higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N6

6 APRIL 2016

This marking guideline consists of 17 pages.

TOTAL: $\frac{200}{2} = 100$

NOTE: Do NOT subtract marks for incorrect units or units omitted.

QUESTION 1

1.1 $z = -5x^3y^2 - y^4 + 3x^2y$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-10x^3y - 4y^3 + 3x^2) \quad \checkmark \checkmark \checkmark$$

$$= -30x^2y + 6x \quad \checkmark$$

(4)

1.2 $I = \frac{V}{R}$

$I = VR^{-1}$

$$\Delta I = \frac{\partial I}{\partial V}\Delta V + \frac{\partial I}{\partial R}\Delta R \quad \checkmark \quad \checkmark$$

$$= R^{-1}\Delta V - VR^{-2}\Delta R \quad \checkmark \quad \checkmark$$

$$= \frac{1}{R}\Delta V - \frac{V}{R^2}\Delta R \quad \checkmark$$

$$= \frac{1}{(10)}(-5) - \frac{30}{(10)^2}(-8) \quad \checkmark \quad \checkmark$$

$= 1{,}9 \text{ A} \quad \checkmark$

(8)
[12]

QUESTION 2

2.1 $y = \int \sin^4 5x \cos^3 5x dx$

✓

$= \int \sin^4 5x \cos^2 5x . \cos 5x dx$

✓

$= \int \sin^4 5x (1 - \sin^2 5x) \cos 5x dx$

$= \frac{1}{5} \int \sin^4 5x (1 - \sin^2 5x) 5 \cos 5x dx$ $\quad u = \sin 5x$, $du = 5 \cos 5x dx$

✓ ✓ ✓

$= \frac{1}{5} \int u^4 (1 - u^2) du$

✓

$= \frac{1}{5} \int u^4 - u^6) du$

✓ ✓

$= \frac{1}{5} \left[\frac{u^5}{5} - \frac{u^7}{7} \right] + c$

✓ ✓

$= \frac{1}{5} \left[\frac{\sin^5 5x}{5} - \frac{\sin^7 5x}{7} \right] + c$

OR

$= \frac{1}{25} \sin^5 5x - \frac{1}{35} \sin^7 5x + c$

OR

$y = \int \sin^4 5x \cos^3 5x dx$

✓

$= \int \sin^4 5x \cos^2 5x . \cos 5x dx$

✓

$= \int \sin^4 5x (1 - \sin^2 5x) \cos 5x dx$

✓ ✓ ✓ ✓

$= \int \sin^4 5x . \cos 5x dx - \int \sin^6 5x . \cos 5x dx$ $\quad u = \sin 5x$, $du = 5 \cos 5x dx$

✓ ✓ ✓ ✓

$= \frac{1}{5} . \frac{\sin^5 5x}{5} - \frac{1}{5} . \frac{\sin^7 5x}{7} + c$ (10)

2.2 $$y = \int \frac{1}{\sqrt{16x - x^2}} dx$$

$$= -x^2 + 16x$$

$$= -(x^2 - 16x) \quad \checkmark$$

$$\checkmark \quad \checkmark$$

$$= -[(x-8)^2 - 64]$$

$$= 64 - (x-8)^2 \quad \checkmark$$

$$= \int \frac{1}{\sqrt{64 - (x-8)^2}} dx \quad \checkmark$$

$$= \sin^{-1} \frac{(x-8)}{8} + c \quad \checkmark$$

(6)

2.3 $$y = \int \sin^4 mx dx$$

$$= \int (\sin^2 mx)^2 dx \quad \checkmark$$

$$= \int (\frac{1}{2} - \frac{1}{2}\cos 2mx)^2 dx \quad \checkmark$$

$$= \int (\frac{1}{2} - \frac{1}{2}\cos 2mx)(\frac{1}{2} - \frac{1}{2}\cos 2mx) dx$$

$$= \int (\frac{1}{4} - \frac{1}{2}\cos 2mx + \frac{1}{4}\cos^2 2mx) dx \quad \checkmark$$

$$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= \frac{1}{4}x - \frac{1}{2}.\frac{\sin 2mx}{2m} + \frac{1}{4}\left[\frac{x}{2} + \frac{\sin 4mx}{8m}\right] + c$$

$$= \frac{1}{4}x - \frac{\sin 2mx}{4m} + \frac{x}{8} + \frac{\sin 4mx}{32m} + c$$

(8)

2.4 $y = \int e^{\frac{x}{2}}.\cos 3x dx$

✓ ✓

$\int y dx = e^{\frac{x}{2}}(\frac{\sin 3x}{3}) - \int \frac{1}{2}e^{\frac{x}{2}}(\frac{\sin 3x}{3})dx$

$f(x) = e^{\frac{x}{2}} \quad g'(x) = \cos 3x$

$f'(x) = \frac{1}{2}e^{\frac{x}{2}} \quad g(x) = \frac{\sin 3x}{3}$

✓

$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x - \frac{1}{6}\int e^{\frac{x}{2}}.\sin 3x dx$

$f(x) = e^{\frac{x}{2}} \quad g'(x) = \sin 3x$

$f'(x) = \frac{1}{2}e^{\frac{x}{2}} \quad g(x) = -\frac{\cos 3x}{3}$

✓ ✓

$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x - \frac{1}{6}\left[e^{\frac{x}{2}}.-\frac{\cos 3x}{3} - \int \frac{1}{2}e^{\frac{x}{2}}.-\frac{\cos 3x}{3}dx\right]$

✓ ✓

$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x - \frac{1}{36}\int e^{\frac{x}{2}}.\cos 3x dx$

$I = \frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x - \frac{1}{36}I$ ✓

✓

$\therefore \frac{37}{36}I = \frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x$ ✓

✓ ✓

$I = \frac{36}{37}(\frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x) + c$

$= 0{,}973(0{,}333e^{\frac{x}{2}}\sin 3x + 0{,}054e^{\frac{x}{2}}.\cos 3x) + c$

$= 0{,}324e^{\frac{x}{2}}\sin 3x + 0{,}054e^{\frac{x}{2}}.\cos 3x + c$

OR

$y = \int e^{\frac{x}{2}}.\cos 3x dx$

✓ ✓

$\int y dx = 2e^{\frac{x}{2}}\cos 3x - \int(-3\sin 3x)2e^{\frac{x}{2}}dx$

$f(x) = \cos 3x \quad g'(x) = e^{\frac{x}{2}}$

$f'(x) = -3\sin 3x \quad g(x) = \frac{e^{\frac{x}{2}}}{\frac{1}{2}} = 2e^{\frac{x}{2}}$

✓

$= 2e^{\frac{x}{2}}\cos 3x + 6\int \sin 3x.e^{\frac{x}{2}}dx$

✓ ✓

$= 2e^{\frac{x}{2}}\cos 3x + 6\left[\sin 3x.2e^{\frac{x}{2}} - \int 3\cos 3x.2e^{\frac{x}{2}}dx\right]$

$f(x) = \sin 3x \quad g'(x) = e^{\frac{x}{2}}$

$f'(x) = 3\cos 3x \quad g(x) = 2e^{\frac{x}{2}}$

✓ ✓

$= 2e^{\frac{x}{2}}\cos 3x + 12\sin 3x.e^{\frac{x}{2}} - 36\int \cos 3x.e^{\frac{x}{2}}dx$

$$I = 2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x - 36I \quad \checkmark \checkmark$$

$$\therefore 37.I = 2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x \quad \checkmark \checkmark$$

$$I = \frac{1}{37}(2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x) + c$$

$$= 0{,}054e^{\frac{x}{2}}\cos 3x + 0{,}324e^{\frac{x}{2}}\sin 3x + c \qquad (12)$$

[36]

QUESTION 3

3.1 $\int \frac{-x^2+3x+4}{x(1-2x)^2}\,dx$

$$\frac{-x^2+3x+4}{x(1-2x)^2} = \frac{A}{x} + \frac{B}{(1-2x)^2} + \frac{C}{(1-2x)} \quad \checkmark$$

$$-x^2+3x+4 = A(1-2x)^2 + Bx + Cx(1-2x) \quad \checkmark$$

$$Let\ \ x = 0; \quad \therefore A = 4 \quad \checkmark$$

$$Let\ \ x = \frac{1}{2}; \quad \therefore B = \frac{21}{2} \quad (10{,}5) \quad \checkmark$$

$$-x^2+3x+4 = A - 4Ax + 4Ax^2 + Bx + Cx - 2Cx^2 \quad \checkmark$$

Equate coeff of x^2: $\quad C = \frac{17}{2} \quad (8{,}5) \quad \checkmark$

$$= \int \frac{4}{x}dx + \int \frac{\frac{21}{2}}{(1-2x)^2}dx + \int \frac{\frac{17}{2}}{(1-2x)}dx \quad \checkmark$$

$$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= 4\ln x - \frac{21}{4}.\frac{(1-2x)^{-1}}{-1} - \frac{17}{4}\ln(1-2x) + c$$

$$= 4\ln x + \frac{21}{4(1-2x)} - \frac{17}{4}\ln(1-2x) + c$$

$$= 0{,}444\ln x + \frac{5{,}25}{(1-2x)} - 1{,}444\ln(1-2x) + c \qquad (12)$$

3.2 $\int \frac{10x^2+7x+1}{(2x^2+1)(4x-1)}\,dx$

$$\frac{10x^2+7x+1}{(2x^2+1)(4x-1)} = \frac{Ax+B}{2x^2+1} + \frac{C}{4x-1} \quad \checkmark$$

$$10x^2+7x+1 = (Ax+B)(4x-1) + C(2x^2+1) \quad \checkmark$$

$$10x^2+7x+1 = 4Ax^2+4Bx-Ax-B+2Cx^2+C \quad \checkmark$$

$$let \quad x=\frac{1}{4} \qquad \therefore C=3 \quad \checkmark$$

Equate coeff of x^2: $A=1$ ✓ *Equate* x: $B=2$ ✓

$$= \int \frac{x+2}{2x^2+1}dx + \int \frac{3}{4x-1}dx \quad \checkmark$$

$$\checkmark$$

$$= \int \frac{x}{2x^2+1}dx + \int \frac{2}{2x^2+1}dx + \int \frac{3}{4x-1}dx$$

$$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= \frac{1}{4}\ln(2x^2+1) + 2\left(\frac{1}{\sqrt{2}}\right)\arctan\sqrt{2}x + \frac{3}{4}\ln(4x-1) + c$$

(12)
[24]

QUESTION 4

4.1 $2\sin x\frac{dy}{dx} - y(\sin 2x) = \frac{2\sin x}{\sec x}$

$$\frac{dy}{dx} - \frac{y(2\sin 2x)}{2\sin x} = \frac{1}{\sec x} \quad \checkmark$$

$$\checkmark$$

$$\frac{dy}{dx} - \frac{y(2\sin x\cos x)}{2\sin x} = \frac{1}{\sec x}$$

$$\frac{dy}{dx} - y.\cos x = \cos x \quad \checkmark$$

$$\checkmark$$

$$e^{\int -\cos x.dx} = e^{-\sin x} \quad \checkmark$$

$$e^{-\sin x}.y = \int e^{-\sin x}.\cos x dx \quad \checkmark$$

$$= -e^{-\sin x} + c \quad \checkmark$$

$$e^{-\sin 0}.(1) = -e^{-\sin 0} + c \quad \checkmark$$

$$\therefore c = 2 \quad \checkmark$$

$$e^{-\sin x}.y = -e^{-\sin x} + 2 \checkmark$$

(10)

4.2 $\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 6y = 2x + 3$

y_c : $m^2 - 7m + 6 = 0$ ✓

$(m-6)(m-1) = 0$

$m = 6; \quad m = 1$

$y_c = Ae^{6x}$ ✓

To find y_p $\therefore y = Cx + D$ ✓

$\dfrac{dy}{dx} = C$ ✓

$\dfrac{d^2y}{dx^2} = 0$ ✓

$0 - 7C + 6Cx + 6D = 2x + 3$ ✓

$C = \dfrac{1}{3}$ (0,333) ✓

$-7C + 6D = 3 \quad \therefore D = \dfrac{8}{9}$ (0,889) ✓

$\therefore y_p = \dfrac{1}{3}x + \dfrac{8}{9}$

$y = Ae^{6x} + Be^{x} + \dfrac{1}{3}x + \dfrac{8}{9}$ ✓

$1 = A + B + \dfrac{8}{9} \quad \therefore A + B = \dfrac{1}{9}$ ✓

$\dfrac{dy}{dx} = 6Ae^{6x} + Be^{x} + \dfrac{1}{3}$ ✓

$2 = 6A + B + \dfrac{1}{3}$

$\therefore A = 0{,}311 \quad \left(\dfrac{14}{45}\right)$ ✓

and $B = -0{,}2 \quad \left(-\dfrac{9}{45} \text{ or } -\dfrac{1}{5}\right)$ ✓

$\therefore y = \dfrac{14}{45}e^{6x} - \dfrac{1}{5}e^{x} + \dfrac{1}{3}x + \dfrac{8}{9}$ ✓

$y = 0{,}311e^{6x} - 0{,}2e^{x} + 0{,}333x + 0{,}889$

(14)
[24]

QUESTION 5

5.1 5.1.1

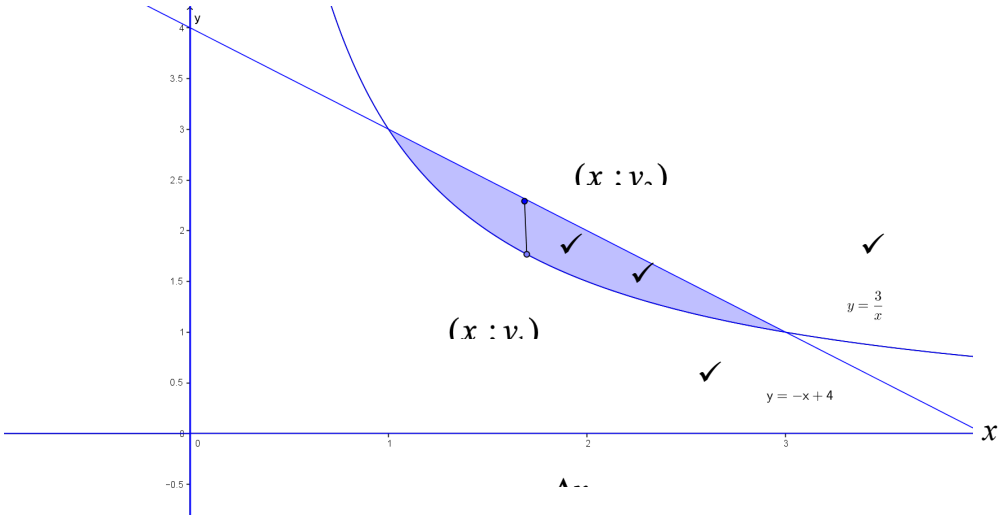
$$\frac{3}{x} = -x + 4$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3; \quad x = 1 \quad ✓$$

$$y = 1; \quad y = 3 \quad ✓ \quad \therefore (3;1) \text{ and } (1;3)$$



✓ ✓ ✓

(6)

5.1.2

$$\Delta V_y = 2\pi x \times (y_2 - y_1) \times \Delta x$$

✓

$$V_y = 2\pi \int_1^3 x(y_2 - y_1)dx$$

Incorrect limits: max 7 marks

✓ ✓

$$= 2\pi \int_1^3 x(-x + 4 - \frac{3}{x})dx$$

$$= 2\pi \int_1^3 (-x^2 + 4x - 3)dx \quad ✓$$

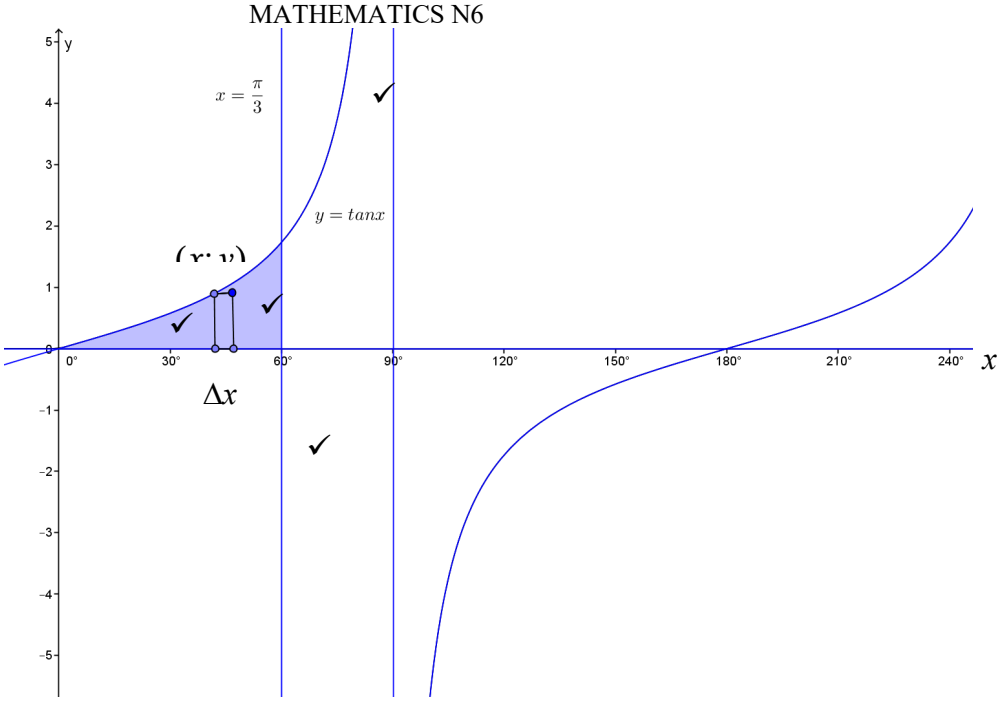
$$= 2\pi \left[-\frac{x^3}{3} + \frac{4x^2}{2} - 3x \right]_1^3 \quad ✓$$

$$= 2\pi \left\{ \left[-\frac{(3)^3}{3} + 2(3)^2 - 3(3) \right] - \left[-\frac{(1)^3}{3} + 2(1)^2 - 3(1) \right] \right\} \quad ✓$$

$$= 2{,}667\pi \quad units^3 \quad or \quad 8{,}278 \quad or \quad \frac{8}{3}\pi \quad units^3 \quad ✓$$

(10)

5.2 5.2.1



(4)

5.2.2 $\Delta V_x = \pi y^2 \Delta x$ ✓

✓

$V_x = \pi \int_0^{\frac{\pi}{3}} y^2 \, dx$

Incorrect limits: max 3 marks

$= \pi \int_0^{\frac{\pi}{3}} (\tan^2 x) \, dx$ ✓

$= \pi \left[(\tan x - x) \right]_0^{\frac{\pi}{3}}$ ✓

$= \pi \left[(\tan \frac{\pi}{3} - \frac{\pi}{3}) - (\tan 0 - 0) \right]$ ✓

$= 0{,}685\pi \quad units^3 \quad or \quad 2{,}152 \; units^3$ ✓

(6)

✓ ✓ ATHEMATICS N6

5.2.3 $\Delta M_y = \pi y^2 \Delta x \times x$

✓

Incorrect limits: max 8 marks

$$\therefore M_y = \pi \int_0^{\frac{\pi}{3}} xy^2 dx$$

$f(x) = x$	$g'(x) = \tan^2 x$
$f'(x) = 1$	$g(x) = \tan x - x$

$$= \pi \int_0^{\frac{\pi}{3}} x(\tan^2 x)dx \quad ✓$$

✓ ✓

$$= \pi \left\{ \left[x(\tan x - x) \right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} (\tan x - x)dx \right\}$$

✓ ✓

$$= \pi \left\{ \left[x(\tan x - x) \right]_0^{\frac{\pi}{3}} - \left[\ln \sec x - \frac{x^2}{2} \right]_0^{\frac{\pi}{3}} \right\}$$

$$= \pi \left\{ \left[\frac{\pi}{3}(\tan \frac{\pi}{3} - \frac{\pi}{3}) \right] - \left[\ln \sec \frac{\pi}{3} - \frac{(\frac{\pi}{3})^2}{2} \right] \right\} \quad ✓$$

$= 0{,}572\pi \quad units^3 \quad or \quad 1{,}798 \quad u^3$ ✓

$\bar{x} = \frac{1{,}798}{2{,}152}$ ✓

$= 0{,}836 \; units$ ✓ (12)

5.3 5.3.1 $2x^2 = 3x$

$2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0; \quad x = \frac{3}{2}$ ✓

$y = 0; \quad y = 4\frac{1}{2}$ ✓ $\therefore (0;0)$ and $(\frac{3}{2};\frac{9}{2})$ (6)

✓

5.3.2 $\Delta A = (y_2 - y_1)\Delta x$ ✓

$A = \int_{\frac{1}{2}}^{1} (y_2 - y_1)dx$

Incorrect limits: max 3 marks

$= \int_{\frac{1}{2}}^{1} (3x - 2x^2)dx$ ✓

$= \left[\frac{3x^2}{2} - \frac{2x^3}{3}\right]_{\frac{1}{2}}^{1}$ ✓

$= \left[\frac{3(1)^2}{2} - \frac{2(1)^3}{3}\right] - \left[\frac{3(\frac{1}{2})^2}{2} - \frac{2(\frac{1}{2})^3}{3}\right]$ ✓

$= 0{,}542 \ \ units^2$ ✓ (6)

5.3.3 ✓ ✓

$\therefore \Delta I_y = (y_2 - y_1)\Delta x \times x^2$

✓ ✓

$= \int_{\frac{1}{2}}^{1} (3x - 2x^2)x^2 dx$

Incorrect limits: max 5 marks

$= \int_{\frac{1}{2}}^{1} (3x^3 - 2x^4)dx$ ✓

$= \left[\frac{3x^4}{4} - \frac{2x^5}{5}\right]_{\frac{1}{2}}^{1}$ ✓

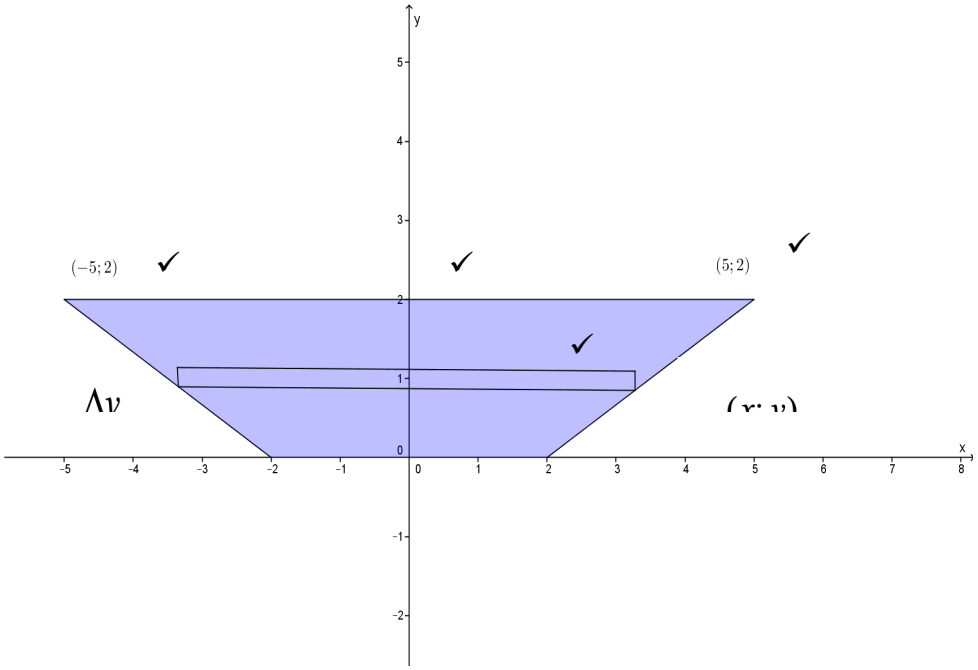
$= \left[\frac{3(1)^4}{4} - \frac{2(1)^5}{5}\right] - \left[\frac{3(\frac{1}{2})^4}{4} - \frac{2(\frac{1}{2})^5}{5}\right]$ ✓

$= 0{,}316 \ \ units^4$ ✓ (8)

5.3.4 $I = \frac{0{,}316}{0{,}542}$ ✓

$= 0{,}553 \ \ A$ ✓ (2)

5.4 5.4.1



$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x-2}=\frac{2-0}{5-2} \quad ✓$$

$$y=\frac{2}{3}(x-2)$$

$$\therefore x=\frac{3}{2}y+2 \quad ✓ \qquad \therefore dA=2(\frac{3}{2}y+2)dy \quad \text{or} \quad \therefore dA=(3y+4)dy$$

(6)

5.4.2 $\int_0^2 rdA$

✓ ✓ ✓

Incorrect limits: max 4 marks

$$=\int_0^2(2-y)(3y+4)dy$$

$$=\int_0^2(6y+8-3y^2-4y)dy \quad ✓$$

$$=\left[\frac{6y^2}{2}+8y-\frac{3y^3}{3}-\frac{4y^2}{2}\right]_0^2 \quad ✓$$

$$=\left[\frac{6(2)^2}{2}+8(2)-\frac{3(2)^3}{3}-\frac{4(2)^2}{2}\right]$$

$$=12 \text{ units}^3 \quad ✓$$

(6)

5.4.3 $\int_0^2 r^2 dA$

✓ ✓

Incorrect limits: max 5 marks

$= \int_0^2 (2-y)^2(3y+4)dy$

$= \int_0^2 (12y - 12y^2 + 3y^3 + 16 - 16y + 4y^2)dy$ ✓

$= \left[\frac{12y^2}{2} - \frac{12y^3}{3} + \frac{3y^4}{4} + 16y - \frac{16y^2}{2} + \frac{4y^3}{3}\right]_0^2$ ✓

$= \left[6(2)^2 - 4(2)^3 + \frac{3(2)^4}{4} + 16(2) - 8(2)^2 + \frac{4}{3}(2)^3\right]$ ✓

$= 14{,}667 \quad \text{units}^4$ ✓

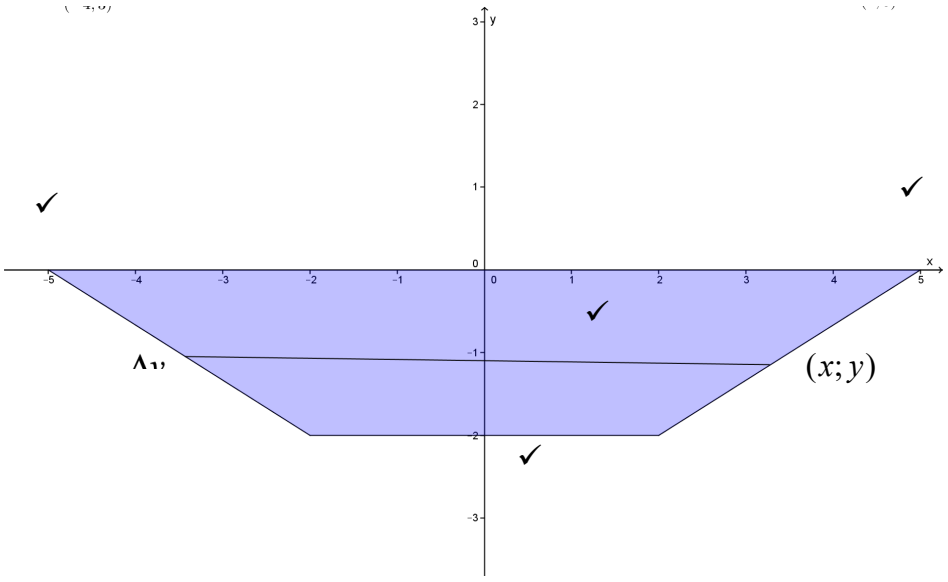
$= y = \frac{14{,}667}{12}$ ✓

$= 1{,}222 \; units$ ✓

(8)

Or alternative method

5.4 5.4.1



✓ ✓ ✓ ✓

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y - 0}{x - 5} = \frac{-2 - 0}{2 - 5}$

$y = \frac{2}{3}(x - 5)$ ✓

$\therefore x = \frac{3}{2}y + 5$ ✓ $\quad \therefore dA = 2(\frac{3}{2}y + 5)dy$ or $\therefore dA = (3y + 10)dy$

(6)

5.4.2 ✓ ✓ ✓

$$= \int_{-2}^{0} y(3y+10)dy$$

Incorrect limits: max 4 marks

$$= \int_{-2}^{0} (3y^2+10y)dy \quad ✓$$

$$= \left[\frac{3y^3}{3}+\frac{10y^2}{2}\right]_{-2}^{0} \quad ✓$$

$$= \left[\frac{3(0)^3}{3}+\frac{10(0)^2}{2}\right]-\left[\frac{3(-2)^3}{3}+\frac{10(-2)^2}{2}\right] ✓$$

$$= -12 \text{ units}^3 \quad ✓$$

(6)

5.4.3 ✓ ✓

$$= \int_{-2}^{0} y^2(3y+10)dy$$

Incorrect limits: max 5 marks

$$= \int_{-2}^{0} (3y^3+10y^2)dy \quad ✓$$

$$= \left[\frac{3y^4}{4}+\frac{10y^3}{3}\right]_{-2}^{0} \quad ✓$$

$$= \left[\frac{3(0)^4}{4}+\frac{10(0)^3}{3}\right]-\left[\frac{3(-2)^4}{4}+\frac{10(-2)^3}{3}\right] ✓$$

$$= 14{,}667 \text{ units}^4 \quad ✓$$

$$\therefore y = \frac{14{,}667}{-12} \quad ✓$$

$$= -1{,}222 \text{ units} \quad ✓$$

(8)

[80]

QUESTION 6

6.1 $x = 5(\cos t + t\sin t)$ and $y = 5(\sin t - t\cos t)$

$\frac{dx}{dt} = 5(-\sin t + t\cos t + \sin t)$ ✓ $\frac{dy}{dt} = 5(\cos t + t\sin t - \cos t)$ ✓

Or $\frac{dx}{dt} = -5\sin t + 5t\cos t + 5\sin t$ OR $\frac{dy}{dt} = 5\cos t + 5t\sin t - 5\cos t$

$= 5t\cos t$ ✓ $= 5t\sin t$ ✓

$\left(\frac{dx}{dt}\right)^2 = (5t\cos t)^2$ $\left(\frac{dy}{dt}\right)^2 = (5t\sin t)^2$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 25t^2\cos^2 t + 25t^2\sin^2 t$ ✓

$= 25t^2(\cos^2 t + \sin^2 t)$

$= 25t^2$ ✓

✓

$S = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Incorrect limits: max 9 marks

$= \int_0^{\pi} \sqrt{25t^2}\,dt$ ✓

$= 5\int_0^{\pi} t.dt$ ✓

$= 5\left[\frac{t^2}{2}\right]_0^{\pi}$ ✓

$= \frac{5}{2}\left[\pi^2\right]$ ✓

$= 2{,}5\,\pi^2$ units or 24,674 units ✓ (12)

6.2 $y = \sqrt{16x}$

$y = 4x^{\frac{1}{2}}$ ✓

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ ✓

$\left(\frac{dy}{dx}\right)^2 = 4x^{-1}$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4}{x}$ ✓

$= \frac{x+4}{x}$ ✓

✓

$A_x = 2\pi\int_1^4 y\sqrt{\frac{x+4}{x}}\,dx$ ✓

Incorrect limits: max 9 marks

✓

$$= 2\pi\int_1^4 4x^{\frac{1}{2}}\sqrt{\frac{x+4}{x}}dx$$

$$= 8\pi\int_1^4 \sqrt{x+4}dx \checkmark$$

$$= 8\pi\int_1^4 (x+4)^{\frac{1}{2}}dx$$

$$= 8\pi\left[\frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 \checkmark$$

$$= \frac{16\pi}{3}\left[(x+4)^{\frac{3}{2}}\right]_1^4$$

$$= \frac{16\pi}{3}\left[(8)^{\frac{3}{2}} - (5)^{\frac{3}{2}}\right]_1^4 \checkmark$$

$= 61{,}051\ \pi$ units^2 or 191,798 units^2 ✓ (12)

[24]

TOTAL: 100