higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

6 April 2016 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = -5x^3y^2 - y^4 + 3x^2y$, determine $\dfrac{\partial^2 z}{\partial x \partial y}$ (2)

1.2 Given: $I = \dfrac{V}{R}$

Calculate the change in I if V decreases with 5 volts and R with 8 ohms. The original value of V is 30 volts and of R is 10 ohms. (4)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \sin^4 5x \cos^3 5x$ (5)

2.2 $y = \dfrac{1}{\sqrt{16x - x^2}}$ (3)

2.3 $y = \sin^4 mx$ (4)

2.4 $y = e^{\frac{x}{2}}.\cos 3x$ (6)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{-x^2 + 3x + 4}{x(1-2x)^2}\,dx$ (6)

3.2 $\displaystyle\int \frac{10x^2 + 7x + 1}{(2x^2+1)(4x-1)}\,dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$2\sin x\frac{dy}{dx} - y(\sin 2x) = \frac{2\sin x}{\sec x} \quad \text{at } (0;1)$$

(5)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2x + 3, \text{ if } y = 1 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 2 \text{ when } x = 0.$$

(7)
[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the two curves $y = \frac{3}{x}$ and $y + x - 4 = 0$. Make a neat sketch of the curves and show the area, in the first quadrant, bounded by the curves. Show the representative strip/element that you will use to calculate the volume (use the SHELL method only) generated if the area bounded by the curves rotates about the y-axis. (3)

5.1.2 Use the SHELL method to calculate the volume generated if the area, described in QUESTION 5.1.1, bounded by the two curves $y = \frac{3}{x}$ and $y + x - 4 = 0$, rotates about the y-axis. (5)

5.2 5.2.1 Make a neat sketch of the graph $y = \tan x$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, the ordinates $y = 0$ and $x = \frac{\pi}{3}$ rotates about the x-axis. (2)

5.2.2 Calculate the volume generated if the area, described in QUESTION 5.1.1, rotates about the x-axis. (3)

5.2.3 Calculate the volume moment about the y-axis as well as the distance of the centre of gravity from the y-axis. (6)

5.3 5.3.1 Calculate the points of intersection of the two curves $y = 2x^2$ and $x = \frac{y}{3}$. Make a neat sketch of the curves and show the area bounded by the curves. Show the representative strip/element, PERPENDICULAR to the x-axis, that you will use to calculate the area bounded by the curves. (3)

5.3.2 Calculate the area described in QUESTION 5.3.1, bounded by the two curves $y = 2x^2$ and $x = \frac{y}{3}$. (3)

5.3.3 Calculate the second moment of area of the area described in QUESTION 5.3.1 about the y-axis. (4)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the area. (1)

5.4 5.4.1 A weir in the form of a trapezium is 2 m high, 10 m wide at the top and 4 m wide at the bottom. The top of the weir is in the water surface.

Sketch the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the retaining wall.

Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate, by using integration, the area moment of the weir about the water level. (3)

5.4.3 Calculate, by using integration, the second moment of area of the weir about the water level, as well as the depth of the centre of pressure on the weir. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve described by the parametric equations, $x = 5(\cos t + t\sin t)$ and $y = 5(\sin t - t\cos t)$, between the points $t = 0$ and $t = \pi$. (6)

6.2 Calculate the surface area generated when the curve of $y = \sqrt{16x}$, over the interval $1 \le x \le 4$, is rotated about the x-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\,.\,f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\,f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\,f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x)\,.\,f'(x)$	-
$\sec f(x)$	$\sec f(x)\tan f(x)\,.\,f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x)\cot f(x)\,.\,f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\,f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2 dx \,;\, V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right) dx \,;\, V_x = 2\pi\int_a^b xydy$$

$$V_y = \pi\int_a^b x^2 dy \,;\, V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right) dy \,;\, V_y = 2\pi\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL

$$I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore y e^{\int P dx} = \int Q e^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$