higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1040**(E)**(M29)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

29 March 2019 (X-Paper)
09:00–12:00

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Keep subsections of questions together.
5. Round ALL calculations off to THREE decimal places.
6. Use the correct symbols and units.
7. Start each new question on a new page.
8. Write neatly and legibly.

QUESTION 1

1.1 Given: $z = \tan(xy)$

Determine $\frac{\partial^2 z}{\partial x^2}$

1.2 A rectangle has length x and width y.

Calculate the change in length of the diagonal r of the rectangle if the length changes from 20 cm to 22 cm and the width from 10 cm to 8 cm.

(2 × 3) **[6]**

QUESTION 2

Determine $\int y dx$ if

2.1 $y = 1 - \tan^4 3x$ (4)

2.2 $y = x(\ln x)^2$ (4)

2.3 $(1 - 2sin^2 2x)^2$ (4)

2.4 $y = \frac{\sin 2x}{e^{2x}}$ (4)

2.5 $y = \frac{x^2 - 2}{x^4 - 4}$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^2 + x - 7}{x^2 + x - 6} dx$

3.2 $\int \frac{7x^2 - 12x + 8}{(2x-1)(x^2 - 2x + 2)} dx$

(2 × 6) **[12]**

QUESTION 4

4.1 Determine the particular solution of $\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 3x - \sin x + \cos x$

if $x = 1$ when $y = 1$

4.2 Determine the general solution of $6\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = x^2$

(2 × 6) **[12]**

QUESTION 5

5.1 5.1.1 Sketch the graphs of $x^2 + 4y^2 = 16$ and $x^2 + y^2 = 16$.
Show the area bounded by the graphs in the first quadrant.
Show a representative strip perpendicular to the *x*-axis. (3)

5.1.2 Calculate the area described in QUESTION 5.1.1. (5)

5.1.3 Calculate the distance of the centroid from the y-axis of the area described in QUESTION 5.1.1. (5)

5.2 5.2.1 Make a neat sketch of the graph of $y = \sin x$ for $0 \le x \le \pi$.
Show the area bounded by the graph with the lines $y = 1$ and $x = 0$.
Show the representative strip/element that you will use to calculate the volume generated when this area rotates about the x-axis. (2)

5.2.2 Calculate the volume generated when the area described in QUESTION 5.2.1 rotates about the x-axis. (4)

5.3 5.3.1 Make a neat sketch of the curve $y = e^x$ and show the area bounded by the curve, the x-axis and the lines $x = -1$ and $x = 1$.
Show the representative strip that you will use to calculate the volume when this area rotates about the x-axis. (3)

5.3.2 Calculate the volume when the area described in QUESTION 5.3.1 rotates about the x-axis. (3)

5.3.3 Calculate the moment of inertia about the x-axis of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x-axis. Express the answer in terms of mass. (5)

5.4 5.4.1 A vertical plate in the shape of a trapezium is installed in a dam wall. The top of the plate is 10 m wide and lies 1 m below the water level. The bottom of the plate is 8 m wide and the height of the plate is 3 m.

Draw a sketch of the plate and show the representative strip that you will use to calculate the first moment of area and the second moment of area about the water level. (2)

5.4.2 Calculate the relationship between x and y as well as the area moment of the plate about the water level. (4)

5.4.3 Calculate the second moment of area of the plate about the water level as well as the depth of the centre of pressure on the plate. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve given by $x = e^{\theta} \sin\theta$ *and* $y = e^{\theta}\cos\theta$ from $\theta = 0$ to $\theta = \dfrac{\pi}{3}$

6.2 Calculate the surface area generated when the curve $y = \dfrac{3}{2}x$ between the points (2;3) and (4;6) rotates about the x-axis.

(2 × 6) **[12]**

TOTAL 100

MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \qquad (n\neq -1)$
ax^n	$a\ \frac{d}{dx}\ x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}\ (ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}\ (dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}\ ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\ \frac{d}{dx}\ f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}\ f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}\ f(x)$	-
$\sin ax$	$a\cos ax$	$-\ \frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln\,[\sec\,(ax)]+C$
$\cot ax$	$-a\,\text{cosec}^2\,ax$	$\frac{1}{a}\ln\,[\sin\,(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln\,[\sec ax+\tan ax]+C$
$\text{cosec}\,ax$	$-a\,\text{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\ dx$
$\sin f(x)$	$\cos f(x)\ .\ f'(x)$	-
$\cos f(x)$	$-\sin f(x)\ .\ f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\ .\ f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x)\ .\ f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x)\ .\ f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x)\ .\ f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\ dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\ g'(x)\ dx = f(x)\ g(x) - \int f'(x)\ g(x)\ dx$

$\int [f(x)]^n\ f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\ dx = \ln\ f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\ dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) + C$

$\int \sqrt{x^2 \pm b^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$

Applications of integration

AREAS

$A_x = \int_a^b y dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$

$A_y = \int_a^b x dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \, ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \, ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \, ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \, ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \, ; \, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\ (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_u^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int Pdx} = \int Qe^{\int Pdx} dx$$

$$y = Ae^{r_1 x} + Be^{r2x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$