higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

28 MARCH 2018

This marking guideline consists of 15 pages.

QUESTION 1

1.1 $\frac{\partial z}{\partial x} =$ ✓ ✓

$\frac{\partial z}{\partial y} = xy^{x-1}$ ✓ ✓

1.2 $z = \tan x \sec y$

$\frac{\partial z}{\partial x} = \sec^2 x \sec$ ✓

$\frac{\partial^2 z}{\partial y \partial x} = \sec^2 x \sec y \tan y$ ✓

$\frac{\partial z}{\partial y} = \tan x \sec y \tan y$ ✓

$\frac{\partial^2 z}{\partial x \partial y} = \sec^2 x \sec y \tan y$ ✓

$\therefore \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$

1.3 $y = 2\sin\theta$ $\quad x = 2\cos\theta$

$\frac{dy}{d\theta} = 2\cos\theta$ ✓ $\quad \frac{dx}{d\theta} = -2\sin\theta$ ✓

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos\theta}{-2\sin\theta} = -\cot\theta$ ✓

When $x = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = -\frac{1}{\tan\frac{\pi}{4}} = -1$ ✓

(3 × 2) **[6]**

QUESTION 2

2.1 $\int \sin^4 4x dx$

$= \int (\sin^2 4x)^2 dx$ ✓

$= \int (\frac{1}{2} - \frac{1}{2}\cos 8x)^2 dx$ ✓ ✓

$= \int (\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{4}\cos^2 8x) dx$ ✓

$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{4}(\frac{1}{2} + \frac{1}{2}\cos 16x) \right) dx$ ✓

$= \int (\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{8} + \frac{1}{8}\cos 16x) dx$ ✓

$= \int (\frac{3}{8} - \frac{1}{2}\cos 8x + \frac{1}{8}\cos 16x) dx$ ✓

$= \frac{3}{8}x - \frac{1}{2}.\frac{1}{8}\sin 8x + \frac{1}{8}.\frac{1}{16}\sin 16x + C$ ✓ ✓ ✓

$= \frac{3}{8}x - \frac{1}{16}\sin 8x + \frac{1}{128}\sin 16x + C$ (5)

> Or use
> $\frac{1}{4}\int \cos^2 8x dx = \frac{1}{4}\left(\frac{x}{2} - \frac{\sin 16x}{32} \right)$
> $= \frac{x}{8} - \frac{1}{128}\sin 16x$

2.2 $\int \frac{\cos 3x}{e^{2x}} dx$

$\int e^{-2x}\cos 3x dx \qquad f(x) = e^{-2x} \qquad g'(x) = \cos 3x$

$= e^{-2x}\frac{\sin 3x}{3} - \int -2e^{-2x}\frac{\sin 3x}{3} dx$ ✓ ✓

$= e^{-2x}\frac{\sin 3x}{3} + \frac{2}{3}\int e^{-2x}\sin 3x dx$ ✓ $\qquad f(x) = e^{-2x} \qquad g'(x) = \sin 3x$

$= e^{-2x}\frac{\sin 3x}{3} + \frac{2}{3}\left[e^{-2x}.-\frac{\cos 3x}{3} - \int -2e^{-2x} - \frac{\cos 3x}{3} dx \right]$ ✓ ✓

$= e^{-2x}\frac{\sin 3x}{3} - \frac{2}{9}\cos 3x - \frac{4}{9}\int e^{-2x}\cos 3x dx$ ✓ ✓

$\therefore \int e^{-2x}\cos 3x dx + \frac{4}{9}\int e^{-2x}\cos 3x dx = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x$ ✓ ✓

$\frac{13}{9}\int e^{-2x}\cos 3x dx = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x$

$\int e^{-2x}\cos 3x dx = \frac{9}{13}\left[\frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x \right] + C$ ✓

$or = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x) + C$

Alternative for 2.2 : Starting with $f(x) = \cos 3x$ and $g'(x) = e^{-2x}$ (5)

2.3 $\int \tan^3 5x(\sec^2 5x - 1)dx$

$\int (\tan^3 5x \sec^2 5x - \tan^3 5x)dx$ ✓

✓

$\int (\tan^3 5x \sec^2 5x - \tan 5x \tan^2 5x)dx$

$\int (\tan^3 5x \sec^2 5x - \tan 5x(\sec^2 5x - 1))dx$ ✓

✓

$\int (\tan^3 5x \sec^2 5x - \tan 5x \sec^2 5x + \tan 5x)dx$

✓ ✓ ✓ ✓

$\frac{1}{5}\frac{\tan^4 5x}{4} - \frac{1}{5}\frac{\tan^2 5x}{2} + \frac{1}{5}\ln\left(\sec 5x\right) + C$ (4)

2.4 $\int \frac{1}{\sqrt{1+3x-x^2}}dx$

$-x^2 + 3x + 1$

$= -\left[x^2 - 3x - 1\right]$ ✓

✓ ✓

$= -\left[\left\{x^2 - 3x + \left(\frac{3}{2}\right)^2\right\} - \left(\frac{3}{2}\right)^2 - 1\right]$

$= -\left[\left(x - \frac{3}{2}\right)^2 - \frac{13}{4}\right]$ ✓

$= \frac{13}{4} - \left(x - \frac{3}{2}\right)^2$ ✓

$$\int \frac{1}{\sqrt{\frac{13}{4}-\left(x-\frac{3}{2}\right)^2}}dx \checkmark$$

$$\frac{1}{b^2}\sin^{-1}\frac{bx}{a}+C$$

$$=\frac{1}{1}\sin^{-1}\frac{1\left(x-\frac{3}{2}\right)\checkmark}{\sqrt{\frac{13}{4}}\checkmark}+C$$

$$=\sin^{-1}\frac{2\left(x-\frac{3}{2}\right)}{\sqrt{13}}+C$$

$$=\sin^{-1}\frac{(2x-3)}{\sqrt{13}}+C$$

Or using the formula $ax^2+bx+c=\frac{4ac-b^2}{4a}+a\left(x+\frac{b}{2a}\right)^2$

$$-x^2+3x+1$$

$$=\frac{4(-1)1-(3)^2}{4(-1)}\checkmark+(-1)\left(x+\frac{3}{2.-1}\right)^2\checkmark$$

$$=\checkmark\frac{-4-9}{-4}-\left(x-\frac{3}{2}\right)^2\checkmark$$

$$=\frac{13}{4}-\left(x-\frac{3}{2}\right)^2$$

(4)
[18]

QUESTION 3

3.1 $$\int \frac{(x+3)(x+4)}{(x+1)(x-2)}dx$$

using long division

$$\frac{(x+3)(x+4)}{(x+1)(x-2)} = \frac{x^2+7x+1\!:}{x^2-x-2} \checkmark = 1+\frac{8x+14}{(x+1)(x-2)} \checkmark \checkmark \checkmark$$

$$\frac{8x+14}{(x+1)(x-2)} = \frac{A}{x+1}+\frac{B}{x-2} \checkmark$$

$$8x+14 = A(x-2)+B(x+1) \checkmark$$

$x=2$ $\quad 8(2)+14 = 0+B(2+1)$ $\quad B=10$ ✓

$x=-1$ $\quad 8(-1)+14 = A(-1-2)$ $\quad A=-2$ ✓

$$\therefore \int \frac{(x+3)(x+4)}{(x+1)(x-2)}dx = \int 1+\frac{8x+14}{(x+1)(x-2)}dx$$

$$= \int 1+\frac{-2}{x+1}+\frac{10}{x-2}\,dx \checkmark$$

$$\checkmark \quad \checkmark \quad \checkmark \checkmark$$

$$= x-2\ln(x+1)+10\ln(x-2)+C$$

<u>Alternative 3.1</u> (Note : Since the numerator and denominator are of second degree with coefficients 1 , the quotient will be 1 and the remainder will be linear. The rational fraction obtained can be resolved into partial fractions without long division. Thus the following can be fully credited)

$$\frac{(x+3)(x+4)}{(x+1)(x-2)} = 1+\frac{A}{x+1}+\frac{B}{x-2} \quad \checkmark \checkmark \checkmark \checkmark$$

$$(x+3)(x+4) = 1(x+1)(x-2)+A(x-2)+B(x+1) \quad \checkmark \checkmark$$

$x=-1$ $\quad (-1+3)(-1+4) = +A(-1-2)$ $\quad A=-2$ ✓

$x=2$ $\quad (2+3)(2+4) = 1B(2+1)$ $\quad B=10$ ✓

$$\therefore \int \frac{(x+3)(x+4)}{(x+1)(x-2)}dx = \int 1+\frac{-2}{x+1}+\frac{10}{x-2}\,dx \quad \checkmark$$

$$\checkmark \quad \checkmark \quad \checkmark$$

$$= x-2\ln(x+1)+10\ln(x-2)+C$$

(6)

3.2

$$\frac{3x^2-2x-4}{(2x^2 1)(x-1)}=\frac{Ax+B}{2x^2+1}+\frac{C}{x-1}$$

$$3x^2-2x-4=(Ax+B)(x-1)+C(2x^2+1) \quad ✓$$

$$x=1 \qquad 3-2-4=C(2+1) \qquad C=-1 \quad ✓$$

$$3x^2-2x-4=Ax^2+Bx-Ax-B+2Cx^2+C$$

$$x^2: \quad A+2C=3 \qquad \therefore A-2=3 \qquad A=5 \quad ✓$$

$$x: \qquad B-A=-2 \qquad \therefore B=A-2=5-2=3 \qquad B=3 \quad ✓$$

$$\int\frac{3x^2-2x-4}{(2x^2 1)(x-1)}dx=\int\frac{5x+3}{2x^2+1}dx+\int\frac{-1}{x-1}dx \quad ✓$$

$$=\int\frac{5x}{2x^2+1}dx+\int\frac{3}{2x^2+1}dx+\int\frac{-1}{x-1}dx \quad ✓$$

✓ ✓ ✓ ✓

$$=\frac{5}{4}\ln(2x^2+1)+\frac{3}{\sqrt{2}}\tan^{-1}(\sqrt{2}x)-\ln(x-1)+C$$

(6)
[12]

QUESTION 4

4.1

$$x\frac{dy}{dx}+2y=xe^x$$

$$\frac{dy}{dx}+2\frac{y}{x}=e^x \quad ✓$$

✓ ✓ ✓ ✓

$$e^{\int pdx}=e^{\int\frac{2}{x}dx}=e^{2\ln x}=e^{\ln x^2}=x^2$$

$$\int Qe^{\int pdx}dx=\int e^x x^2 dx$$

✓ ✓

$$=x^2e^x-\int 2xe^x dx$$

✓

$$=x^2e^x-\left[2xe^x-\int 2e^x dx\right]$$

$$=x^2e^x-2xe^x+2e^x \quad ✓$$

$$yx^2=x^2e^x-2xe^x+2e^x+C \quad ✓$$

(5)

4.2

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}-5y=2e^x$$

$r^2+4r-5=0$ ✓

$(r+5)(r-1)=0$

✓ ✓

$r=-5 \quad r=1$

$y_c=Ae^{-5x}+Be^x$ ✓

$y=Cxe^x$ ✓

$\frac{dy}{dx}=Cxe^x+Ce^x$ ✓

$\frac{d^2y}{dx^2}=Cxe^x+Ce^x+Ce^x$

$=Cxe^x+2Ce^x$ ✓

$\therefore Cxe^x+2Ce^x+4\left(Cxe^x+Ce^x\right)-5Cxe^x=2e^x$ ✓

$Cxe^x+2Ce^x+4Cxe^x+4Ce^x-5Cxe^x=2e^x$

$6Ce^x=2e^x \quad \therefore C=\frac{1}{3}$ ✓

$y_p=\frac{1}{3}xe^x$

$y=y_c+y_p$

$y=Ae^{-5x}+Be^x+\frac{1}{3}xe^x$ ✓

$x=0 \text{ and } y=1 \quad 1=A+B................(1)$

$\frac{dy}{dx}=-5Ae^{-5x}+Be^x+\frac{1}{3}xe^x+\frac{1}{3}e^x$ ✓

$x=0 \text{ and } \frac{dy}{dx}=0 \quad 0=-5A+B+\frac{1}{3} \quad 5A-B=\frac{1}{3}..........(2)$

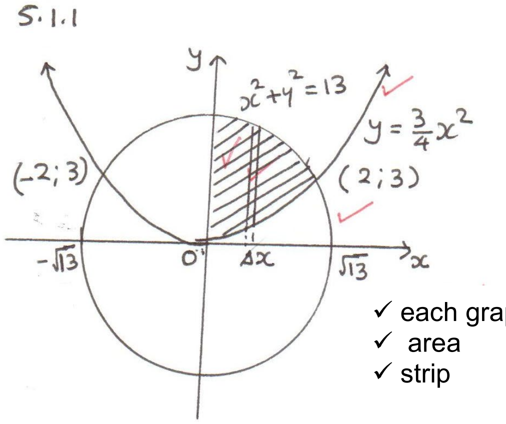
$(2)+(1) \quad 6A=\frac{4}{3} \quad A=\frac{2}{9}$ ✓ $\quad \therefore B=1-A=\frac{7}{9}$ ✓

$y=\frac{2}{9}e^{-5x}+\frac{7}{9}e^x+\frac{1}{3}xe^x$ ✓

or $y=Ae^x+Be^{-5x}$ *Also* $y=2Cxe^x$ which will give $C=\frac{1}{6}$ Thus $y_p=2Cxe^x=2\frac{1}{6}xe^x=\frac{1}{3}xe^x$

(7)
[12]

5.1 5.1.1



✓ each graph
✓ area
✓ strip

(2)

5.1.2

$$A=\int_a^b y_1-y_2\,dx$$

$$=\int_0^2 \sqrt{13-x^2}-\frac{3}{4}x^2 \quad ✓\ ✓\ ✓$$

$$=\left[\frac{13}{2}\sin^{-1}\frac{x}{\sqrt{13}}+\frac{x}{2}\sqrt{13-x^2}-\frac{3}{4}\frac{x^3}{3}\right]_0^2 \quad ✓\ ✓$$

$$=\frac{13}{2}\sin^{-1}\frac{2}{\sqrt{13}}+\frac{2}{2}\sqrt{13-2^2}-\frac{3}{4}\frac{x^3}{3}-0 \quad ✓$$

$$=4{,}822\,units^2 \quad ✓ \quad ✓$$

$y=4x-x^2$ $\qquad$ $y=x$

$\therefore x=4x-x^2$

$x^2-3x=0$

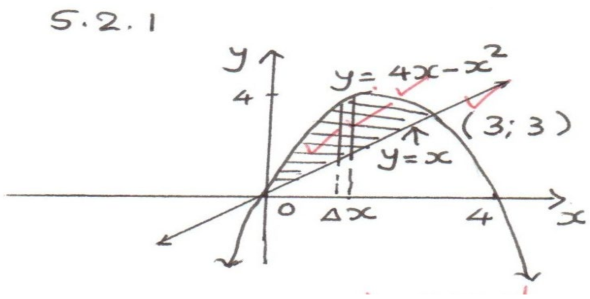
$x(x-3)=0$

$x=0 \quad x=3$

$y=0 \quad y=3$ $\qquad$ $(0;0)$ ✓ $\quad$ $(3;3)$ ✓

(4)

5.2 5.2.1



✓ each graph
✓ area
✓ strip

5.2.2

$$V_x = \pi\int_a^b y_1^2 - y_2^2\,dx$$

$$= \pi\int_0^3 \left(4x - x^2\right)^2 - x^2\,dx \quad ✓✓✓$$

$$= \pi\int_0^3 16x^2 - 8x^3 + x^4 - x^2\,dx \quad ✓$$

$$= \pi\int_0^3 15x^2 - 8x^3 + x^4\,dx \quad ✓$$

$$= \pi\left[\frac{15x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5}\right]_0^3 \quad ✓$$

$$= \pi\left[5(3)^3 - 2(3)^4 + \frac{(3)^5}{5} - 0\right]$$

$$= 21,6\pi \qquad = 21\frac{3}{5}\pi \qquad = \frac{108}{5}\pi \quad ✓$$

$$= 67,858\,units^3 \quad ✓$$

(3)

5.2.3

$$V_{m-y} = \int_a^b r\,dv$$

$$= \int_a^b x2\pi\left(\frac{y_1 + y_2}{2}\right)\left(y_1 - y_2\right)dv$$

$$= \pi\int_a^b x(y_1^2 - y_2^2)\,dx \quad ✓ ✓ ✓$$

1tick for each step or 3 ticks even if only this last step

(4)

$$= \pi\int_0^3 x\left(15x^2 - 8x^3 + x^4\right)dx \quad ✓ \qquad note:\ y_1^2 - y_2^2\ \ from\ \ 5.2.2$$

$$= \pi\int_0^3 \left(15x^3 - 8x^4 + x^5\right)dx \quad ✓$$

$$= \pi\left[\frac{15x^4}{4} - \frac{8x^5}{5} + \frac{x^6}{6}\right]_0^3 \quad ✓$$

$$= \pi\left[\frac{15(3)^4}{4} - \frac{8(3)^5}{5} + \frac{(3)^6}{6} - 0\right] \quad ✓$$

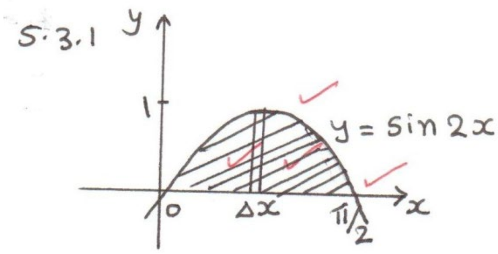
$$= \pi 36,45\,units^4 \qquad = \pi\frac{729}{20}units^4 \qquad = \pi 39\frac{9}{20}units^4$$

$$= 114,511\,units^4$$

$$\bar{x} = \frac{\pi 36,45\,units}{21,6\pi\,units^3} \quad ✓ ✓ \quad 1,688\,units \quad ✓$$

(5)

5.3 5.3.1



✓ shape
✓ *x* intercept
✓ stoip

(2)

$$A = \int_0^{\frac{\pi}{2}} \sin 2x dx \quad ✓ ✓$$

$$= \left[-\frac{\cos 2x}{2} \right]_0^{\frac{\pi}{2}} \quad ✓$$

$$= -\frac{\cos 2\frac{\pi}{2}}{2} - \left(-\frac{\cos 2(0)}{2} \right) \quad ✓ \quad ✓$$

$$= 1 \; units^2 \quad ✓$$

5.3.2

$$I_x = \int_a^b r^2 dA$$

$$I_x = \int_0^{\frac{\pi}{2}} x^2 \sin 2x dx \quad ✓ ✓ ✓$$

$$= \left[x^2 (-\frac{1}{2} \cos \angle x) - \int 2x(-\frac{1}{2} \cos 2x) dx \right]_0^{\frac{\pi}{2}} \quad ✓ ✓$$

$$= \left[-\frac{1}{2} x^2 \cos 2x + \int x \cos 2x dx \right]_0^{\frac{\pi}{2}}$$

(3)

5.3.3

$$= \left[-\frac{1}{2} x^2 \cos 2x + \left\{ x \frac{1}{2} \sin 2x - \int \frac{1}{2} \sin 2x \right\} \right]_0^{\frac{\pi}{2}} \quad ✓ \quad ✓$$

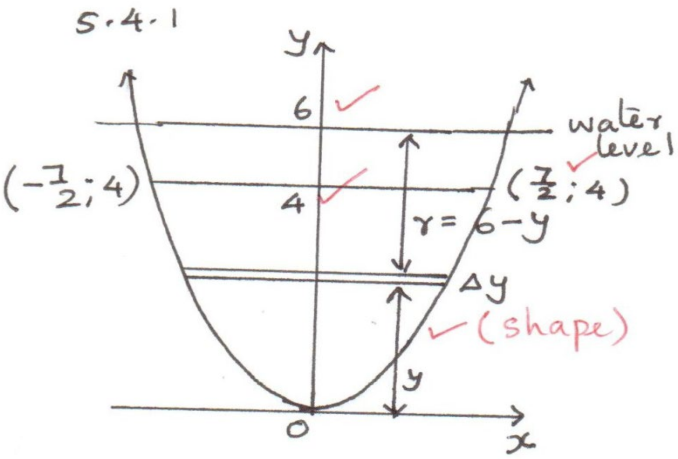
$$= \left[-\frac{1}{2} x^2 \cos 2x + \left\{ x \frac{1}{2} \sin 2x + \frac{1}{4} \cos 2x \right\} \right]_0^{\frac{\pi}{2}} \quad ✓$$

$$= -\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \cos 2\left(\frac{\pi}{2}\right) + \left\{ \left(\frac{\pi}{2}\right) \frac{1}{2} \sin 2\left(\frac{\pi}{2}\right) + \frac{1}{4} \cos 2\left(\frac{\pi}{2}\right) \right\} - \left\{ 0 + 0 + \frac{1}{4} \right\} \quad ✓$$

$$= 0,7337 units^4 \quad ✓$$

(5)

5.4.1



$y = ax^2$ ✓ $\quad 4 = a\left(\frac{7}{2}\right)^2$ ✓ $\quad 4 = a\left(\frac{49}{4}\right) \quad a = \frac{16}{49}$ ✓

$\therefore y = \frac{16}{49}x^2 \quad x^2 = \frac{49}{16}y \quad x = \frac{7}{4}y^{\frac{1}{2}}$ ✓

First moment of area $= \int_a^b r\,dA = \int_0^4 (6-y)2x\,dy$
✓

$= \int_0^4 (6-y)2\frac{7}{4}y^{\frac{1}{2}}dy$ ✓

$= \frac{7}{2}\int_0^4 (6-y)\,y^{\frac{1}{2}}dy$

$= \frac{7}{2}\int_0^4 \left(6y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy$ ✓

$= \frac{7}{2}\left[\frac{6y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4$

$= \frac{7}{2}\left[4y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right]_0^4$ ✓

$= \frac{7}{2}\left[4(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}} - 0\right]$ ✓ $\quad = 67,2m^3$ ✓

Second moment=

$\int_a^b r^2 dA \quad = \quad \int_0^4 (6-y)^2 2.\frac{7}{4}y^{\frac{1}{2}}dy$ ✓ ✓ ✓

$= \frac{7}{2}\int_0^4 (36 - 12y + y^2).y^{\frac{1}{2}}dy$ ✓

$= \frac{7}{2}\int_0^4 (36y^{\frac{1}{2}} - 12y^{\frac{3}{2}} + y^{\frac{5}{2}})dy$ ✓

$$=\frac{7}{2}\left[36y^{\frac{3}{2}}\frac{2}{3}-12y^{\frac{5}{2}}\frac{2}{5}+y^{\frac{7}{2}}\frac{2}{7}\right]_0^4 \quad ✓$$

$$=\frac{7}{2}\left[36(4)^{\frac{3}{2}}\frac{2}{3}-12(4)^{\frac{5}{2}}\frac{2}{5}+(4)^{\frac{7}{2}}\frac{2}{7}-0\right] \quad ✓$$

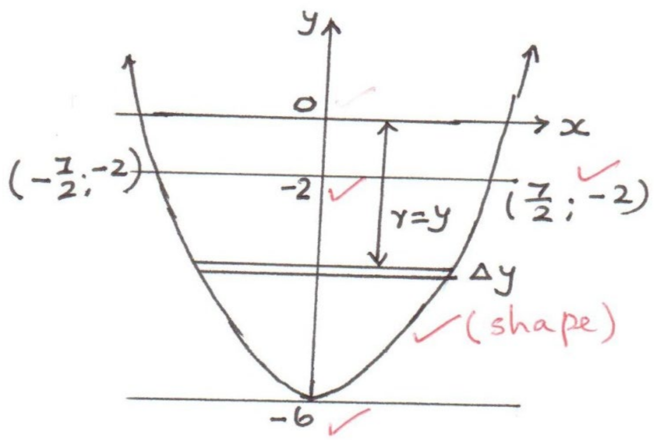
$$=\frac{7}{2}\left[24(4)^{\frac{3}{2}}-\frac{24}{5}(4)^{\frac{5}{2}}+(4)^{\frac{7}{2}}\frac{2}{7}-0\right]$$

$$=262,4m^4 \quad ✓$$

$$\bar{\bar{y}}=\frac{262,4}{67,2}m \quad ✓$$

=3,905 m

5.4.1 (alternative)



$$y=ax^2-6 \quad ✓$$

$$-2=a\left(\frac{7}{2}\right)^2-6 \quad ✓ \qquad a=\frac{16}{49} \quad ✓$$

$$y=\frac{16}{49}x^2-6 \qquad x^2=\frac{49}{16}\,(y+6) \qquad x=\frac{7}{4}(y+6)^{\frac{1}{2}} \quad ✓$$

(4)

5.4.2 **<u>(Alternative)</u>**

First moment

$$= \int_{-6}^{-2} y2\left\{\frac{7}{4}(y+6)^{\frac{1}{2}}\right\}dy \quad ✓ \quad ✓$$

$$= \frac{7}{2}\int_{-6}^{-2} y(y+6)^{\frac{1}{2}}dy \qquad u = y+6 \quad y = u-6 \quad dy = du$$

$$y = -2 \ \ then \ u = 4 \ and \ y = -6 \ \ then \ u = 0$$

$$= \frac{7}{2}\int_{0}^{4}(u-6)u^{\frac{1}{2}}du \quad ✓$$

$$= \frac{7}{2}\int_{0}^{4}\left(u^{\frac{3}{2}} - 6u^{\frac{1}{2}}\right)du \quad ✓$$

$$= \frac{7}{2}\left[\frac{2}{5}u^{\frac{5}{2}} - 6\frac{2}{3}u^{\frac{3}{2}}\right]_4^0 ✓ = \frac{7}{2}\left[0 - \left\{\frac{2}{5}(4)^{\frac{5}{2}} - 4(4)^{\frac{3}{2}}\right\}\right]_0^4 \quad ✓$$

$= $ --67,2 m^3 ✓

(3)

5.4.3 **<u>Second moment (Alternative)</u>**

$$= \int_a^b r^2 dA$$

$$= \int_{-6}^{-2} y^2 2\left\{\frac{7}{4}(y+6)^{\frac{1}{2}}\right\}dy \quad ✓ \quad ✓ \qquad u = y+6 \qquad y = u-6 \qquad dy = du$$

$$y = -2 \ then \ u = 4 \ \ and \ \ y = -6 \ \ then \quad ✓$$

$$= \frac{7}{2}\int_0^4 (u-6)^2 u^{\frac{1}{2}}du \quad ✓$$

$$= \frac{7}{2}\int_0^0 (u^2 - 12u + 36)u^{\frac{1}{2}}du \quad ✓$$

$$= \frac{7}{2}\int_4^0 (u^{\frac{5}{2}} - 12u^{\frac{3}{2}} + 36u^{\frac{1}{2}})du \quad ✓$$

$$= \frac{7}{2}\left[\left(\frac{2}{7}u^{\frac{7}{2}} - 12u^{\frac{5}{2}}\frac{2}{5} + 36u^{\frac{3}{2}}\frac{2}{3}\right)\right]_4^0 \quad ✓$$

$$= \frac{7}{2}\left[0 - \left\{\frac{2}{7}(4)^{\frac{7}{2}} - 12(4)^{\frac{5}{2}}\frac{2}{5} + 36(4)^{\frac{3}{2}}\frac{2}{3}\right\}\right] \quad ✓$$

$$= 262,4m^4 \quad ✓$$

$$\bar{y} = \frac{262,4}{-67,2} = -3,905m \quad ✓ \quad ✓$$

(5)

[40]

QUESTION 6

6.1 $y = \ln(\sec x)$

$$\frac{dy}{dx} = \frac{1}{\sec x}\sec x \tan x \quad = \tan x \; ✓✓✓$$

$$\left(\frac{dy}{dx}\right)^2 = (\tan x)^2 \; ✓$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2} \; ✓$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{1+\tan^2 x}\,dx \; ✓$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x}\,dx \; ✓$$

$$= \int_0^{\frac{\pi}{3}} \sec x\,dx \; ✓$$

$$= \left[\ln(\sec x + \tan x)\right]_0^{\frac{\pi}{3}} \; ✓$$

$$= \ln\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) - \ln(\sec 0 + \tan 0) \; ✓$$

$$= 1{,}317\,units \; ✓✓$$

(6)

6.2 $x = 2\cos\theta$ $\qquad$ $y = 2\sin\theta$

$$\frac{dx}{d\theta} = -2\sin\theta \qquad \frac{dy}{d\theta} = 2\cos\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 = 4\sin^2\theta \qquad \left(\frac{dy}{d\theta}\right)^2 = 4\cos^2\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4\sin^2\theta + 4\cos^2\theta$$

$$= 4(1) = 4$$

$$A_y = 2\pi\int_{\theta_1}^{\theta_2} x\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta$$

$$= 2\pi\int_0^{\frac{\pi}{2}} 2\cos\theta\sqrt{4}\;d\theta \; ✓$$

$$= 8\pi\left[\sin\theta\right]_0^{\frac{\pi}{2}} \; ✓$$

$$= 8\pi\left[\sin\frac{\pi}{2} - \sin 0\right] \; ✓$$

$$= 8\pi = 25{,}133\,units^2 \; ✓$$

(6)
[12]

TOTAL: 100