



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1040(E)(M28)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

28 March 2018 (X-Paper)

09:00–12:00

Nonprogrammable calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA
NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
 2. Read ALL the questions carefully.
 3. Number the answers according to the numbering system used in this question paper.
 4. Questions may be answered in any order but subsections of questions must be kept together.
 6. Show ALL the intermediate steps. Round calculations off to THREE decimal places.
 7. ALL the formulae used must be written down.
 8. Questions must be answered in BLUE or BLACK ink.
 9. Write neatly and legibly.
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QUESTION 1

1.1 Given: $z = y^x$ determine $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

1.2 Given: $z = \tan x \sec y$ prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$

1.3 If $y = 2 \sin \theta$ and $x = 2 \cos \theta$ determine $\frac{dy}{dx}$ where $\theta = \frac{\pi}{4}$

(3 × 2) [6]

QUESTION 2

Determine: $\int y \, dx$ if:

2.1 $y = \sin^4 4x$ (5)

2.2 $y = \frac{\cos 3x}{e^{2x}}$ (5)

2.3 $y = \tan^3 5x (\sec^2 5x - 1)$ (4)

2.4 $y = \frac{1}{\sqrt{1+3x-x^2}}$ (4)

[18]

QUESTION 3

Use partial fractions to integrate the following:

3.1 $\int \frac{(x+3)(x+4)}{(x+1)(x-2)} dx$

3.2 $\int \frac{3x^2 - 2x - 4}{(2x^2 + 1)(x-1)} dx$

(2 × 6) [12]

QUESTION 4

4.1 Determine the general solution of:

$$x \frac{dy}{dx} + 2y = xe^x \quad (5)$$

4.2 Find the particular solution of:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 5y = 2e^x \quad \text{if } y=1 \text{ when } x=0 \quad \text{and} \quad \frac{dy}{dx}=0 \text{ when } x=0 \quad (7)$$

[12]

QUESTION 5

5.1 5.1.1 The points of intersection of $x^2 + y^2 = 13$ and $y = \frac{3}{4}x^2$ are given as (-2;3) and (2;3).

Make a neat sketch of the two graphs and show the area bounded by the two curves and the y -axis in the first quadrant. Show the representative strip/element that you will use to calculate the area. (2)

5.1.2 Calculate the area described in QUESTION 5.1.1. (4)

5.2 5.2.1 Determine the point of intersection of $y = 4x - x^2$ and $y = x$.

Make a neat sketch of the two graphs and show the area enclosed by the graphs. Show the representative strip/element that you will use to calculate the volume when the area is rotated about the x -axis. (3)

5.2.2 Calculate the volume generated when the area described in QUESTION 5.2.1 is rotated about the x -axis. (4)

5.2.3 Calculate the distance of the centre of gravity from the y -axis of the solid of revolution obtained when the area in QUESTION 5.2.1 rotates about the x -axis. (5)

5.3 5.3.1 Sketch the graph of $y = \sin 2x$ for $0 \leq x \leq \frac{\pi}{2}$. Show the area enclosed by the graph and the x -axis. Show the representative strip/element to be used to calculate the area and the second moment of area. (2)

5.3.2 Calculate the area described in QUESTION 5.3.1. (3)

5.3.3 Calculate the second moment of area about y -axis of the area described in QUESTION 5.3.1. (5)

- 5.4 5.4.1 A vertical sluice gate in the form of a parabola is placed in a dam wall. The top of the gate is 7 m wide and lies 2m below the water level. The sluice gate is 4m high.
- Make a neat sketch of the sluice gate and show the representative strip you will use to calculate the first and second moment of area of the gate about the water level. Calculate the relationship between the variables. (4)
- 5.4.2 Calculate the first moment of area of the gate about the water level. (3)
- 5.4.3 Calculate the second moment of area of the gate about the water level and hence the depth of centre of pressure on the sluice gate. (5)
- [40]**

QUESTION 6

- 6.1 Calculate the length of the curve $y = \ln(\sec x)$ from $x=0$ to $x = \frac{\pi}{3}$.
- 6.2 Calculate the surface area generated when the curve given by $x = 2\cos\theta$ and $y = 2\sin\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$ rotates about y-axis.
- (2 × 6) **[12]**
- TOTAL: 100**

MATHEMATICS N6**FORMULA SHEET**

Any other applicable formula may also be used.

Trigonometry

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\tan x = \frac{\sin x}{\cos x}; \quad \sin x = \frac{1}{\operatorname{cosec} x}; \quad \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n dx$
e^{ax+b}	$e^{ax+b} \cdot \frac{d}{dx} (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx} (ax+b)} + C$
a^{dx+e}	$a^{dx+e} \cdot \ln a \cdot \frac{d}{dx} (dx+e)$	$\frac{a^{dx+e}}{\ln a \cdot \frac{d}{dx} (dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} \cdot \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} \cdot \ln a \cdot \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x) dx$
$\sin f(x)$	$\cos f(x) \cdot f'(x)$	-
$\cos f(x)$	$-\sin f(x) \cdot f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \cdot f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \cdot f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \cdot f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cot f(x) \cdot f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2 + 1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2 + 1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x) \sqrt{[f(x)]^2 - 1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x) \sqrt{[f(x)]^2 - 1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a} \tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x) dx$
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$$\cot^2(ax) \quad - \quad -\frac{1}{a} \cot(ax) - x + C$$

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

$$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2 x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2 x^2} dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2 x^2} + C$$

$$\int \frac{dx}{a^2 - b^2 x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2} dx = \frac{x}{2} \sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2 x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2 x^2 \pm a^2} \right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y dx; A_x = \int_a^b (y_1 - y_2) dx$$

$$A_y = \int_a^b x dy; A_y = \int_a^b (x_1 - x_2) dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx; V_x = \pi \int_a^b (y_1^2 - y_2^2) dx; V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy; V_y = \pi \int_a^b (x_1^2 - x_2^2) dy; V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = r dA \quad A_{m-y} = r dA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b r dA}{A}; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b r dA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b r dV; \quad V_{m-y} = \int_a^b r dV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b r dV}{V}; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b r dV}{V}$$

MOMENTS OF INERTIA

Mass = Density \times volume

$$M = \rho V$$

DEFINITION: $I = m r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \quad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{y} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots + \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$A_x = \int_u^{u_2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$A_y = \int_{u_1}^{u_2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$S = \int_{u_1}^{u_2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx} dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A \cos bx + B \sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx}$$