in terms of the radius (r) of the circle, and angle A.



A) AB = $2r\cos A$ B) AB = $2r\sin A$ C) AB = $2 + r\cos A$ D) AB = $r\cos 2A$

7. The sequence with general term $T_n = \frac{0{,}5}{2^n}$ will

A) have a constant first difference B) have a constant second difference
C) be a geometric sequence D) be an arithmetic sequence

8. Find the value of $\frac{3{,}010\times3010}{30{,}10\times301}$.

A) 100 B) 1 C) 0,1 D) 0,01

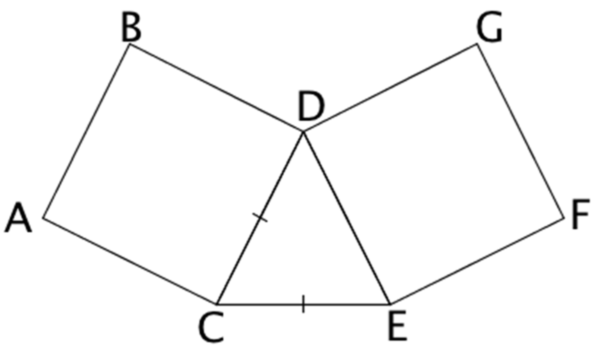
9. $\sqrt{36^{16x^8}} =$

A) 6^{4x^4} B) 18^{8x^4} C) 36^{4x^4} D) 36^{8x^8}

10. Four actors audition for four parts in a play, and they can each play any of the roles. In how many different ways can the actors be assigned to the four parts?

A) 4 B) 10 C) 12 D) 24

11. The figure shows two squares drawn on the sides of an isosceles triangle. If angle DCE = *x,* find the size of obtuse angle BDG.



A) $180° - x$ B) $180° - 2x$ C) $90° + \frac{x}{2}$ D) $90°$ - $\frac{x}{2}$

12. $8 \times 2^{100} + 4 \times 2^{101} =$

A) 2^{206} B) 2^{104} C) 2^{703} D) 4^{206}

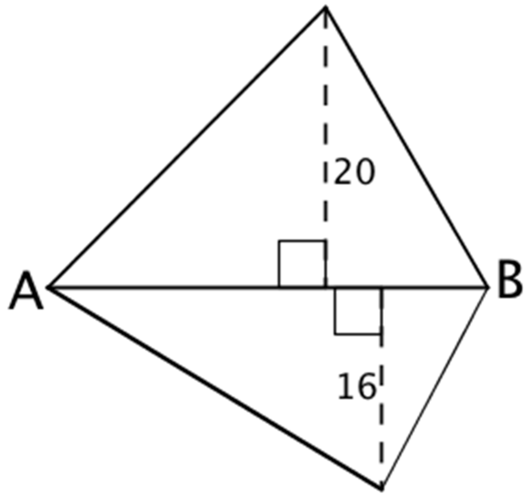
13. If $f(x) = \log x$, which statement is true?

A) $f(a+b) = f(ab)$ B) $f(2a) = 2f(a)$
C) $f(ab) = f(a) + f(b)$ D) $f(a+b) = f(a) + f(b)$

14. The length of the diagonal of a square is *d*. Find an expression for the area of the square in terms of *d*.

A) d^2 B) $\sqrt{2}d$ C) $\frac{d}{\sqrt{2}}$ D) $\frac{d^2}{2}$

15. In the quadrilateral, the dotted lines are 20 cm and 16 long. Find the length of AB in terms of the area of the quadrilateral.



A) $\frac{\text{Area}}{18}$ B) $\frac{\text{Area}}{36}$ C) 18Area D) $6\sqrt{\text{Area}}$

16. Find the inverse of the function $f(x) = \log_4 \sqrt{x}$.

A) $f^{-1}(x) = 16^x$ B) $f^{-1}(x) = 4^x$ C) $f^{-1}(x) = 8^x$ D) $f^{-1}(x) = x^{16}$

17. If the equation $x^2 + mx + n = 0$ has equal roots, find the value of $\frac{n}{m^2}$.

A) –4 B) –1 C) $-\frac{1}{4}$ D) $\frac{1}{4}$

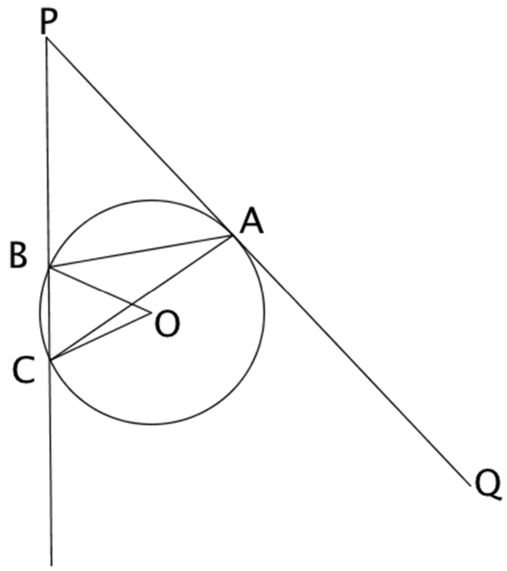
18. If $f(x) = 1 - \frac{1}{x}$, find f$\left(1 + \frac{1}{x}\right)$.

A) $\frac{1}{x+1}$ B) $\frac{x^2-1}{x^2}$ C) $\frac{1}{x-1}$ D) $\frac{x}{x+1}$

19. A circle has a radius of 5 and centre O(2; *a*). Find the value(s) of *a* if the point P(–1; 6) lies on the circumference of the circle.

A) $a = 2$ or $a = 10$ B) $a = 1$ or $a = 11$ C) $a = 2$ D) $a = 1$

20. In the diagram (not drawn to scale), PAQ is a tangent to the circle centre O at A, PBC is a secant to the circle, and PB = BA. If ∠CAQ = 100°, find the size of ∠BOC.



A) 30° B) 40° C) 60° D) 65°

21. The equation for one of the vertical asymptotes of $f(x) = 3\tan(2[x - 15°])$ is $x =$

A) 15° B) 30° C) 60° D) 75°

22. In triangle ABC, AB = 6 cm, AC = 8 cm and angle A = 30°. Find the length of BC.

A) $2\sqrt{25 + 12\sqrt{3}}$ B) $2\sqrt{25 - 12\sqrt{3}}$ C) $2\sqrt{13}$ D) $2\sqrt{37}$

23. A square ABCD has a diagonal AC with vertices A(–4 ; 3) and C(2; 11). A circle is drawn so that all four vertices of the square lie on the circle. Find the equation of the circle.

A) $(x + 1)^2 + (y - 7)^2 = 25$ B) $(x - 1)^2 + (y + 7)^2 = 5$
C) $(x + 3)^2 + (y - 8)^2 = 10$ D) $(x + 1)^2 + (y + 8)^2 = 100$

24. Simplify the expression $\left(\sqrt{2} - \sqrt{3}\right)^4\left(\sqrt{2} + \sqrt{3}\right)^2$.

A) $12\sqrt{2} + 7 - 6\sqrt{3}$ B) $6\sqrt{2} + 12\sqrt{3}$ C) $1 + \sqrt{6}$ D) $5 - 2\sqrt{6}$

25. The polynomial $x^3 - ax^2 + bx + 2$ is exactly divisible by $x^2 - 1$. Find the relationship between b and a.

A) $a + 2b = 0$ B) $a - 2b = 0$ C) $2a - b = 0$ D) $2a + b = 0$

26. In a geometric series, the second term is 12 and the sum to infinity is 54. Find the possible value of the common ratio r.

A) $r = \frac{1}{2}$ or $r = \frac{2}{3}$ B) $r = \frac{1}{3}$ or $r = \frac{1}{2}$ C) $r = \frac{1}{3}$ or $r = \frac{2}{3}$ D) $r = \frac{1}{2}$ or $r = \frac{3}{2}$

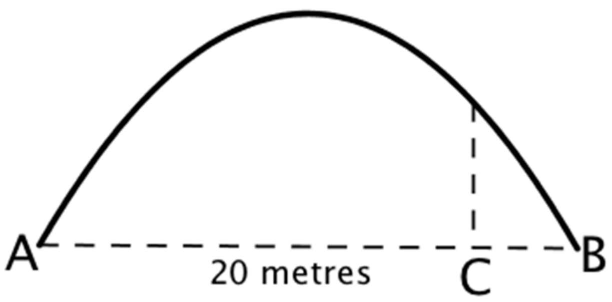
27. If $p = \sin\theta + \cos\theta$, then $\frac{1}{(p+1)(p-1)} =$

A) $\frac{1}{\sin 2\theta}$ B) $\frac{1}{2(\sin\theta+\cos\theta)}$ C) $\frac{1}{2\sin\theta}$ D) $\frac{1}{\sin 2\theta+\cos 2\theta-1}$

28. Solve for x if $2^x(x^2+4)(5^x-1) < 0$.

A) $x < 1$ B) $2 < x < 5$ C) $x > 1$ D) $x < 0$

29. A parabolic bridge spans a distance of 20 m across its base AB, and is 8 m tall at its highest point. From a 4 m high point on the bridge arch, a rope hangs vertically to touch the ground at point C. Find the distance from C to the centre of the base line AB.



A) $10 - 5\sqrt{2}$ B) $5\sqrt{2}$ C) $20 - 5\sqrt{2}$ D) $10\sqrt{2}$

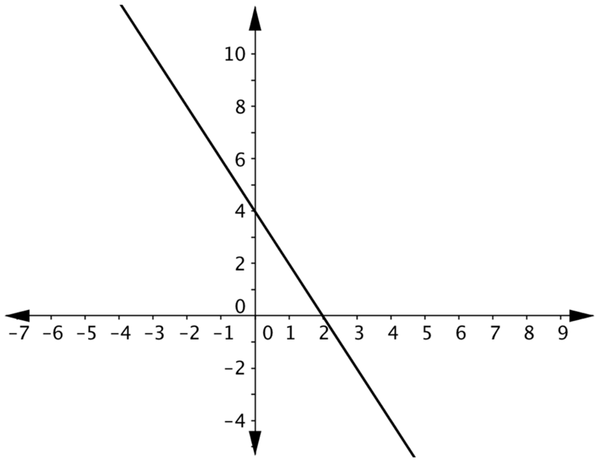
30. Evaluate $\sum_{x=6}^{35} \log\frac{x+1}{x}$

A) log 6 B) log 2 C) log 12 D) log 30

31. If $\cos 36° = m$, find $\tan 216°$ in terms of m.

A) $\frac{1-m}{m}$ B) $\frac{m-1}{m}$ C) $-\frac{\sqrt{1-m^2}}{m}$ D) $\frac{\sqrt{1-m^2}}{m}$

32. The linear graph is the graph of $f'(x)$. On which interval will the graph of $f(x)$ be decreasing?



A) $(-\infty; 2)$ B) $(2; \infty)$ C) $(0; 2)$ D) $(-\infty; \infty)$

33. The range of the function f is [–2; 8]. Find the range of the function g defined by $g(x) = 2f(-x)$.

A) [4; –16]   B) [–16; 4]   C) [2; –8]   D) [–4; 16]

34. Find the derivative of the function $f(x) = \frac{a^2}{3\sqrt{x}}$ if a is a constant.

A) $f'(x) = -\frac{a^2}{6\sqrt{x^3}}$ B) $f'(x) = -\frac{2a}{3\sqrt{x^3}}$ C) $f'(x) = \frac{2a}{3x}$ D) $f'(x) = \frac{a^2}{6x}$

35. Which one of the following would be a solution to the equation $\cos x(\sin x - 1) = 0$ for every integer n?

A) $x = n.180°$ B) $x = 90° + n.180°$ C) $x = n.90°$ D) $x = 90° + n.360°$

36. A car depreciates linearly by an annual rate of r. A machine worth half as much depreciates at the same annual rate but according to a reducing balance. If they have the same value after two years, find an equation satisfied by r.

A) $r^2 + 2r - 1 = 0$ B) $r = \frac{1-3r}{\sqrt{2r}}$ C) $\log_2 r = 2r$ D) $2r^2 - 2r + 1 = 0$

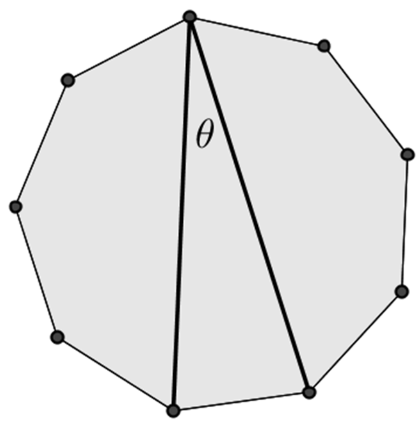
37. Three cards are drawn from a shuffled deck of 52 cards, without replacing the cards each time. What is the probability of drawing three black cards?

A) $\frac{75}{26^2}$ B) $\frac{1}{8}$ C) $\frac{27}{52^3}$ D) $\frac{2}{17}$

38. A parabola graph has its turning point at (–4; 2), with its axis of symmetry parallel to the *y*-axis. Find the value of *p* if the two points (–7; 5) and (*p*; 5) lie on the graph.

A) –10 B) –5 C) –1 D) 1

39. The diagram shows a regular polygon. Find the size of angle θ.



A) 10° B) 20° C) 24° D) 36°

40. Triangle ABC is similar to triangle DEF (in order of equal angles). Find the length of EF, if AB = 3, BC = 6, and $\frac{\text{Area } \Delta\text{DEF}}{\text{Area } \Delta\text{ABC}} = \frac{10{,}89}{9}$.

A) 6,6 B) 54 C) 27 D) 3,3

41. Five eggs with different masses are put into a basket one by one. Each time an egg is added, the average mass of the eggs in the basket increases by one gram. If the mass of the first egg is 50 grams, find the mass of the last egg.

A) 55 g B) 56 g C) 57 g D) 58 g

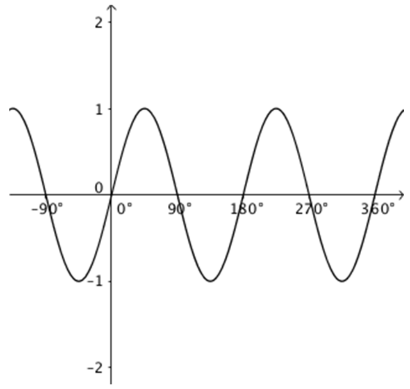
42. Find the value of *a* if $f(x) = ax + 3$ and $f\big(f(2)\big) - 3a = 11$.

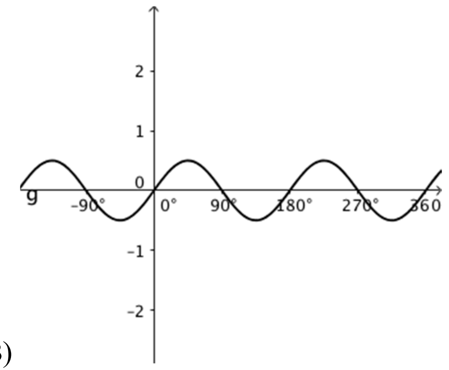
A) 2 B) 8 C) –8 D) ± 2

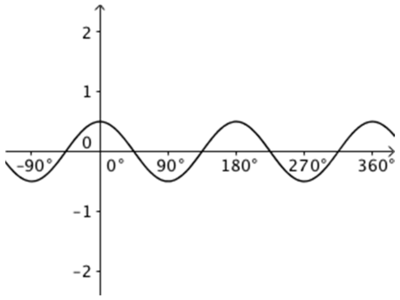
43. The shadow of a tree increases by 10 metres when the angle of elevation of the sun decreases from 60° to 30°. How tall is the tree?

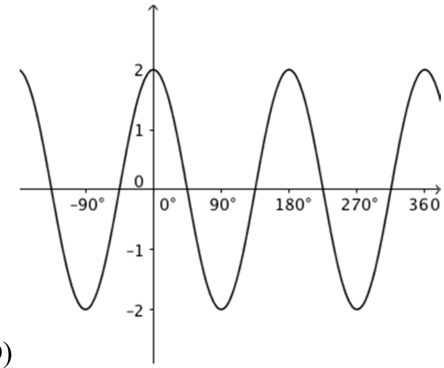
A) $5\sqrt{3}$ B) $\frac{5}{\sqrt{3}}$ C) $15\sqrt{3}$ D) $\frac{\sqrt{2}}{20}$

44. If $f(x) = \sin x$ and $g(x) = \cos x$, which of the following is the graph of $h(x) = f(x).g(x)$?

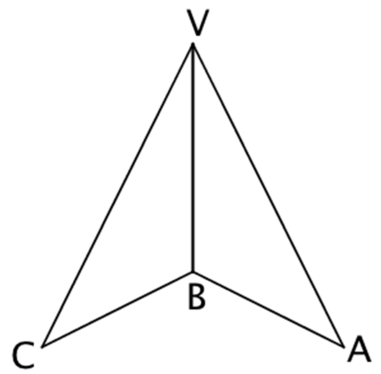
A) 

B) 

C) 

D) 

45. In the diagram, two cables AV and VC are 12 m long each, supporting a vertical pole BV. A, B and C are points in the same horizontal plane, with ∠VCB = ∠VAB = α and ∠ABC = 120° Find the length of AC in terms of α.



A) $12(2 - \sqrt{3})\cos\alpha$
B) $12(2 + \sqrt{3})\cos\alpha$
C) $12\sqrt{3}\cos\alpha$
D) $12\cos\alpha$

46. If sin θ = –0,4 and cos θ < 0, find the value of sin2θ.

A) $-\frac{4\sqrt{29}}{5}$
B) $\frac{25}{4\sqrt{29}}$
C) $-\frac{4\sqrt{21}}{25}$
D) $\frac{4\sqrt{21}}{25}$

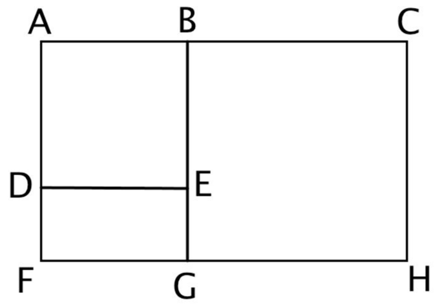
47. A farmyard collection of sheep and chickens have a total of 91 heads and legs. If there are twice as many sheep as chickens, what is the total number of chickens and sheep in the farmyard?

A) 13 B) 17 C) 21 D) 32

48. If $2^2 \times 3^3 \times 4^3 \times 5^9$ is multiplied out, what will be the sum of the digits in the answer?

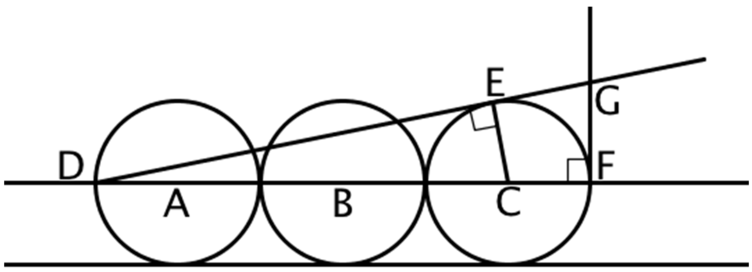
A) 5 B) 7 C) 8 D) 9

49. In the diagram, ABED and BCHG are squares. Find EG if FH = $\sqrt{7}$ and AF = 2.



A) $4-\sqrt{7}$ B) $3\sqrt{7}-4$ C) $\sqrt{7}-2$ D) $5-\sqrt{3}$

50. Three circles with centres A, B and C are tangent to each other. Each circle has a radius of 5. Lines DE and FG are tangent to circle C and intersect at G. Find the length of FG.

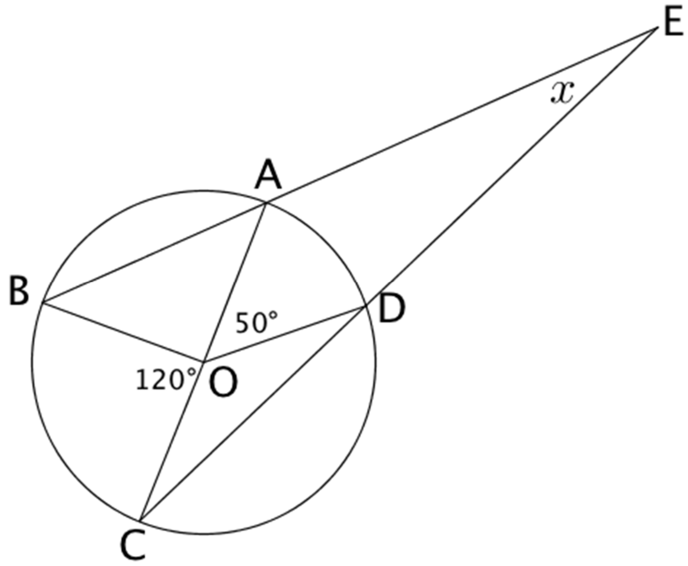


A) $5\sqrt{2}$ B) $\frac{15}{\sqrt{6}}$ C) $5\sqrt{13}$ D) $2\sqrt{5}$

51. In ΔABC, AB = AC, and point F lies on AB so that AF = FC = CB. Find the size of angle A.

A) 60° B) 30° C) 36° D) 45°

52. Use the given diagram, with circle centre O, to find the size of *x*.



A) 25° B) 28° C) 30° D) 35°

53. David only wears black or brown shoes to work. He is twice as likely to choose black shoes rather than brown shoes. He has seven white shirts and three blue shirts. Each day he chooses a shirt at random. What is the probability that he chooses to wear a blue shirt with black shoes or a white shirt with brown shoes?

A) $\frac{7}{150}$ B) $\frac{1}{15}$ C) $\frac{13}{30}$ D) $\frac{7}{30}$

54. How many real solutions does the equation $x + \sqrt{x^2 + \sqrt{x^3 + 1}} = 1$ have?

A) none B) 1 C) 2 D) 3

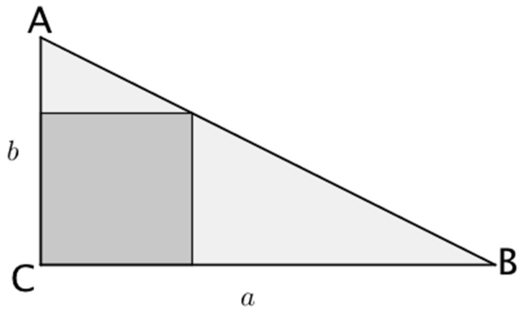
55. A hollow cubic metal box is formed using 1801 cubic centimetres of metal. If the steel walls of the box are 0,5 cm thick, the outer side length of the box will be:

A) less than 22,5 cm B) between 22,5 cm and 24,5 cm
C) between 24,5 cm and 26,5 cm D) more than 26,5 cm

56. The roots of the equation $2x^2 - 5x - 9 = 0$ are *a* and *b*. Find a quadratic equation with roots 2a and 2b.

A) $x^2 - 10x - 18 = 0$ B) $4x^2 - 10x - 18 = 0$
C) $x^2 - 5x - 18 = 0$ D) $4x^2 - 5x - 18 = 0$

57. The diagram shows right-angled triangle ABC with lengths AC = b and CB = a. Find the ratio of the area of the inscribed square to the area of triangle ABC, in terms of a and b.

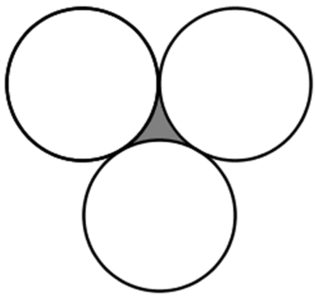


A) $\frac{ab}{a+b}$ B) $\frac{ab}{(b-1)^2}$ C) $\frac{2ab}{a^2+b^2}$ D) $\frac{2ab}{(a+b)^2}$

58. A container weighs M kg when it is full of water, and L kg when it holds one third of the volume of water. Find an expression for the weight of the empty container in terms of M and L.

$\frac{2L+M}{3}$ B) $\frac{3L-M}{2}$ C) $\frac{L-3M}{2}$ D) $\frac{L+2M}{3}$

59. The diagram shows three circles, each of radius 2 cm. The circles touch each other so that at each point of contact, two of the circles have a common tangent. Find the area of the shaded region enclosed by the three discs.



A) $12\pi - 4\sqrt{3}$ B) $4 - 2\pi$ C) $4\sqrt{3} - 2\pi$ D) $4 - 12\pi$

60. A runner runs 30 km/h slower than a cyclist, and takes 18 minutes longer to cover a distance of 10 km. The cyclist's speed will be

A) less than 45 km /h
B) between 45 and 48 km/h
C) between 49 km/h and 52 km/h
D) more than 52 km/h

NBT Exemplar questions

1. The function f defined by $y = f(x) = -x^2 + 6x - 5$ has

   A) A minimum y value and a negative y −intercept
   B) A maximum y value and a positive y −intercept
   C) A minimum y value and a positive y −intercept
   D) A maximum y value and a negative y −intercept

2. The sum of the roots of the equation $-x^2 + 6x - 5 = 0$ is

   A) -5 B) -4 C) 3 D) 6

3. The expression $\sqrt{-x^2 + 6x - 5}$ has a

   A) maximum value of 2 B) minimum value of 2
   C) maximum value of 3 D) minimum value of 3

4. If the graph of $y = -x^2 + 6x - 5$ is reflected in the x −axis and then the resulting graph is then reflected in the y −axis, the new equation is

   A) $y = -(x-3)^2 + 4$ B) $y = -x^2 - 6x - 5$
   C) $y = (x+3)^2 + 4$ D) $y = x^2 + 6x + 5$

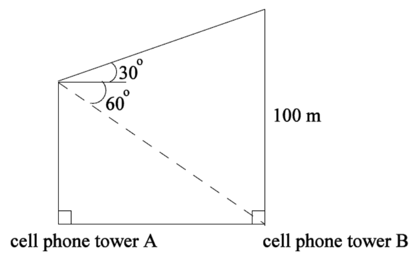
5. For any real number x, which one of the following statements is **always** true?

   A) $-x < 0$ B) $\frac{1}{x}$ is rational
   C) $\frac{x}{x+1} < 1$ D) $\frac{1}{x} > 1$ if $0 < x < 1$

6. $\sin 43° \cos 23° - \cos 43° \sin 23°$ is equal to

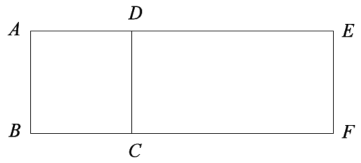
   A) $\cos 66°$ B) $\cos 20°$
   C) $\sin 66°$ D) $\sin 20°$

7. The angle of elevation of the top of cell phone tower B from the top of cell phone tower A is 30°. The angle of depression of the foot of cell phone tower B from the top of cell phone tower A is 60°. The height of cell phone tower B is $100m$. The foot of cell phone tower A and the foot of cell phone tower B are in the same horizontal plane. The height of cell phone tower A is



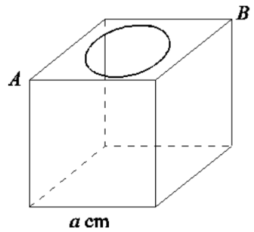
   A) $60m$ B) $65m$ C) $70m$ D) $75m$

8. Suppose $ABCD$ is a square with side length $(x-1)cm$. If the area of the rectangle $ABFE$ is $(x^2+x-2)cm^2$, then the length of FC, in cm, is



A) 2 B) 3 C) 4 D) 5

9. The figure represents an empty cube with a circular opening at the top. The diameter of the opening is half the length of the diagonal AB. The outer surface of the area of the cube (in square centimeters) is:



A) $6a^2 - \frac{\pi a^2}{2}$ B) $6a^2 - 2\pi a^2$

C) $6a^2 - \frac{\pi a^2}{8}$ D) $6a^2 - \frac{\pi a^2}{2}$

10. An amount of R1000 is invested at an annual interest rate of 6%. Interest is compounded **every 3 months** (quarterly). After 5 years the investment, in Rands, will be worth

A) $1000(1{,}015)^{20}$ B) $1000(1{,}02)^{15}$ C) $1000(1{,}03)^{20}$ D) $1000(1{,}06)^5$

These questions were taken from The National Benchmark Tests project: The NBT Mathematics (MAT) test: Exemplar Questions.

Solutions

Section A

Algebra and Exponents

Grades 10,11 and 12

1. Option A)

$$\frac{1+2\ \div 3}{\frac{2}{3}-\frac{1}{4}}=\frac{1+\frac{2}{3}}{\frac{8-3}{12}}=\frac{5}{3}\times\frac{12}{5}=4$$

2. Option D)

Each term is the sum of the two terms on its right, so $q+2=\ -1$ so $q=-3$ and then $r+(-3)=\ 2$ so $r=5$

3. Option A)

$2^{1001}-2^{1000}-2\times 2^{999}=2^{1001}-2^{1000}-2^{1000}=2^{1001}-2(2^{1000})=2^{1001}-2^{1001}=0$

4. Option A)

$$\frac{s-2}{t}=\frac{1}{p}+\frac{1}{r}\Rightarrow pr(s-2)=tr+tp$$

$$r(ps-2p-t)=tp\Rightarrow r=\frac{tp}{ps-2p-t}$$

5. Option C)

$$\left(\sqrt{x}\right)^{6}=\left(4\times\sqrt[3]{25}\right)^{3}=2^{6}\times 5^{2}=2^{4}\times 2^{2}\times 5^{2}=1\,600$$

6. Option B)

Let the distance of each part of the trip be d. The times for each part of the trip are given by $t_1=\frac{d}{y};\ t_2=\frac{d}{x}$. Then $Av\ speed=\frac{total\ distance}{total\ time}=\frac{2d}{\frac{d}{y}+\frac{d}{x}}=\frac{2dxy}{dx+dy}=\frac{2xy}{x+y}$. (Note that you cannot average the two speeds directly, as she spends different periods of time driving each of the two different speeds.)

7. Option B)

Let the integers be x and y so that $x+y=m$ and $xy=n$. Then $x^2+y^2=(x+y)^2-2xy=m^2-2n$.

8. Option B)

$2^{x+2}+2^{x}=5^{y+1}-5^{y}\Rightarrow 2^{x}(2^{2}+1)=5^{y}(5^{1}-1)\Rightarrow 5\times 2^{x}=4\times 5^{y}$

Hence, $2^{x-2}=5^{y-1}$. Since 2 and 5 are prime numbers, this is only possible if $x-2=y-1=0$, which yields $x=2, y=1$ and $x-y=1$.

9. Option D)

$\frac{n+3}{n-1}=1+\frac{4}{n-1}$, which is only an integer if $n-1=\pm1,\pm2$ or ± 4. Hence, there are 6 different values for n.

10. Option B)

Rate of brick laying for four men: 3 200 bricks: 6 hr = 800 bricks: 1,5 hr.
Time for three men to lay 800 bricks is $1{,}5\times\frac{4}{3}=\frac{3}{2}\times\frac{4}{3}=2$ hours.
OR: One man can lay $\frac{3200}{4\times6}$ bricks in one hour. Three men can lay $\frac{3\times3200}{4\times6}=400$ bricks in one hour. Hence three men will take two hours to lay 800 bricks.

11. Option D)

Let the original amount of money be x. Then

$0{,}2x+0{,}1x+0{,}4(x-0{,}2x-0{,}1x)+168=x$

$0{,}3x+0{,}28x+168=x$

$30x+28x+16800=100x$

$x=\frac{16800}{42}=\frac{8400}{21}=400$

The money spent on paper plates is $0{,}1\times 400=$ R40.

12. Option D)

$4(9^{n})-9^{n}=3^{801}\Rightarrow 3(3^{2})^{n}=3^{801}\Rightarrow 2n+1=801\Rightarrow n=400$

13. Option A)

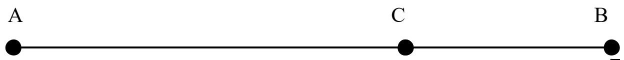
$5\otimes 2=5-2\times 2=1; 7\otimes 1=7-2\times 1=5$

14. Option B)
Substitute the solutions to obtain the equations $8 + 2b = c$ and $-27 - 3b = c$. Solve simultaneously to find that $b = -7, c = -6$, and $b + c = -13$.

15. Option A)
Let $1005 = x$, so that $1005^2 - 1004 \times 1006 = x^2 - (x-1)(x+1) = 1$

16. Option C)
Let Sipho have x Rand and Maria have y Rand, so that $\frac{x}{y} = \frac{3}{4}$ or $4x = 3y$. After their spending, we have $\frac{x-100}{y-100} = \frac{1}{2}$, so that $2x - 200 = y - 100$. Solving simultaneously yields $4x = 3(2x - 100)$, from which we obtain $x = 150, y = 200$ and the original total is R350.

17. Option A)



Let AC = x. Then $\frac{x+1}{x} = \frac{x}{1}$. Simplify to obtain $x^2 - x - 1 = 0$, hence $x = \frac{1+\sqrt{5}}{2}$. (Only one root provides a positive answer.)

18. Option D)
x = (199 – 197) + (195 – 193) + (191 – 189) + ... (7 – 5) + (3 – 1) = 50 × 2 = 100. (Notice that the number of brackets is also the number of terms in the arithmetic sequence 1; 5; 9; ...; 197.)

19. Option B)
Solve the equations simultaneously. From equation 1), $y = 2 + x$, so equation 2) becomes $2 + x - 3x + 2 = 0$, hence $x = 2$ and $y = 2 + x = 4$. Equation 3) then yields $3(4) - k(2) - 5 = 0$, so $k = 3{,}5$.

20. Option B)
Let $n + 1 = k^2$, so that $n = k^2 - 1 = (k+1)(k-1)$. As n is prime, its factors can only be itself and 1, so we deduce that $k - 1 = 1$ and $k + 1 = n$, which yields a unique solution $k = 2, n = 3$. (Notice that the alternative choice $k + 1 = 1$ and $k - 1 = n$ yields $k = 0, n = -1$ but prime numbers are natural numbers by definition.)

Grades 11 and 12

21. Option C)
$4x^2 < 8x \Rightarrow 4x(x-2) < 0 \Rightarrow 0 < x < 2$

22. Option D)
$4x^2 - 16x + 20 = 4(x^2 - 4x + 5) = 4[(x-2)^2 + 1] = 4(x-2)^2 + 4$
This quadratic expression has a minimum value of 4, hence the square root expression has a minimum value of 2.

23. Option C)
As $(3 - x)^2 \geq 0$, we deduce that $\left(\frac{-4}{x}\right) < 0$. This fraction can only be negative if the denominator is positive, hence $x > 0$.

24. Option A)
$$\frac{3\sqrt{12} - \sqrt{27}}{\sqrt{24}} = \frac{6\sqrt{3} - 3\sqrt{3}}{2 \times \sqrt{2} \times \sqrt{3}} = \frac{3\sqrt{3}}{2 \times \sqrt{2} \times \sqrt{3}} = \frac{3}{2\sqrt{2}}$$

25. Option A)
$x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$. The product of the roots is $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$.

26. Option C)
$(\sqrt{x})^4 = 3\sqrt{x} \Rightarrow x^2 - 3x^{1/2} = 0$
$x^{1/2}(x^{3/2} - 3) = 0$
$x = 0 \text{ or } x = 3^{2/3} = \sqrt[3]{9}$

27. Option D)
The maximum value of $(-2 - (x-3)^2)$ is –2, so this expression is always negative. The exponential expression 3^x is always positive, so the solution will be found by $x(x+2)(x-2) \geq 0$. By a table of signs, this expression is positive for $-2 \leq x \leq 0$ or $x \geq 2$.

Grade 12

28. Option A)

The series is the sum of the first p odd numbers: $1 + 3 + 5 + \ldots (2p - 1)$. By the arithmetic sum formula, $S_p = \frac{p}{2}\{2(1) + 2(p-1)\} = p^2$.

29. Option B)

$f(-1) = 2 - k - b = -3$ and $f(1) = 2 - k + b = 5$. Adding the two equations yields $4 - 2k = 2$, so $k = 1$.

30. Option D)

$\log_x y^0 = 0$ (written as $\log_x 1$ or as $0 \times (\log_x y)$), hence the equation becomes $\log_y(x^4 - 3) = 0$; $x^4 - 3 = 1 \Rightarrow x^4 = 4 \Longrightarrow x = \sqrt[4]{4} = \pm\sqrt{2}$. But we can't have a negative base so $x = \sqrt{2}$.

31. Option B)

$a = \log_2 3 = \frac{\log 3}{\log 2}$ and $b = \log_9 16 = \frac{\log 16}{\log 9}$; hence $ab = \frac{\log 3}{\log 2} \times \frac{4 \log 2}{2 \log 3} = 2$

32. Option B)

By factorising, the series becomes $(52 + 51)(52 - 51) + (50 + 49)(50 - 49) + (48 + 47)(48 - 47) + \cdots + (2 + 1)(2 - 1)$. As the second bracket in each term is always 1, the series becomes $52 + 51 + 50 + 49 + \cdots + 1$. The arithmetic sum is $\frac{52}{2}[2(1) + 51(1)] = (26)(53)$.

33. Option A)

In 10 years' time, the account will have $12\,000\left(1 + \frac{0{,}07}{12}\right)^{120} - 8\,000$.

In another four years' time, the account will have

$\left[\left(12\,000\left(1 + \frac{0{,}07}{12}\right)^{120} - 8\,000\right)\left(1 + \frac{0{,}07}{12}\right)^{48}\right] - 8\,000$.

34. Option C)

$a = b^{x^2} \Rightarrow x^2 = \log_b a = \frac{\log a}{\log b}$, hence $x = \pm\sqrt{\frac{\log a}{\log b}}$

35. Option C)

$2 + 4 + 6 + \cdots + 2p = 420$, hence $1 + 2 + 3 + \cdots + p = 210$. Use the formula for the sum of an arithmetic series: $210 = \frac{p}{2}\{2 + (p-1)(1)\}$ yields $420 = p(1 + p)$, so $p^2 + p - 420 = 0$ or $(p - 20)(p + 21) = 0$, so $p = 20$ $(p > 0)$.

36. Option D)

$\frac{\log x}{\log 2} = \frac{\log(x + 12)}{2 \log 2} \Rightarrow 2 \log x = \log(x + 12) \Rightarrow x^2 = x + 12$

Solve $(x - 4)(x + 3) = 0$ to find $x = 4$ $(x \neq -3$ since you can't have the log of a negative number).

37. Option A)

The ninth term is the sum of the first nine terms subtract the sum of the first eight terms.

$$\sum_{n=1}^{9} T_n = 3(9)^2 - 1 = 242; \sum_{n=1}^{8} T_n = 3(8)^2 - 1 = 191; \; 242 - 191 = 51$$

Functions and Graphs

Grades 10,11 and 12

38. Option B)

For p>-6, the distance between the axis of symmetry and the given x-intercept is $p - (-6) = p + 6$. Add this distance to the axis of symm value to find the other x-intercept: $p + (p + 6) = 2p + 6$. For p<-6, the distance between the axis of symm and the given x-intercept is $-6 - p$. Subtract this distance from the axis of symmetry value to find the other x-intercept: $p - (-6 - p) = 2p + 6$.

OR: The axis of symmetry lies in the middle of the two x-intercepts, so $\frac{x_2 + (-6)}{2} = p$, hence $x_2 = 2p + 6$.

39. Option D)

a must be negative to reflect the increasing graph of $y = 3^x$ in the x-axis, and b must be positive to shift the horizontal asymptote above the x-axis.

40. Option A)

$$f(f(x)) = 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{x}{x - 1} = \frac{x - 1 - x}{x - 1} = \frac{1}{1 - x}$$

41. Option C)

The gradient of line EF is –1 (parallel to AB). AB = $\sqrt{16 + 16} = 4\sqrt{2}$. Hence, E is the point $(8 + 4\sqrt{2}; 4 + 4\sqrt{2})$. Substituting into the equation of line EF $(y = -x + c)$ yields $4 + 4\sqrt{2} = -(8 + 4\sqrt{2}) + c$, hence $c = 12 + 8\sqrt{2}$.

42. Option C)

$yf(xy) = f(x) \Rightarrow f(xy) = \frac{f(x)}{y}$. So $f(40) = f\left(30 \times \frac{4}{3}\right) = \frac{f(30)}{4/3} = \frac{3}{4} \times 20 = 15$

OR: Choose a value for x and work from there; for example, let $x = 1$ so that $f(y) = \frac{f(1)}{y}$; then $20 = \frac{f(1)}{30} \Rightarrow f(1) = 600$, and $f(40) = \frac{600}{40} = 15$.

Grades 11 and 12

43. Option C)

$a > 0$ to have a range going to positive infinity; $b > 0$ so that the axis of symmetry has $\frac{-b}{2a} < 0$; $b^2 - 4ac > 0$ to have real roots

44. Option D)

The reflection replaces y with $-y$, and the translation replaces x with $(x+3)$. The equation becomes $3(-y) - 2(x+3) + 4 = 0$, or $3y + 2x + 2 = 0$.

45. Option B)

The range of $y = \sin x$ will be doubled to lie between –2 and 2, and then the graph will be shifted down by 1 unit to lie between –3 and 1.

46. Option B)

$f(x) = 4px^2 + x(-6p - 6) + 3$; the turning point of the parabola occurs at $x = -\frac{b}{2a} = \frac{6p+6}{8p} = \frac{3p+3}{4p}$.

47. Option B)

The graph of $y = \cos x$ has been vertically stretched by a factor of 2, and shifted to the right by 30°.

48. Option A)

Write the equation in the form $y = -(x^2 - 6x + 9) - 2 = -(x-3)^2 - 2$. This is a parabola graph with a maximum value of –2, hence the range is $y \in (-\infty;\ -2]$.

49. Option A)

The original equation had 3 units added to x, and the sign of y was changed. Undo these to obtain the equation $-y = \frac{2}{(x-3)-1} \Rightarrow y = \frac{-2}{x-4} = \frac{2}{4-x}$.

50. Option D)

The range of $g(x) = \cos 2x$ is $y \in [-1; 1]$; max(f) = 3÷min(g) and min(f) = 3÷max(g); the graph of $f(x)$ will tend to infinity at the zeros of $g(x)$; the range of the reciprocal function will be $y \in (-\infty; -3]$ or $[\,3;\ \infty)$.

Grade 12

51. Option B)

$$f(x) = \frac{3}{2\sqrt{x}} = \frac{3}{2}x^{-\frac{1}{2}}$$
$$f'(x) = -\frac{3}{4}x^{-\frac{3}{2}}$$
$$f'(9) = -\frac{3}{4} \times \frac{1}{9^{\frac{3}{2}}} = -\frac{3}{4} \times \frac{1}{27} = -\frac{1}{36}$$

52. Option A)

Swop variables and solve for y: $x = \frac{2}{\frac{1}{y}-3} \times \frac{y}{y} \Rightarrow x = \frac{2y}{1-3y} \Rightarrow x - 3xy - 2y = 0$, hence $y = \frac{x}{3x+2}$.

53. Option D)

From the turning points of the graph, $f'(-1) = f'(2) = 0$, and hence $x - 2$ is a factor of $f'(x)$.

54. Option B)

$f'(x) = 3kx^2 + 1$, and the gradient of the tangent line is given by $f'(-1)$. Hence, $3k(-1)^2 + 1 = 7$, so $k = 2$ and $f(-1) = 2(-1)^3 + (-1) = -3$. Substituting the point $(-1;\ -3)$ we get $-3 = 7(-1) + p$ so $p = 4$

55. Option B)

From $f^{-1}(4) = 0$, we have $f(0) = 4$. Hence, if $f(f(0)) = 2$, then $f(4) = 2$. Substitute both of these facts into $y = mx + c$: $4 = 0 + c$ and $2 = 4m + c$, giving $c = 4, m = -\frac{1}{2}$, so that the linear function is $f(x) = -\frac{1}{2}x + 4$, and $f(6) = -\frac{1}{2}(6) + 4 = 1$.

56. Option C)
$4x - 6 > 0 \Rightarrow x > 1{,}5$

57. Option C)
$f'(k) = 0$ at the three turning points of the graph.

58. Option C)
$x^2 + 2x + 2 = (x+1)^2 + 1$, which is always positive (with a minimum value of 1), and the sign of $f'(x)$ will not change at $x = -4$, so the only turning point will occur at $x = 2$.

59. Option D)

Solve simultaneously: $\frac{-4}{x} = 5 - x^2 \Rightarrow x^3 - 5x - 4 = 0$. Factorise using the factor theorem to obtain $(x+1)(x^2 - x - 4) = 0 \Rightarrow x = -1$ or $\frac{1 \pm \sqrt{17}}{2}$. The positive root is $\frac{1+\sqrt{17}}{2}$.

60. Option C)

The circle's centre is at the origin, and will intersect the hyperbola along the line of symmetry $y = x$, i.e. at the points $(\pm 2\sqrt{2};\ \pm 2\sqrt{2})$. The radius of the circle is therefore $\sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{8+8} = 4$, and the equation is $x^2 + y^2 = 16$

61. Option A)
$f'(x) = 8x^3 - 8 = 8(x^3 - 1) = 8(x-1)(x^2 + x + 1)$

62. Option A)
$x = \log(t-1); y = 3t$; hence $x = \log\left(\frac{y}{3} - 1\right) \Rightarrow 10^x = \frac{y}{3} - 1 \Rightarrow y = 3(10^x + 1)$

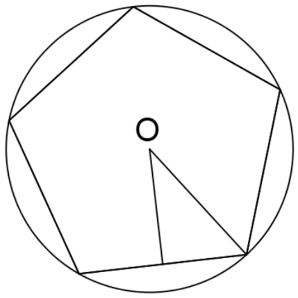
Trigonometry

Grades 11 and 12

63. Option A)
$\cos^2 20° + \sin 70°.\cos 200° = \cos^2 20° + \cos 20°(-\cos 20°) = 0$

64. Option A)
Area = ½ $b.c$.sinA = ½ × $\sqrt{3}$ × $\sqrt{3}$ × sin 120° $= \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$.
OR: Use trig to first find the base and height of the triangle.

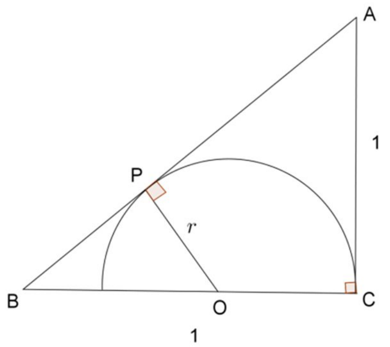
65. Option D)



The sides of the pentagon are each 50 ÷ 5 = 10 cm. Each interior angle of the pentagon will be $\frac{3 \times 180°}{5} = 108°$. In the right-angle triangle, the base is 5 cm and angle O = 180° – 90° – ½ (108°) = 36°. Then radius $r = \frac{5}{\sin 36°}$.

66. Option C)
$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 2 \Rightarrow \frac{1+\sin\theta}{\cos\theta} = 2$. From this we obtain $\sin\theta = 2\cos\theta - 1$. We also obtain $\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = 2 \Rightarrow \frac{\cos\theta}{1-\sin\theta} = 2$. Hence, $\cos\theta = 2 - 2\sin\theta = 2 - 2(2\cos\theta - 1)$, and solving yields $\cos\theta = \frac{4}{5}$.

67. Option D)

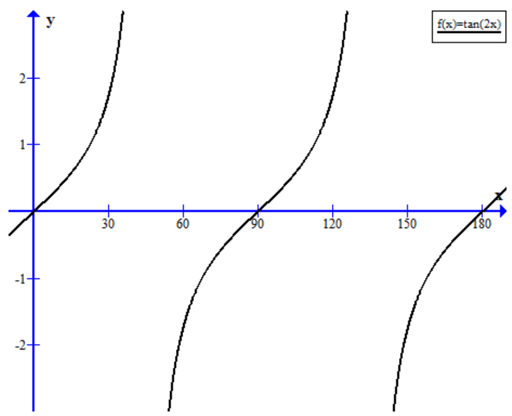


Draw radius OP perpendicular to tangent APB. Then AP = AC = 1 (equal tangents), and ΔPBO is isosceles as ∠B = 45°. Then by Pythagoras, $AB = \sqrt{2}$, and $r = OP = BP = AB - AP = \sqrt{2} - 1$.

68. Option A)
$\cos 110° = -\cos 70° = -\sin 20° = -p$

69. Option C)
$$\frac{\sin 300°}{\tan 240°} = \frac{-\sin 60°}{\tan 60°} = -\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = -\frac{1}{2}$$

70. Option D)



Choose the intervals where the graph of $y = \tan 2x$ lies below the x-axis:

$x \in (45; 90)$ or $(135; 180)$.

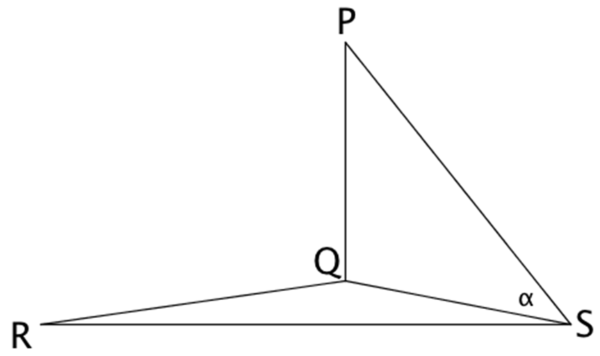
71. Option A)
$$\tan(180° + x) - \frac{\cos x}{\sin(180° + x)} = \tan x - \frac{\cos x}{-\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{2}{\sin 2x}$$

Grade 12

72. Option C)
$\sin\theta = \frac{1}{m} = \frac{y}{r}$; $x = \pm\sqrt{m^2 - 1}$; $\cos 2\theta = 2\cos^2\theta - 1 = 2.\frac{m^2-1}{m^2} - 1 = \frac{m^2-2}{m^2}$

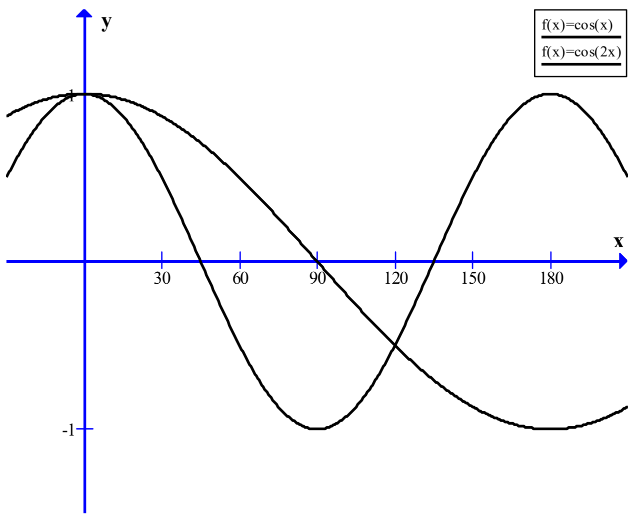
73. Option C)
$\cos 2x \cos x - \sin 2x \sin x = 0 \Rightarrow \cos(2x + x) = 0 \Rightarrow 3x = 90° + k.180°$ (using the zeros of the cosine graph), hence $x = 30° + k.60°$. (A similar result can be obtained by writing the equation in the form $\tan 2x = \cot x$ and solving for tangent in $\frac{2t}{1-t^2} = \frac{1}{t}$ to find $\tan x = \pm\frac{1}{\sqrt{3}}$; *however*, this division form of the equation will exclude angles which create zero divisors or an undefined tangent function.)

74. Option B)



In ΔPQS, $h = \text{PQ} = \text{QS}\tan\alpha$. Using the cosine rule in ΔRQS with RQ = RS = x, we obtain $\text{QS}^2 = 2x^2 - 2x^2\cos(180° - 2\theta) = 2x^2(1 + \cos 2\theta)$. Substituting into the first result yields $h = \sqrt{[2x^2(1 + \cos 2\theta)]}\tan\alpha = x\sqrt{2(1 + \cos 2\theta)}\,\tan\alpha$.

75. Option D)



Choose the interval where the two graphs have opposite signs (one above and one below the x-axis) or a zero value: $x \in [45°; 90°]$ or $x \in [135°; 180°]$

76. Option B)
$\sin(2x) - 2\sin^3 x\cos x = 2\sin x\cos x - 2\sin^3 x\cos x = 2\sin x\cos x\,(1 - \sin^2 x) = 2\sin x\cos x\cos^2 x = 2\sin x\cos^3 x$

77. Option B)
$\tan\beta = 0{,}3 = \frac{3}{10} = \frac{y}{x}$, so $r = \sqrt{109}$, and $\cos 2\alpha = 2\cos^2\alpha - 1 = 2\sin^2\beta - 1$ (as $\cos\alpha = \sin(90° - \alpha) = \sin\beta$). Hence, $\cos 2\alpha = 2\left(\frac{3}{\sqrt{109}}\right)^2 - 1 = \frac{18}{109} - 1 = -\frac{91}{109}$.

Geometry and Measurement

Grades 10,11 and 12

78. Option D)
Area of triangle ABC = ½ BC.AN = ½ (10)(8) = 40 cm^2. Also, Area of triangle ABC = ½ AC.BM, so that ½ (12)(BM) = 40, so BM $= \frac{40}{6} = 6\frac{2}{3}$ cm.

79. Option D)
The sprinkler waters a circular region with an area of πr^2. The area of the square field is $(2r)(2r) = 4r^2$, so the unwatered part is $4r^2 - \pi r^2$. As a fraction of the square area, this is $\frac{r^2(4-\pi)}{4r^2} = \frac{4-\pi}{4}$.

80. Option D)
Using the scale factors, the width of the larger cuboid must be $3k \times \frac{2}{4}$, and the length must be $3k \times \frac{10}{4}$. Hence, the volume is $3k \times 3k \times \frac{1}{2} \times 3k \times \frac{5}{2} = \frac{135k^3}{4}$.

81. Option C)
Let the length of the lawn be x. Then the area of the path is $(x + 2)^2 - x^2 = 40$
Hence, $4x + 4 = 40$, so $x = 9$ and the area of the lawn is 81 m^2.

82. Option B)

When the tank is tipped forward, Volume = triangular cross-section area × length = $\left(\frac{1}{2}\times\frac{2}{3}.30\times 7\right)\times 50 = 70\times 50$. Hence, Initial Volume = $50\times 30\times$ depth = 70×50. Solve for depth: $\frac{70\times 50}{50\times 30} = \frac{7}{3}$ metres.

83. Option B)

The large cube will have 4 smaller cubes along each length. In the middle of each of the 12 edges of the large cube will be two cubes painted on exactly two faces: $12\times 2 = 24$ cubes.

84. Option D)

The equation of the circle is $(x-3)^2+y^2=3^2$. Substitute $y=2$: $(x-3)^2+4=9$ yields $(x-3)^2=5$, so that $x=3\pm\sqrt{5}$. Point C has the larger x-value of $3+\sqrt{5}$.

85. Option D)

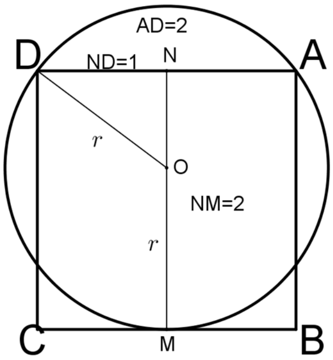
The radius of the circle is $\frac{10}{2}=5$, and the centre is the midpoint $\left(0;\ \frac{8+(-2)}{2}\right)=(0;3)$, so the circle equation is $x^2+(y-3)^2=25$. Let $y=0$ to find B(–4 ; 0) and C(4; 0). Then AC $=\sqrt{8^2+4^2}=4\sqrt{5}$.

86. Option B)

The possible dimensions of the cuboid are: $1\times 1\times 12$; $2\times 1\times 6$; $3\times 1\times 4$; $2\times 2\times 3$

87. Option A)

Draw NOM through midpoints N, M and circle centre O, with radius = r.



In ΔNOD, ON = $\sqrt{r^2-1}$; hence as NM = 2 and NM = NO + OM, we have $\sqrt{r^2-1}+r=2$, or $\sqrt{r^2-1}=2-r$

Square and solve $r^2-1=4-4r+r^2$ to obtain $r=\frac{5}{4}$. (Check by substitution that this does satisfy the original equation and is not an extraneous root introduced by squaring.)

88. Option D)

The height of the cylinder will be x, and the radius will be $\frac{x}{2}$, so the volume is

$$V=\pi r^2 h=\pi\frac{x^2}{4}.x=\pi\frac{x^3}{4}$$

89. Option C)

The radius of the circle will be the length AC, which is the diagonal of the rectangle ABCD. AC = $\sqrt{6^2+2^2}=\sqrt{40}$. The area of the circle is $\pi r^2=40\pi$.

90. Option C)

The number of triangles formed within the polygon is $\frac{2340}{180}=13$, so there are 15 sides.

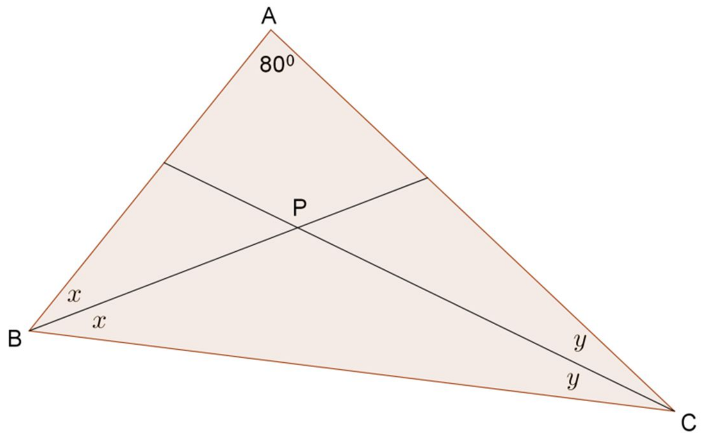
91. Option B)

In ΔABC, BC $=\frac{9}{\tan 30°}=9\div\frac{1}{\sqrt{3}}=9\sqrt{3}$. In ΔBDC, BD $=\sqrt{\left(9\sqrt{3}\right)^2+36}=\sqrt{3(81)+36}=3\sqrt{27+4}=3\sqrt{31}$.

92. Option A)

The area of the square is s^2, so the circle area = $\pi r^2=s^2\Rightarrow r=\sqrt{\frac{s^2}{\pi}}=\frac{s}{\sqrt{\pi}}$.

93. Option C)



From the fact that $80° + 2x + 2y = 180$, we have $x + y = 50°$, hence ∠BPC = 130°.

94. Option A)

189/7 cubes = 27 cm^3, so each cube has a side of 3 cm and a face area of 9 cm^2. The surface area of the solid consists of 30 square faces, so the surface area is 270 cm^2.

95. Option B)

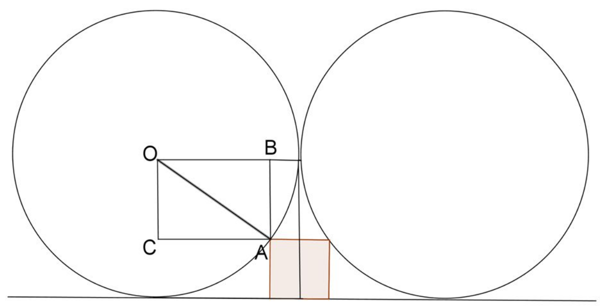
Let the radius of each circle be x. Then the length of the smaller square is $2x$ and the length of the larger square is $4x$. The sides are in the ratio 1 : 2, so the areas will be in the ratio $1^2 : 2^2$ = 1 : 4.

96. Option A)

The area of a rhombus is the half the product of the diagonals, so that $x(x - 8) = 48$, or $(x - 12)(x + 4) = 0$, so that the longer diagonal is 12 ($x \neq -4$), the shorter diagonal is 12 – 8 = 4, and the sum of the lengths is 16.

OR: Use the half of the diagonal lengths (x and x – 8) as the bases of the triangles formed by the diagonals.

97. Option B)



Draw a vertical line from the point of contact of the two circles, through the middle of the square, and draw rectangle OBAC to touch the square at A. Then OA = 1, and if the length of the square is x, OC = 1 – x and OB = 1 – ½ x. By Pythagoras we have $(1 - x)^2 + \left(1 - \frac{x}{2}\right)^2 = 1$, so $x^2 + \frac{x^2}{4} - 3x + 2 = 1$ or $5x^2 - 12x + 4 = 0$. This factorises as $(5x - 2)(x - 2) = 0$, from which it follows that $x = \frac{2}{5}$ (x = 2 is too large).

Grades 11 and 12

98. Option B)

The area of the square is $4x$, so the side AB will be $\sqrt{4x} = 2\sqrt{x}$. Then by Pythagoras, the diagonal is given by $\sqrt{\left(2\sqrt{x}\right)^2 + \left(2\sqrt{x}\right)^2} = \sqrt{8x} = 2\sqrt{2x}$.

99. Option D)

∠BDC = 30° (angle between tan and chord = angle in alternate segment)

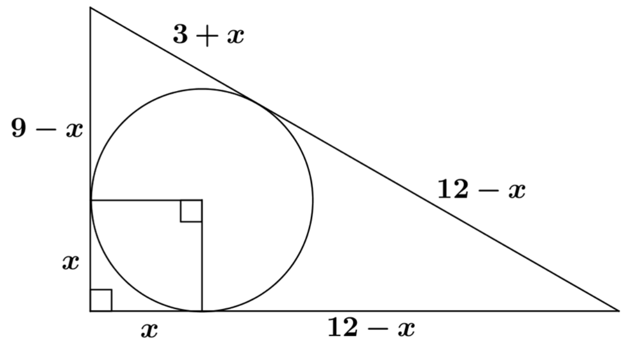
∠ABD = ∠BDC = 30° (alternate angles, BA // CD)

∠A = 100° (opp angles of a cyclic quad are supplementary)

∠ADB = 50° (180° in triangle ABD)

100. Option B)

The triangle is right-angled, with sides in the ratio 3: 4: 5.



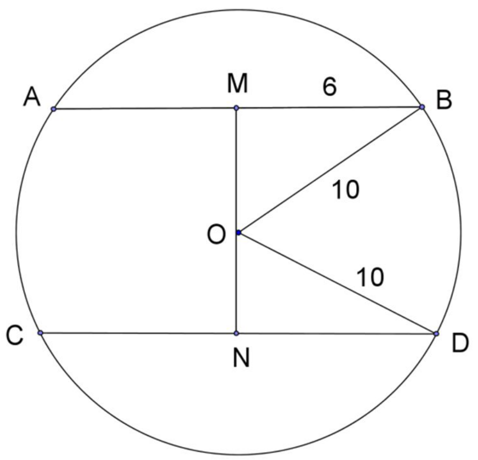
Let the radius of the circle be x and notice the square that is formed, as each radius is perpendicular to the tangent line. Using equal tangents to the circle, we have $9-x+12-x=15$, and hence $x = 3$.

101. Option D)

$\angle$DBC = 90° (subtended by a diameter)
$\angle$ADB = 40° (opp angles of cyclic quad ABCD are supplementary)
$\angle$PAB = $\angle$ADB = 40° (angle bet tan and chord = angle in alternate segment)

102. Option D)

Construct a line perpendicular to the parallel chords, through the centre of the circle so that M and N are midpoints of the chords. Then by Pythagoras, OM = 8 cm, ON = 14 – 8 = 6, and ND = 8 cm, so CD = 16 cm.



Grade 12

103. Option A)

With DE // CF in ΔACF, $\frac{\text{AD}}{\text{DC}} = \frac{\text{AE}}{\text{EF}} \Rightarrow \text{EF} = \frac{3\times 3}{4{,}5} = 2$. With ΔBEG /// ΔBFC, $\frac{\text{FC}}{\text{EG}} = \frac{\text{BF}}{\text{BE}} = \frac{5}{7} = 5:7$.

104. Option C)

From ΔABC /// ΔMPC, we have $\frac{\text{MP}}{\text{MC}} = \frac{\text{AB}}{\text{AC}}$, so $\text{MP} = \frac{1\times a}{\sqrt{(2a)^2-1}} = \frac{a}{\sqrt{4a^2-1}}$

105. Option D)

By the distance formula, $\text{OP}^2 = 13^2 = (-1-4)^2 + (b+2)^2$, hence $b^2+4b+4 = 169-25$, so $b^2+4b-140=0$, or $(b-10)(b+14)=0$. The sum of the roots 10 and –14 is –4 .

106. Option C)

If $\angle$B = x, then $\angle$BAC = 90° – x = $\angle$D, so triangles BAC and ADC are similar. Hence, $\frac{\text{AC}}{\text{BC}} = \frac{\text{DC}}{\text{AC}}$, so $\text{AC}^2 = \text{BC} \times \text{DC} = 72$, and AC = $6\sqrt{2}$.

107. Option C)

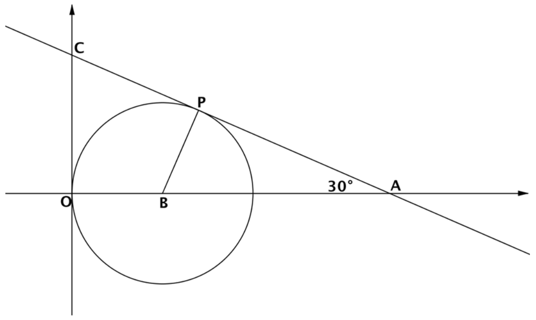
The circle will have its centre at the midpoint of diagonals BD and AC, which is $\text{M}\left(\frac{12+8}{2}; \frac{12+6}{2}\right) = \text{M}(10; 9)$. The radius of the circle will be MC = MD = $\sqrt{(10-8)^2+(9-6)^2} = \sqrt{13}$. Hence, the circle equation is $(x-10)^2+(y-9)^2 = 13$.

108. Option D)

From BC // EA in triangle EMA, we have $\frac{MC}{CA} = \frac{MB}{BE} = \frac{1}{2}$, or CA = $\frac{2}{3}$MA. Also, MA = $\frac{3}{8}$MN, so that MA : AN = 3 : 5. Hence, CA = $\frac{2}{3}$MA = $\frac{2}{3} \times \frac{3}{5}$AN = $\frac{2}{5}$AN. As BC // DA in triangle BCN, we thus have BD : DN = CA : AN = 2 : 5.

109. Option D)

The circle has its centre at B(2; 0) and a radius of 2. Draw radius BP = 2 perpendicular to the tangent line AC.



In right-angled triangle BPA, AB = $\frac{2}{\sin 30°} = 2 \div \frac{1}{2} = 4$. In right-angled triangle OCA, OC = $OA\tan 30° = 6.\frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$.

Data and Probability

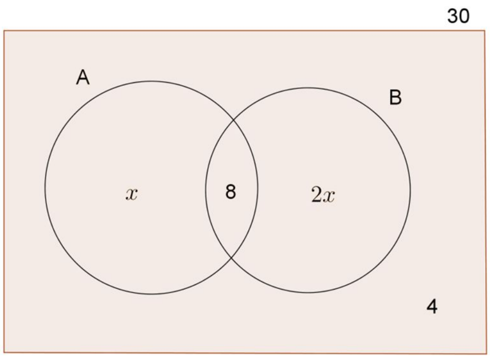
Grades 10,11 and 12

110. Option D)

Find the 15th number, which is 27.

111. Option A)

Use a Venn diagram to form the equation $3x + 12 = 30$, so that $x = 6$ and the number of people reading newspaper A is $x + 8 = 14$.



112. Option A)

For a winning round, the third card must be 7, 8 or 9 of any of the four suits. The probability is thus $\frac{3\times 4}{50} = \frac{6}{25}$.

113. Option B)

Let the number of boys be B and the number of girls be G. Then $\frac{Mass_B + Mass_G}{B+G} = 165$; $\frac{Mass_B}{B} = 172$; $\frac{Mass_G}{G} = 160$. Hence $Mass_B + Mass_G = 165(B+G) = 172B + 160G$. Simplify to obtain $5G = 7B$, and the ratio B : G = 5 : 7.

114. Option B)

The middle number will remain in the same position, but will be increased by 2.

Logic

Grades 10,11 and 12

115. Option A)

From the ratio $x - y : x + y : xy = 1 : 3 : 4$, we can form two equations $\frac{x+y}{x-y} = \frac{3}{1}$ and $\frac{xy}{x+y} = \frac{4}{3}$. From the first equation we obtain $x + y = 3x - 3y$, hence $x = 2y$. Substituting into the second equation yields $\frac{2y^2}{3y} = \frac{4}{3}$, or $6y(y-2) = 0$. As y is positive, we obtain the smaller number $y = 2$ $(x = 4)$.

116. Option C)

As B is common, we need A + C = D + E. The possible combinations are 2 + 3 = 1 + 4, 2 + 5 = 3 + 4 and 1 + 5 = 2 + 4. Hence there are correspondingly three possibilities for the middle number B: 5, 1 and 3. For each of these three choices, there will be four choices for A (with C following) and then two choices for D (with E following). The total number of possibilities is thus 3 × 4 × 2 = 24. (The same result will follow if, for each choice of B, the arrangement is rotated into four positions and each result reflected into four new arrangements, so that the total number of arrangements is 3 × 8 = 24.)

117. Option A)

Look for a pattern in the last digits of the powers: $7^1 = 7; 7^2 = 49;\ 7^3 = \cdots 3;\ 7^4 = \cdots 1;\ 7^5 = \cdots 7$. This repeats the pattern of end digits 7; 9; 3; 1; 7; 9; 3; 1… in cycles of 4 different digits. 77 = 4(19) + 1, so the last digit will be a 7.

118. Option B)

The smallest value will have the smallest numerator and largest denominator: $\frac{3}{9}$.

The largest value will have the smallest denominator and largest numerator: $\frac{6}{7}$.

The sum of the two values is $\frac{3}{9} + \frac{6}{7} = \frac{75}{63}$.

Section B

NBT Prep test 1

1. Option D)

$3^{n+1} - 3^n + 3^{n-1} = 3^n\left(3^1 - 1 + \frac{1}{3}\right) = \frac{7}{3}.3^n = 7.3^{n-1}$

2. Option B)

$Area = \frac{1}{2}AC.BC.\sin C;\ 18 = \frac{1}{2}m^2 \sin C;\ \sin C = \frac{36}{m^2}$

3. Option B)

m% of $r = \frac{mr}{100}$; Cars left = $r - \frac{mr}{100} = \frac{100r - mr}{100}$

4. Option A)

$$\frac{x^2 + x + 3x - 5 + 5x + 6 - 7x}{4} = 1$$

$(x+1)^2 = 4; x = \pm 2 - 1 = 1$ or –3. Substitute to find the data: 6; –14; –15; 27

The median value is $\frac{(6)+(-14)}{2} = -4$

5. Option C)

Solve $f'(x) = 0$ (*with a change of sign*), which only occurs when $x = 3$.

6. Option C)

4 x 3 x 2 = 24

7. Option D)

$cos^2x + cos^2x tan^2x = cos^2x(1 + tan^2x) = cos^2x.sec^2x = 1$

8. Option C)

Option A): $p^2 - q^2 = (p+q)(p-q)$ which is a product of two odd numbers, which is odd.

Option B): $p^2 + q^2$ is the sum of an odd and an even number, therefore odd.

Option C): $(qp + q)^{q+1} = even^{odd}$ which is even.

Option D): $(p-q)^2$ is the square of an odd number which is odd.

9. Option C)

In triangle ABC, $BC = \frac{16}{\tan 60°} = \frac{16}{\sqrt{3}}$. In triangle ABD, $BD = \frac{16}{\tan 30°} = \frac{16}{1/\sqrt{3}}$

CD = BD – CD = $\sqrt{3}.16 - \frac{16}{\sqrt{3}} = \frac{32}{\sqrt{3}}$

10. Option A)

$2P = P\left(1 + \frac{0{,}12}{12}\right)^{12n}$ hence $2 = 1{,}01^{12n}$ and $n = \left(\log_{1,01} 2\right) \div 12$

11. Option D)

Each face of the cube has an area of $A \div 6$, so each side has a length of $\sqrt{\frac{A}{6}}$.

12. Option B)

From $f(-2) = 0$, we have $2(-2)^3 - (-2)^2 + 12 - k = 0$, which yields $k = -8$.

13. Option A)

$$x = \frac{\log 3}{\log 10} \div \frac{\log 3}{\log 2}$$

$x = \log_{10} 2$

14. Option C)

$$\frac{1}{a+b} \times \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{1}{a+b} \times \frac{b+a}{ab} = \frac{1}{ab}$$

15. Option B)

$f'(x) = 3x^2 + b$, hence $3(3)^2 + b = gradient = 3$, and $b = -24$

16. Option D)

∠PQR = ∠TPR = 40° (angle between tangent and chord)

∠POR = 80° (angle at centre)

∠OPR = 50° (180° in isosceles triangle POR)

17. Option A)

The exponential graph of $y = 2^x$ has been reflected in the *x*-axis, and the horizontal asymptote shows that the graph has been shifted up by two units. The points (0; 1) and (1; 0) both satisfy the equation in Option A).

18. Option D)

When a linear dimension is doubled, this has double the effect on the area.

19. Option C)

When $x = 4$:

$$\sum_{k=3}^{12} (10 - 2x)^k = \sum_{k=3}^{12} (2)^k$$

The third term has $k = 5$, which yields the term $2^5 = 32$.

20. Option A)

$-2(x+3)(5-x) < 0 \Rightarrow (x+3)(5-x) > 0$. To solve the inequality, use the associated parabola graph, or a table of signs, or a sign line to find that $-3 < x < 5$.

21. Option D)

The roots of the equation $x^2 - x - 1 = 0$ are $x = \frac{1 \pm \sqrt{1+4}}{2}$, so the sum of the roots is $\frac{(1+\sqrt{5})+(1-\sqrt{5})}{2} = 1$

22. Option B)

The restriction $2x + 4 > 0$ yields $x > -2$.

23. Option C)

The graph of $f(x)$ has been stretched vertically by a factor of 2, and shifted 30° to the right.

24. Option B)

$a < 0$ so that the parabola has a maximum value; $b > 0$ so that the axis of symmetry value $\left(x = -\frac{b}{2a}\right)$ is positive; $c > 0$ to provide a positive *y*-intercept.

25. Option A)

The diagonals of a rhombus bisect each other at right angles. Using the side of the rhombus and half of the diagonal of length 6 cm, we have $\sqrt{6^2 - 5^2} = \sqrt{11}$. The other diagonal will therefore have a length of $2\sqrt{11}$ cm.

26. Option C)

Since $x > 0$, it follows that the expression is greater than zero, and since $x < x + 1$ it follows that the expression will be less than 1.

27. Option D)

If $x = 80°$is a solution, then $k = \cos 60° = \frac{1}{2}$. Since $\cos(-60°) = \frac{1}{2}$, another solution is given by $x - 20° = -60°$, or $x = -40°$.

28. Option C)

The sequence repeats a cycle of 7 letters; 2 000/7 leaves a remainder of 5, which takes us to the letter C in the last cycle of numbers.

29. Option B)

From the tangent information, $r = 9$; $(x - \sqrt{17})^2 + (y - 9)^2 = 81$;

Let $x = 0$: $(y-9)^2 = 81 - 17 = 64$; $y = \pm 8 + 9 = 17$ or 1

30. Option D)

The graph must be shifted down by 4 units to touch the x-axis at one point.

31. Option D)

This statement incorrectly asserts that the (positive) y-value of the function at $x = -1$ is less than the (zero) function value at $x = 2$.

32. Option B)

First find AC = 5.

$\sin \angle BAD = \sin(\angle BAC + \angle CAD)$

$= \sin \angle BAC . \cos \angle CAD + \cos \angle BAC . \sin \angle CAD$

$= \frac{3}{5} . \frac{5}{\sqrt{29}} + \frac{4}{5} . \frac{2}{\sqrt{29}} = \frac{23}{5\sqrt{29}}$

33. Option A)

$(\sin 3x + \cos 3x)^2 = (\sin^2 3x + \cos^2 3x) + 2\cos 3x \sin 3x = 1 + \sin 6x = \frac{5}{3}$

34. Option D)

∠ADB = 20° (equal radii)

∠BDC = 52° – 20° = 32° (isos ΔADC)

∠BAC = 64° (angle at centre)

∠BOC = 84° (ext angle of ΔBAO)

35. Option A)

$5 \otimes 2 = 5^2 - 2 = 23$; $3 \otimes 23 = 3^2 - 23 = -14$

36. Option B)

Let n be the number of jumps: $18 + 2n = 4n$; $n = 9$

37. Option D)

$xy + 2(x + y) + 4 = 18$; $xy = 18 - 2(7) - 4 = 0$

38. Option D)

By inspection, the sides must be given by 1, $2x$ and $\frac{y}{2}$. The volume is the product xy.

39. Option A)

The distance between the two centres must be greater than the sum of the two radi

40. Option A)

In this interval, the y-values of the graphs have opposite signs.

41. Option C)

$6 \sin x \cos x + 4 = 3 \sin 2x + 4$

The minimum value of $3\sin 2x$ is –3, so the minimum value of the expression is 1.

42. Option B)

$\log_{10} \frac{100!}{99!} = \log_{10} \frac{(100)(99)(98)\dots(1)}{(99)(98)\dots(1)} = \log_{10} 10^2 = 2$

43. Option D)

$\sin B = \frac{3}{5} = \frac{y}{r}$; $\tan A = \tan(90° - B) = \cot B = \frac{x}{y} = \frac{4}{3}$

$\tan(180° - A) = -\tan A = -\frac{4}{3}$

44. Option A)

The graph of $y = k$ is a horizontal line, which will only intersect the given cubic function at three distinct points if it lies between the two turning points.

45. Option C)

$r^2 = 4 + 9$; $r = \sqrt{13}$; shaded area = ¼ $\pi r^2 - (2)(3) = \frac{13\pi}{4} - 6$

46. Option D)

The radius of the largest circle is 12, divided into 6 equal parts, so the smallest circle has a radius of 2 and centre of (10; 0). The equation is $(x - 10)^2 + y^2 = 4$.

47. Option A)

By Pythagoras, the diagonal of the base is $4\sqrt{2}$. The height will be

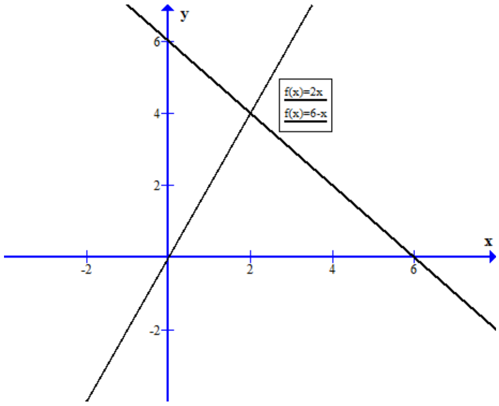
$\sqrt{5^2 - (2\sqrt{2})^2} = \sqrt{25-8} = \sqrt{17}$

48. Option D)

In triangle OAC, AC = $m \sin\alpha$ = CB; The x-coordinate of B = OA + CB;

OA = $m \cos\alpha$. Hence, $B_x = m(\cos\alpha + \sin\alpha)$

49. Option B)



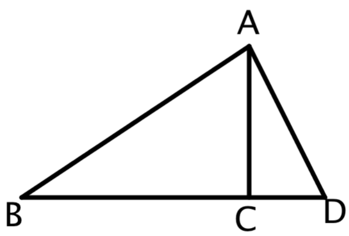
By simultaneous equations, the graphs of $y = 2x$ and $y = 6 - x$ intersect at the point (2; 4). The area of the enclosed triangle is $\frac{1}{2} \times$ $base \times height = \frac{1}{2} \times 6 \times 4$ = 12 square units.

50. Option B)

$$\frac{2}{\sqrt{-2x^2+4x+2}} = \frac{2}{\sqrt{-2(x-1)^2+4}}$$

The denominator has a maximum value of $\sqrt{4}$, so the fraction will have a minimum value of $\frac{2}{\sqrt{4}} = 1$.

51. Option C)



Using the right-angles, ΔCDA /// ΔCAB; $\frac{AC}{BC} = \frac{CD}{AC}$; $AC^2 = 36$; AC= 6 cm

52. Option D)

From $x = z + y$, $\frac{2x}{z+y} = \frac{2(z+y)}{z+y} = 2$

53. Option B)

Notice that $2(2^{r-1}) = 2^r = \left(\frac{1}{2}\right)^{-p}$, so the general terms are the same, using different ‘dummy’ variables. The left sum adds terms from T_5 up to T_{18}, which is the same as adding the terms from T_1 to T_{18}, except for the missing terms from T_1 to T_4. Hence, a = 1 and b = 4.

54. Option B)

Subst x = 1: $f(0) + 2f(1) = 3$; Subst x = 0: $f(1) + 2f(0) = 0$

Hence $f(1) = \frac{3-f(0)}{2} = -2f(0)$, which solves to give $f(0) = -1$.

55. Option C)

The axis of symmetry will be the perpendicular bisector of the line AB.

Midpoint of AB = (3; 3); Gradient AB = –1 ; Axis of symmetry: $y = x$

56. Option B)

Solve $6x + 3 = -9$ to find $x = -2$

If the lines intersect at (–2; – 9), then the second line will be $y = 4\frac{1}{2}x$. To intersect below this point, the second line must have a steeper gradient than $4\frac{1}{2}$, but not be steeper than the first line.OR solve algebraically: $6x + 3 = mx \Leftarrow x = \frac{3}{m-6}$, so $y = mx = \frac{3m}{m-6}$.

Solve $\frac{3m}{m-6} < -9$, i.e. $\frac{12m-54}{m-6} < 0$

		$4\frac{1}{2}$	6
$12m - 54$	-	+	+
$m - 6$	-	-	+
$\frac{12m-54}{m-6}$	+	-	+

So $4\frac{1}{2} < m < 6$.

57. Option C)

Angle C is a right angle. Given: ½ (AC)(CB) = 11.

Find perimeter = AC + CB + BA = AC + CB + 10

$(AC + CB)^2 = AC^2 + 2.AC.CB + CB^2$, but $AC^2 + CB^2 = AB^2 = 100$ (Pyth.)

Hence, $(AC + CB)^2 = 100 + 44 = 144$, AC + CB = 12, and perimeter = 22

58. Option C)

$P(G) = 1 - \frac{1}{3} - \frac{1}{5} = \frac{7}{15}$. The least number of green marbles is 7 (and the least number of marbles is 15).

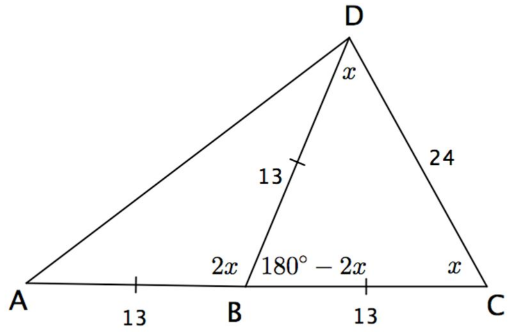
59. Option A)

$A_I = \frac{5}{8}B_I; A_E = \frac{1}{2}B_E$ and $A_S = A_I - A_E = 0{,}4A_I$;

$\frac{5}{8}B_I - \frac{1}{2}B_E = 0{,}4\frac{5}{8}B_I$ so $\frac{5}{8}B_I - 0{,}4\frac{5}{8}B_I = \frac{1}{2}B_E$ so $\frac{3}{4}B_I = B_E$

So his savings are ¼ of his income.

60. Option B)



In ΔCBD, $\frac{24}{\sin(180°-2x)} = \frac{13}{\sin x} \Rightarrow \frac{24}{\sin 2x} = \frac{13}{\sin x}; \frac{24}{2\sin x \cos x} = \frac{13}{\sin x} \Rightarrow \cos x = \frac{24}{26}$ ($\sin x \neq 0$). In ΔACD, $AD^2 = 26^2 + 24^2 - 2(26)(24).\cos x = 26^2 + 24^2 - 2(26)(24)\frac{24}{26}$. This becomes $AD^2 = 26^2 - 24^2 = (26 + 24)(26 - 24) = (50)(2) = 100$, so AD = 10.

NBT Prep test 2

1. Option D)
$\log_{10} 0{,}009 = \log_{10}\frac{9}{1\,000} = 2\log_{10} 3 - 3\log_{10} 10 = 2a - 3$
2. Option D)
New area = (0,8 length)(1,2 breadth) = 0,96 of original area, which is a decrease of 4%.
3. Option B)
$1 - 2\cos^2 15° = -\cos 2(15°) = -\frac{\sqrt{3}}{2}$
4. Option A)
The equation is satisfied when $x = 0$ or $x^3 = -1$ ($x^2 \neq -9$ for real numbers), i.e. $x = 0$ or $x = -1$.
5. Option A)
Each odd numbered term will be 0.
6. Option A)
In triangle AOC, AC = $r\cos$A; double this to find AB, as the two triangles are congruent.
7. Option C)
The sequence will have a constant ratio of ½, as the denominator increases by a factor of two each time.
8. Option B)
$\frac{3{,}010 \times 3010}{30{,}10 \times 301} = \frac{(301 \times 301) \times 0{,}01 \times 10}{(301 \times 301) \times 0{,}1} = 1$
9. Option D)
Apply the exponent rule $\sqrt{a^b} = a^{b \div 2}$
10. Option D)
There is a choice of four actors for the first part, then three actors for the second part, two actors for the third part and one actor for the fourth part. The number of possibilities is $4 \times 3 \times 2 \times 1 = 24$.
11. Option C)
Angle CDE = ½ (180° – x) = 90° – $\frac{x}{2}$, so angle BDG = 360 – (90° – $\frac{x}{2}$) – 2(90°) = 90° + $\frac{x}{2}$.
12. Option B)
$8 \times 2^{100} + 4 \times 2^{101} = 2^{103} + 2^{103} = 2(2^{103}) = 2^{104}$
13. Option C)
This is a fundamental log law
14. Option D)
Let the side length of the square be x. Then $x^2 + x^2 = d^2; Area = x^2 = \frac{d^2}{2}$
15. Option A)
Area = ½ AB(20) + ½ AB(16), so 18AB = Area; AB = Area/18
16. Option A)
Swop the variables: $x = \log_4 \sqrt{y} \Rightarrow x = \frac{1}{2}\log_4 y \Rightarrow 2x = \log_4 y$. Hence, $y = 4^{2x} = 16^x$
17. Option D)
For equal roots, $b^2 - 4ac = 0$, hence $m^2 - 4n = 0$, so $\frac{n}{m^2} = \frac{1}{4}$.
18. Option A)

$$f\left(1 + \frac{1}{x}\right) = 1 - \frac{1}{1 + \frac{1}{x}} = 1 - \frac{x}{x+1} = \frac{x + 1 - x}{x + 1} = \frac{1}{x+1}$$

19. Option A)
From the distance formula, OP² = 5² = $(-1-2)^2 + (6-a)^2$. This yields the equation $(a - 10)(a - 2) = 0$, so $a = 2$ or $a = 10$.
20. Option C)
∠ABC = ∠CAQ = 100° (angle between tangent and chord)
∠PAB = 50° (equal interior angles of isosceles triangle PAB)
∠BAC = 30° (180° on straight line)
∠BOC = 60° (angle at centre)
21. Option C)
The graph of $f(x) = 3\tan(2[x - 15°])$ has half the period of $y = \tan x$, and is shifted 15° to the right (to compensate for the subtraction of 15°). The original vertical asymptote at $x = 90°$ will shift first to 45° and then to 45° + 15° = 60°.
22. Option B)
By the cosine rule, BC² = 36 + 64 – 2(6)(8)cos30° = 100 – 96 $\left(\frac{\sqrt{3}}{2}\right)$. Hence, BC = $\sqrt{100 - 48\sqrt{3}} = 2\sqrt{25 - 12\sqrt{3}}$.
23. Option A)
The diagonal AC of the square is the diameter of the circle, with centre (midpoint) O (–1; 7) and $r = AO = \sqrt{3^2 + 4^2} = 5$. The equation of the circle is $(x + 1)^2 + (y - 7)^2 = 25$.
24. Option D)
Change the order of the factors to obtain $\left(\sqrt{2} - \sqrt{3}\right)^2\left([\sqrt{2} - \sqrt{3}][\sqrt{2} + \sqrt{3}]\right)^2$ which simplifies to $\left(\sqrt{2} - \sqrt{3}\right)^2(2 - 3)^2 = \left(2 - 2\sqrt{6} + 3\right)(1) = 5 - 2\sqrt{6}$
25. Option A)
If $f(x) = x^3 - ax^2 + bx + 2$, then $f(\pm 1) = 0$. Solving $1 - a + b + 2 = 0$ and $-1 - a - b + 2 = 0$ simultaneously yields $a = 2, b = -1$, and hence $a = -2b$.
26. Option C)
Solve simultaneously $ar = 12$ and $\frac{a}{1-r} = 54$ to obtain $\frac{12}{r} \times \frac{1}{1-r} = 54$. This yields $9r^2 - 9r + 2 = 0$, hence $(3r - 1)(3r - 2) = 0$, so that $r = \frac{1}{3}$ or $r = \frac{2}{3}$. (Both of the infinite geometric series 18; 12; 8; ... and 36; 12; 4; ... have a sum of 54.)

27. Option A)

$$\frac{1}{(p+1)(p-1)} = \frac{1}{p^2-1} = \frac{1}{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1} = \frac{1}{2\sin\theta\cos\theta}$$
$$= \frac{1}{\sin 2\theta}$$

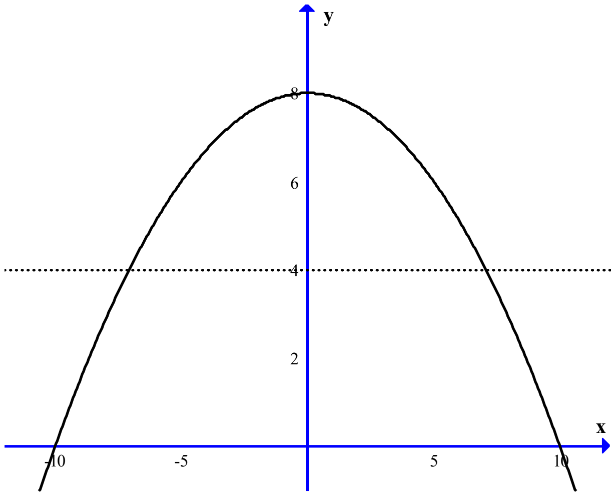
28. Option D)

The first two factors are positive, so only the third factor can be negative. Hence, $5^x < 1$, so $x < 0$.

29. Option B)

Sketch a representative parabola with its maximum value on the y-axis, with equation $y = 8 - ax^2$. By substitution, we have $0 = 8 - a(100) \Rightarrow a = \frac{2}{25}$.

Now solve $4 = 8 - \frac{2}{25}x^2$, or $x^2 = 4 \times \frac{25}{2} = 50$, to obtain $x = 5\sqrt{2}$.



30. Option A)

$$\sum_{x=6}^{35} \log\frac{x+1}{x} = \log\frac{7}{6} + \log\frac{8}{7} + \log\frac{9}{8} + \cdots + \log\frac{36}{35}$$
$$= \log\left(\frac{7}{6}\times\frac{8}{7}\times\frac{9}{8}\times \ldots \times\frac{36}{35}\right) = \log\frac{36}{6} = \log 6$$
$$OR: (\log 7 - \log 6) + (\log 8 - \log 7) + \cdots + (\log 36 - \log 35)$$
$$= \log 36 - \log 6 = \log\frac{36}{6} = \log 6$$

31. Option D)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, and from $\sin^2\theta + \cos^2\theta = 1$ we have $\sin\theta = \pm\sqrt{1-\cos^2\theta}$. Hence, $\tan\theta = \frac{\pm\sqrt{1-\cos^2\theta}}{\cos\theta}$. When $\theta = 36°$, we have $\tan\theta = \frac{+\sqrt{1-\cos^2\theta}}{\cos\theta}$ as the angle is in the first quadrant. Finally, as $\tan 216° = \tan 36°$, we have $\tan 216° = \tan 36° = \frac{\sqrt{1-\cos^2 36°}}{\cos 36°} = \frac{\sqrt{1-m^2}}{m}$.

32. Option B)

The graph of $f(x)$ will have a negative derivative over its interval of decrease. The given derivative function has negative values when x is greater than 2.

33. Option D)

The graph will be stretched vertically by a factor of 2 and reflected in the y-axis. Only the first transformation will affect the range of the function.

34. Option A)

Write the function in the form $f(x) = \frac{a^2}{3}.x^{-1/2}$ .Differentiate to obtain $f'(x) = \frac{a^2}{3}.\left(-\frac{1}{2}x^{-3/2}\right)$ and simplify.

35. Option B)

The solution of $\cos x = 0$ is $x = 90° + n.180°$ and the solution of $\sin x = 1$ is $x = 90° + n.360°$. The combined solution is $x = 90° + n.180°$.

36. Option A)

$P(1-rn) = \frac{P}{2}(1-r)^n$ when $n = 2$ so $2(1-2r) = (1-r)^2$;

hence $r^2 + 2r - 1 = 0$.

37. Option D)

Without replacing, the number of cards available decreases each time. The events are dependent and the probabilities are multiplied: $\frac{26}{52}\times\frac{25}{51}\times\frac{24}{50} = \frac{2}{17}$

38. Option C)

By symmetry, the second point lies 3 units to the right of the axis of symmetry, so $p = -4 + 3 = -1$.

39. Option B)

The regular nonagon can be inscribed in a circle. Each chord will subtend an angle of $\frac{360°}{9} = 40°$ at the centre of the circle. Hence the angle at the circumference will be $\frac{40°}{2} = 20°$.

40. Option A)

$$\frac{Area\ \Delta DEF}{Area\ \Delta ABC} = \frac{10{,}89}{9} = 1{,}21 = 1{,}1^2$$

The sides will compare in the ratio 1 : 1,1 so EF = 1,1 × 6 = 6,6

41. Option D)

Use inspection each time. The first two eggs have average mass 51 g, so the second egg is 52 g. The first three eggs have average mass 52 g (dividing by 3), so the third egg is 54 g (to make 156 g). The first four eggs have average mass 53 g (dividing by 4), so the fourth egg is 54 g (to make 212 g). In the same way, the last egg will have mass 58 g.

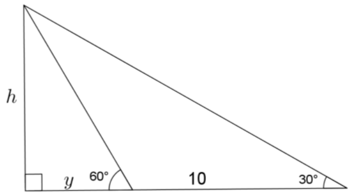
OR use algebra: The total mass of the first 4 eggs is 4 × 53 = 212 g. If the mass of the last egg is x g, then 212 + x = 5 × 54 = 270, which yields x = 58 g.

42. Option D)

$$f(f(2)) = f(2a+3) = a(2a+3)+3 = 2a^2+3a+3$$

The equation $f(f(2)) - 3a = 11$ becomes $(2a^2+3a+3) - 3a = 11$; hence $2a^2 = 8$, so $a = \pm 2$.

43. Option A)



tan60° = $\frac{tree\ height}{shadow\ length}$, so $h = \sqrt{3}y$; also tan 30° $= \frac{h}{y+10}$ yields $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}y}{y+10}$. Solve to find $y = 5$; $h = 5\sqrt{3}$.

44. Option B)

$h(x) = \sin x . \cos x = \frac{1}{2}\sin 2x$

45. Option C)

In ΔVBC, BC = 12cosα; In ΔABC, AB = BC (congruent triangles ABV and CBV), hence by the cosine rule $(AC)^2 = 2(BC)^2 - 2(BC)^2\cos 120°$, which gives $(AC)^2 = 2(144\cos^2\alpha) - 2(144\cos^2\alpha)\left(-\frac{1}{2}\right)$; $(AC)^2 = 3(144\cos^2\alpha)$, and hence $AC = 12\sqrt{3}\cos\alpha$.

46. Option D)

sinθ = $\frac{-2}{5} = \frac{y}{r}$ with θ in the third quadrant (cosine is also negative). Using Pythagoras, $x = -\sqrt{25-4} = -\sqrt{21}$ (negative in third quadrant), so $\sin 2\theta = 2\sin\theta\cos\theta = 2.\frac{-2}{5}.\frac{-\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$

47. Option C)

Let the number of sheep be 2x and the number of chickens be x. Then 2x(5) + x(3) = 91, so x = 7 and the total number of animals is 3x = 21.

48. Option D)

$2^2 \times 3^3 \times 4^3 \times 5^9 = 2^8 \times 3^3 \times 5^9 = 10^8 \times 27 \times 5 = 135 \times 10^8$. The sum of the digits will be 9.

49. Option A)

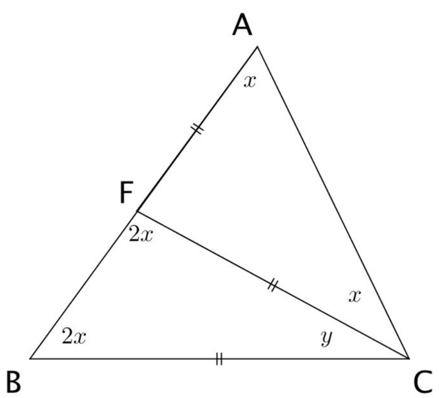
AF = CH = GH = 2, so FG = FH – GH = $\sqrt{7} - 2$, and BE = AB = FG = $\sqrt{7} - 2$. Hence, EG = BG – BE = 2 – ($\sqrt{7} - 2$) = $4 - \sqrt{7}$.

50. Option B)

In ΔDEC, DE = $\sqrt{25^2 - 5^2} = 10\sqrt{6}$. Let EG = GF = x (equal tangents). In ΔDGF,
$(DE + x)^2 - 30^2 = x^2$, which after substituting DE gives = $\frac{15}{\sqrt{6}}$.

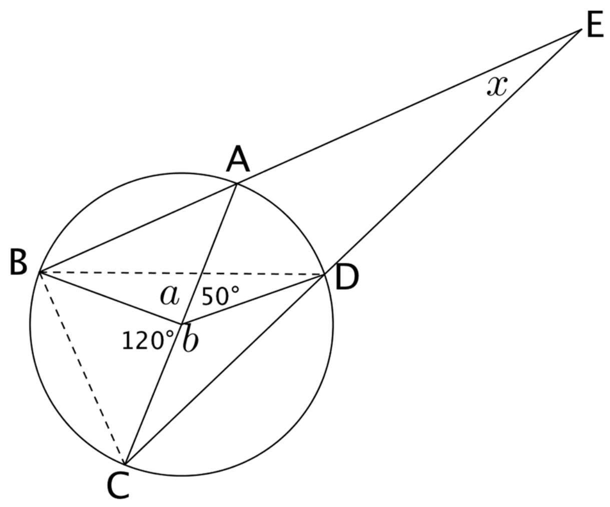
OR: Notice that ΔDGF is similar to ΔDCE (AAA) so that FG/CE = DF/DE. Hence FG/5 = 30/DE so that FG =150/(10$\sqrt{6}$) = $\frac{15}{\sqrt{6}}$.

51. Option C)



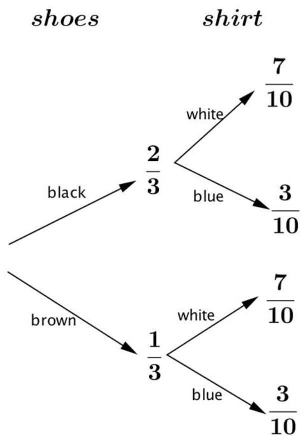
Let ∠A = ∠ACF = x, and ∠FCB = y. Then ∠B = ∠CFB = 2x (ext angle of ΔAFC and isos ΔBFC). Solve simultaneously 4x + y = 180° (in ΔFBC) and x + y = 2x (in isos ΔABC) to find that x = y = 36°.

52. Option D)



From angles around a point, $a + b = 190°$. From isosceles triangles (equal radii), ∠OBC = ∠OCB = 30°, ∠OBA = ∠OAB = 90° $-\frac{a}{2}$ and ∠OCD = ∠ODC = 90° $-\frac{b}{2}$. Hence in triangle EBC, $x = 180° - 2(30°) - \left(180° - \frac{a+b}{2}\right) = -60° + \frac{190°}{2} = 35°$.
OR: Construct line BD. Then ∠BDC = ½ ∠BOC = 60° (angle at circumference); ∠ABD = ½ ∠AOD = 25° (angle at circumference); hence $x = 35°$ (exterior angle of triangle BDE = sum of interior opposite angles).

53. Option C)
Use a tree diagram to identify the probabilities.



Total probability = (2/3 × 3/10) + (1/3 × 7/10) = $\frac{13}{30}$

54. Option B)
Solve by squaring and then check for extra roots introduced by the process: $x^2 + \sqrt{x^3+1} = (1-x)^2 \Rightarrow \sqrt{x^3+1} = 1-2x$; squaring again yields $x^3 + 1 = 1 - 4x + 4x^2$, or $x(x-2)^2 = 0$. However only $x = 0$, not $x = 2$, satisfies the original equation, so there is only one solution.

55. Option C)
Let the outer dimensions be x. Then the inner dimensions are $x - 1$. The volume of metal is Outer volume – Inner volume, so $x^3 - (x-1)^3 = 1801$, which simplifies to
$x^2 - x - 600 = 0$, or $(x-25)(x+24) = 0$, so the outer dimension is 25 cm.

56. Option C)
Simplify:

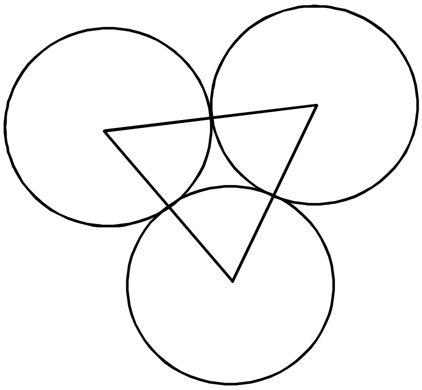
$$2x^2 - 5x - 9 = 0 \Rightarrow x^2 - \frac{5}{2}x - \frac{9}{2} = 0$$

Compare this result to $(x-a)(x-b) = 0 \Rightarrow x^2 - (a+b)x + ab = 0$ to see that $a + b = \frac{5}{2}$ and $ab = -\frac{9}{2}$. The new equation is $(x-2a)(x-2b) = 0$, and $x^2 - 2(a+b)x + 4ab = x^2 - 2\left(\frac{5}{2}\right)x + 4\left(-\frac{9}{2}\right) = x^2 - 5x - 18$. (It is not efficient to try to first find the roots of the original equation.)

57. Option D)
Let the side of the square be x. Then by similar triangles, $\frac{b}{a} = \frac{b-x}{x}$, so $x = \frac{ab}{a+b}$. The ratio of areas is $\frac{x^2}{\frac{1}{2}ab}$, which becomes $\frac{a^2b^2}{(a+b)^2} \times \frac{2}{ab} = \frac{2ab}{(a+b)^2}$.

58. Option B)
Let the container have weight C, and the full volume of water have weight W. Then $C + W = M$ and $C + \frac{W}{3} = L$. Substitution yields $C + (3L - 3C) = M$, hence $C = \frac{3L-M}{2}$.

59. Option C)



Construct an equilateral triangle joining the three centres, with sides of length 4 cm and interior angles of 60°. Area of Δ = ½(4)(4)sin60° = 8.$\frac{\sqrt{3}}{2}$ = $4\sqrt{3}$. Area of each circle sector within the triangle = $\frac{1}{6}\times\pi r^2 = \frac{1}{6}\times\pi(2^2) = \frac{2\pi}{3}$. The area enclosed by the circles = $4\sqrt{3} - 3\left(\frac{2\pi}{3}\right) = 4\sqrt{3} - 2\pi$.

60. Option C)

Let the runner's speed be x, and the cyclist's speed be x + 30. Use the fact that $cyclist's\ time + \frac{18}{60} = runner's\ time$, and $time = \frac{distance}{speed}$. This yields $\frac{10}{x+30} + \frac{3}{10} = \frac{10}{x}$. The equation simplifies to $x^2 + 30x - 1000 = 0$, or $(x+50)(x-20) = 0$, so the runner's speed is 20 km/h and the cyclist's speed is 50 km/h.

NBT exemplar test

1. Option D)
   From the question, the y-intercept is $(0;-5)$ and the negative co-efficient of x^2 indicates that the graph has a maximum turning point.
2. Option D)
   $-x^2 + 6x - 5 = 0$
   $\therefore x^2 - 6x + 5 = 0$

   $\therefore (x-5)(x-1) = 0$

   $\therefore x = 5\ or\ x = 1$

   Therefore, the sum of the roots is 6.
3. Option A)
   $-x^2 + 6x - 5$ has a maximum value of 4, ∴ $\sqrt{-x^2 + 6x - 5}$ has a maximum value of 2.
4. Option D)
   Reflect in x-axis: $-y = -x^2 + 6x - 5$; ∴ $y = x^2 - 6x + 5$
   Reflect resulting graph in y-axis: $y = (-x)^2 - 6(-x) + 5$; ∴ $y = x^2 + 6x + 5$
5. Option D)
   $\frac{1}{x} > 1$; ∴ $\frac{1-x}{x} > 0$ is true for $0 < x < 1$
6. Option D)
   $\sin 43° \cos 23° - \cos 43° \sin 23°$
   $= \sin(43° - 23°)$
   $= \sin 20°$
7. Option D)
   Distance from top of A to foot of B:
   $\frac{d}{100} = \cos 30°$
   $d = 100.\frac{\sqrt{3}}{2}$
   $\therefore d = 50\sqrt{3}$
   Height of A:
   $\frac{h}{50\sqrt{3}} = \sin 60°$
   $h = \frac{\sqrt{3}}{2}.50\sqrt{3}$
   $\therefore h = 75m$
8. Option B)
   $x^2 + x - 2 = (x-1)(x+2)$
   $\therefore BF = x + 2cm$
   $BC = x - 1cm$
   $\therefore FC = BF - BC$
   $\therefore FC = 3cm$

9. Option C)
   $AB^2 = \sqrt{2a^2}$
   Radius of opening: $\frac{\sqrt{2a^2}}{4}$
   Area of opening $= \pi\left(\frac{\sqrt{2a^2}}{4}\right)^2 = \pi\left(\frac{2a^2}{16}\right) = \frac{\pi a^2}{8}$
   SA shape $= 6a^2 - \frac{\pi a^2}{8}$
10. Option A
   Total $= 1000\left(1+\frac{0{,}06}{4}\right)^{20} = 1000(1{,}015)^{20}$

Acknowledgements

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