higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

1 April 2020

This marking guideline consists of 15 pages.

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NOTE: This marking guideline adds up to 200, and the total must be divided by 2 to get a mark out of 100.

QUESTION 1

1.1

$$z = \ln\left(\sqrt{x} + \sqrt{y}\right)$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x} + \sqrt{y}} \frac{1}{2\sqrt{x}} \quad \surd\surd$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{x} + \sqrt{y}} \frac{1}{2\sqrt{y}} \quad \surd\surd$$

$$\sqrt{x}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\sqrt{y} = \sqrt{x}\frac{1}{\sqrt{x} + \sqrt{y}}\frac{1}{2\sqrt{x}} + \sqrt{y}\frac{1}{\sqrt{x} + \sqrt{y}}\frac{1}{2\sqrt{y}} \surd$$

$$= \frac{1}{2\sqrt{x} + \sqrt{y}} + \frac{1}{2\sqrt{x} + \sqrt{y}}$$

$$\frac{1+1}{2\sqrt{x} + \sqrt{y}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{y}} \surd$$

(6)

1.2

$$V = \pi r^2 h$$

$$\Delta V = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h \surd$$

$$= 2\pi rh\Delta r + \pi r^2 \Delta h \surd\surd$$

$$= 2\pi(4)(20)(0,1) + \pi(4)^2(0,5) \surd\surd$$

$$= 75,398cm^3 \surd$$

(6)

[12]

QUESTION 2

2.1

$(x+3)^2 - 8x$

$= x^2 + 6x + 9 - 8x$ √

$= x^2 - 2x + 9$ √

$= x^2 - 2x + 1 + 9 - 1$ √

$= (x-1)^2 + 8$ √ √

$\int \frac{1}{(x+3)^2 - 8x} dx$

$= \int \frac{1}{8 + (x-1)^2} dx$ √

$= \frac{1}{\sqrt{8}.1} \tan^{-1} \frac{x-1}{\sqrt{8}} + C$ √ √

or $ax^2 + bx + c = \frac{4ac - b^2}{4a} + a\left(x + \frac{b}{2a}\right)^2$

$x^2 - 2x + 9 = \frac{4.1.9 - (-2)^2}{4.1} + 1\left(x + \frac{-2}{2.1}\right)^2$

$= 8 + (x-1)^2$

(8)

2.2

$\int \ln 2x \ln x dx = \ln 2x(x\ln x - x) - \int \frac{1}{x}(x\ln x - x)dx$ √ √ √

$f(x) = \ln 2x$, $g'(x) = \ln x$, $f'(x) = \frac{1}{x}$, $g(x) = x\ln x - x$

$= \ln 2x(x\ln x - x) - \int (\ln x - 1)\, dx$ √

$= \ln 2x(x\ln x - x) - (x\ln x - x - x) + C$ √ √ √

$= \ln 2x(x\ln x - x) - x\ln x + 2x + C$ √

__**Alternative**__

$\int \ln 2x \ln x dx$

$= \ln x(x\ln 2x - x) - \int \frac{1}{x}(x\ln 2x - x)dx$ √ √ √

$f(x) = \ln x$, $g'(x) = \ln 2x$, $f'(x) = \frac{1}{x}$, $g(x) = x\ln 2x - x$

$= \ln x(x\ln 2x - x) - \int (\ln 2x - 1)\, dx$ √

$= \ln x(x\ln 2x - x) - (x\ln 2x - x - x) + C$ √ √ √

$= \ln x(x\ln 2x - x) - x\ln 2x + 2x + C$ √

$= x\ln x\ln 2x - x\ln x - x\ln 2x + 2x + C$

$= \ln 2x(x\ln x - x) - x\ln x + 2x + C$

(8)

2.3

$$\int \frac{1+\tan^2 x}{\tan^3 x}dx$$

$$\int (\tan x)^{-3}\sec^2 x dx \surd\surd$$

$$= \frac{(\tan x)^{-2}}{-2} + C \surd\surd$$

$$= -\frac{1}{2}\frac{1}{\tan^2 x} + C$$

$$= -\frac{1}{2}\cot^2 x + C$$

Alternative 1

$$\int \frac{1+\tan^2 x}{\tan^3 x}dx$$

$$= \int \left(\frac{1}{\tan^3 x} + \frac{\tan^2 x}{\tan^3 x}\right)dx$$

$$= \int (\cot^3 x + \cot x)dx \surd$$

$$= \int (\cot x \cot^2 x + \cot x)dx$$

$$= \int \left(\cot x(\cos ec^2 x - 1) + \cot x\right)dx \surd$$

$$= \int \cot x \cos ec^2 x - \cot x + \cot x)dx$$

$$= \int \cot x \cos ec^2 x \, dx \surd$$

$$= -\frac{1}{2}\cot^2 x + C \surd$$

(4)

Alternative 2

$$\int \frac{1+\tan^2 x}{\tan^3 x}dx$$

$$= \int \frac{\sec^2 x}{\tan^3 x}dx$$

$$= \int \frac{1}{\cos^2 x}\frac{\cos^3 x}{\sin^3 x}dx \surd$$

$$= \int \frac{\cos x}{\sin^3 x}dx \surd$$

$$= \int (\sin x)^{-3}\cos x \, dx \surd$$

$$= \frac{(\sin x)^{-2}}{-2} + C \surd$$

$$= -\frac{1}{2}(\cos ec^2 x) + C$$

$$= -\frac{1}{2}(1+\cot^2 x) + C$$

$$-\frac{1}{2}\cot^2 x - \frac{1}{2} + C$$

$= -\frac{1}{2}\cot^2 x + C$ Because $-\frac{1}{2} + C$ is yet another constant

2.4

$$\int (\sin^3 x + \cos^3 x)dx$$

$$= \int (\sin^2 x \sin x + \cos^2 x \cos x)dx \surd\surd$$

$$= \int (1-\cos^2 x)\sin x + (1-\sin^2 x)\cos x \; dx \quad \surd\surd$$

$$= \int \sin x - \cos^2 x \sin x + \cos x - \sin^2 x \cos x \; dx \surd\surd$$

$$= -\cos x + \frac{1}{3}\cos^3 x + \sin x - \frac{1}{3}\sin^3 x + C \surd\surd\surd\surd$$

(10)

Alternative

$$\int(\sin^3 x+\cos^3 x)dx$$
$$=\int(\sin x+\cos x)\left(\sin^2 x-\sin x\cos x+\cos^2 x\right)dx \surd\surd\surd$$
$$=\int(\sin x+\cos x)\left(1-\sin x\cos x\right)dx \surd$$
$$=\int(\sin x+\cos x-\sin^2 x\cos x-\cos^2 x\sin x\ dx \surd\surd$$
$$=-\cos x+\sin x-\frac{1}{3}\sin^3 x+\frac{1}{3}\cos^3 x+C \surd\surd\surd\surd$$

2.5

$$\int 3\tan^{-1}\frac{x}{3}dx$$
$$=3x\tan^{-1}\frac{x}{3}-\int\frac{3}{x^2+9}3xdx \surd\surd$$
$$=3x\tan^{-1}\frac{x}{3}-\frac{9}{2}\ln(x^2+9)+C \surd$$

$f(x)=\tan^{-1}\frac{x}{3}$ $\qquad g'(x)=3$

$f'(x)=\dfrac{\frac{1}{3}}{1+\frac{x^2}{9}}\surd$ $\qquad g(x)=3x\surd$

$=\dfrac{3}{x^2+9}\surd$

(6)

Alternative

$$\int 3\tan^{-1}\frac{x}{3}dx = 3x\tan^{-1}\frac{x}{3}-\int\frac{x}{1+\frac{x^2}{9}}dx ✓✓✓$$
$$=3x\tan^{-1}\frac{x}{3}-\frac{9}{2}\int\frac{\frac{2}{9}x}{1+\frac{x^2}{9}}dx ✓$$
$$=3x\tan^{-1}\frac{x}{3}-\frac{9}{2}\ln\left(1+\frac{x^2}{9}\right)+C ✓✓$$

$f(x)=\tan^{-1}\frac{x}{3}\Rightarrow f'(x)=\dfrac{\frac{1}{3}}{1+\frac{x^2}{9}}$

$g'(x)=3\Rightarrow g(x)=3x$

[36]

QUESTION 3

3.1

$$6x^2+x-1=(3x-1)(2x+1)\surd$$
$$\frac{5-5x}{6x^2+x-1}=\frac{A}{3x-1}+\frac{B}{2x+1}\surd$$
$$5-5x=A(2x+1)+B(3x-1)\surd$$
$$when\quad x=-\frac{1}{2}\quad B=-3\ \surd$$
$$when\quad x=\frac{1}{3}\quad A=2\surd$$
$$\int x+\frac{5-5x}{6x^2+x-1}dx=\int x+\frac{2}{3x-1}+\frac{-3}{2x+1}dx \surd\surd$$
$$=\frac{x^2}{2}+\frac{2}{3}\ln(3x-1)-\frac{3}{2}\ln(2x+1)+C \surd\surd\surd$$

$$5-5x=A(2x+1)+B(3x-1)$$
$$5-5x=2Ax+A+3Bx-B$$
$$2A+B=-5$$
$$A-B=5$$
$$\therefore A=2 \qquad B=-3$$

(10)

3.2 $\dfrac{2x^3+6x^2-12}{x(x+3)(x^2+3x+4)}=\dfrac{A}{x}+\dfrac{B}{x+3}+\dfrac{Cx+D}{x^2+3x+4}$ √

$2x^3+6x^2-12=A(x+3)(x^2+3x+4)+Bx(x^2+3x+4)+(Cx+D)x(x+3)$ √

$Ifx=-3$ $\quad -54+54-12=B(-3)(9-9+4)$ √ $\quad -12=-12B \quad \therefore B=1$ √

$Ifx=0$ $\quad -12=A.3.4$ √ $\quad \therefore A=-1$ √

equating $\quad x^3: \quad A+B+C=2\Rightarrow -1+1+C=2$ √ $\quad \therefore C=2$ √

$Ifx=1$ $\quad 2+6-12=A(1+3)(1+3+4)+B.1.(1+3+4)+(C.1+D).1.(1+3)$ √

$-4=32A+8B+4C+4D \quad .....(1)$

substituting $\quad A=-1; B=1andC=2 \quad -4=32(-1)+8.1+4.2+4D \quad \therefore D=3$ √

$\displaystyle\int\frac{2x^3+6x^2-12}{x(x+3)(x^2+3x+4)}=\int\frac{-1}{x}dx+\int\frac{1}{x+3}dx+\int\frac{2x+3}{x^2+3x+4}dx$ √

$=-\ln x+\ln(x+3)+\ln(x^2+3x+4)+C$ √ √ √ (14)

Alternative for constants A, B, C and D

$2x^3+6x^2-12=A(x^3+3x^2+4x+3x^2+9x+12)+B(x^3+3x^2+4x)+(Cx^2+Dx)(x+3)$

$=Ax^3+6Ax^2+13Ax+12A+Bx^3+3Bx^2+4Bx+Cx^3+Dx^2+3Cx^2+3Dx$ √..........(1)

Equating x^3 $\quad A+B+C=2 \quad ....(2)$

Equating x^2 $\quad 6A+3B+D+3C=6 \quad ....(3)$

Equating x $\quad 13A+4B+3D=0 \quad ....(4)$

Equating constants $\quad 12A=-12 \quad \therefore A=-1$ √

$A=-1$ in (2) $\quad B+C=3 \quad \Rightarrow C=3-B$ √

$A=-1$ in (4) $\quad -13+4B+3D=0\Rightarrow D=\frac{1}{3}(13-4B)$ √

$A=-1;\ C=3-B$ and $D=\frac{1}{3}(13-4B)$ into (3) $\quad -6+3B+\frac{1}{3}(13-4B)+3(3-B)=6$ √

$-18+9B+13-4B+27-9B=18$

$-4B=-4 \quad \therefore B=1$ √ then $C=2$ √ and $D=3$ √

[24]

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QUESTION 4

4.1

$$\frac{dx}{dy}-3y=2x$$

$$\frac{dx}{dy}-2x=3y\ \surd$$

$$e^{\int pdy}=e^{\int -2dy}=e^{-2y}\quad \surd\surd$$

$$\int Qe^{\int pdy}dy=\int 3ye^{-2y}dy\ \surd$$

$$=3y\frac{e^{-2y}}{-2}-\int 3\frac{e^{-2y}}{-2}dy\ \surd$$

$$=3y\frac{e^{-2y}}{-2}+\frac{3}{2}\int e^{-2y}dy$$

$$=3y\frac{e^{-2y}}{-2}+\frac{3}{2}.\frac{e^{-2y}}{-2}+C\surd$$

$$=-\frac{3}{2}ye^{-2y}-\frac{3}{4}e^{-2y}+C$$

$$xe^{-2y}=-\frac{3}{2}ye^{-2y}-\frac{3}{4}e^{-2y}+C\surd$$

$$x=1;\quad y=0\quad 1.e^{-2.0}=-\frac{3}{2}.0.e^{-2.0}-\frac{3}{4}e^{-2.0}+C\surd$$

$$1=-\frac{3}{4}+C\qquad C=\frac{7}{4}\surd$$

$$xe^{-2y}=-\frac{3}{2}ye^{-2y}-\frac{3}{4}e^{-2y}+\frac{7}{4}\surd$$

(10)

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4.2 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 18e^{-3x}$ $y = 1$ $x = 0$ $and \frac{dy}{dx} = 2$; $x = 0$

$r^2 - 6r + 9 = 0$ √

$(r-3)(r-3) = 0$ √

$r = 3$ √

$y_c = e^{3x}(A + Bx)$ or $y_c = Ae^{3x} + Bxe^{3x}$ √

$y = Ce^{-3x}$ √ $\frac{dy}{dx} = -3Ce^{-3x}$ √ $\frac{d^2y}{dx^2} = 9Ce^{-3x}$ √

$\therefore 9Ce^{-3x} - 6(-3Ce^{-3x}) + 9Ce^{-3x} = 18e^{-3x}$

$36Ce^{-3x} = 18e^{-3x}$ $\therefore C = \frac{1}{2}$ √

$\therefore y_p = \frac{1}{2}e^{-3x}$ √

$y = y_c + y_p$ $y = e^{3x}(A + Bx) + \frac{1}{2}e^{-3x}$ √

$y = 1$ $x = 0$ $1 = A + \frac{1}{2}$ $\therefore A = \frac{1}{2}$ √

Differentiating the general solution : $\frac{dy}{dx} = e^{3x}B + 3e^{3x}(A + Bx) - \frac{3}{2}e^{-3x}$ √

$\frac{dy}{dx} = 2$; $x = 0$ $2 = e^{3.0}B + 3e^{3.0}(A + B.0) - \frac{3}{2}e^{-3.0}$

$2 = B + 3A - \frac{3}{2}$

$2 = B + 3\left(\frac{1}{2}\right) - \frac{3}{2}$ $\therefore B = 2$ √

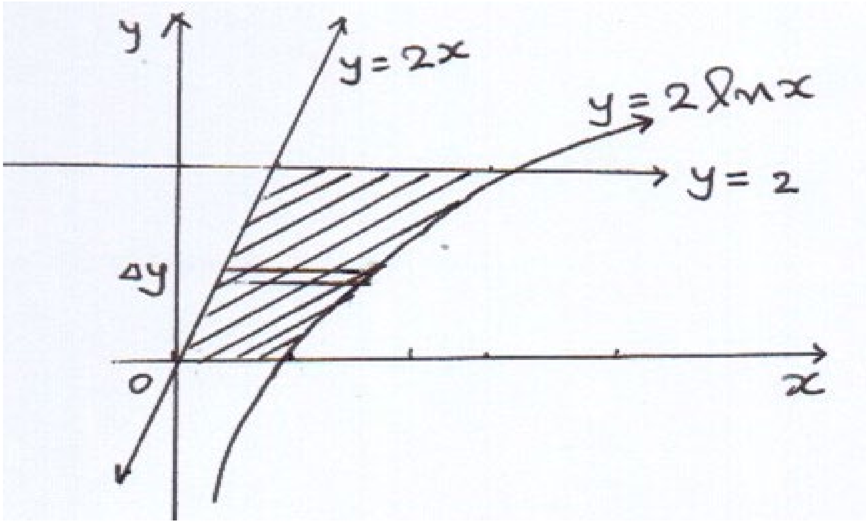
$y = e^{3x}(\frac{1}{2} + 2x) + \frac{1}{2}e^{-3x}$ √

(14)

[24]

QUESTION 5

5.1 5.1.1



✓ for y = 2x
✓ for y = 2lnx
✓ strip
✓ area (4)

5.1.2

$A=\int_a^b (x_1-x_2)dy$ √ $y=2\ln x \Rightarrow \ln x=\frac{y}{2}$ $\therefore x=e^{\frac{y}{2}}$ √ and from $y=2x$ $x=\frac{y}{2}$

$A=\int_0^2 (e^{\frac{y}{2}}-\frac{y}{2})dy$ √√

¶

$=\left[2e^{\frac{y}{2}}-\frac{y^2}{4}\right]_0^2$ √√

$=\left[2e^{\frac{2}{2}}-\frac{2^2}{4}-(2-0)\right]$ √ $=2e-1-2$ $=2,437\text{units}^2$ √

$A_{m-y}=\int_a^b r dA$ √ (9)

5.1.3

$=\int_a^b \frac{x_1+x_2}{2}(x_1-x_2)dA$ √ $=\frac{1}{2}\int_a^b x_1^2-x_2^2 dy$ √

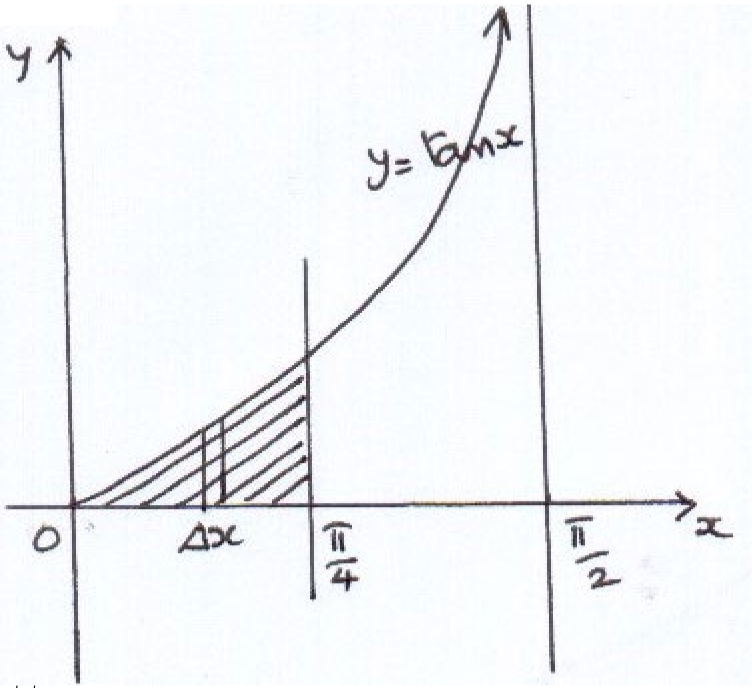
$=\frac{1}{2}\int_0^2 \left(e^{\frac{y}{2}}\right)^2-\left(\frac{y}{2}\right)^2 dy$ √√√

$=\frac{1}{2}\int_0^2 e^y-\frac{y^2}{4}\,dy$ √

$=\frac{1}{2}\left[e^y-\frac{y^3}{12}\right]_0^2$ √ $=\frac{1}{2}\left[e^2-\frac{2^3}{12}-(1-0)\right]_0^2$ √ $=2,861\text{units}^3$ √

$\bar{x}=\frac{2,861}{2,437}=1,174$ √√ (11)

5.2 5.2.1



√√ each graph
√ strip
√ area (4)

5.2.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 dx \qquad or \pi \int_a^b y^2 dx \ \surd$$

$$= \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \ \surd \surd$$

$$= \pi \left[\tan x - x \right]_0^{\frac{\pi}{4}} \ \surd$$

$$= \pi \left[\tan \frac{\pi}{4} - \frac{\pi}{4} \right]_0^{\frac{\pi}{4}} \ \surd$$

$$= 0,674 \text{units}^3 \ \surd$$ (6)

$$\pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

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5.2.3

$$I_x = \frac{1}{2}\pi\rho\int_a^b y^4 dx \ \surd$$

$$= \frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^4 x dx \ \surd\surd$$

$$= \frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^2 x \tan^2 x\, dx \ \surd$$

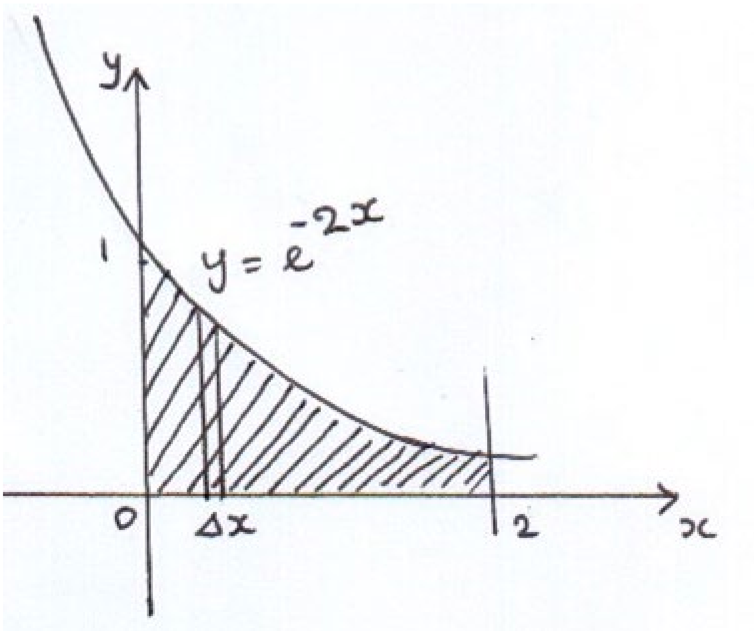
$$= \frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^2 x(\sec^2 x - 1)\, dx \ \surd$$

$$= \frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} (\tan^2 x \sec^2 x - \tan^2 x)\, dx \ \surd$$

$$= \frac{1}{2}\pi\rho\left[\frac{\tan^3 x}{3} - \tan x + x\right]_0^{\frac{\pi}{4}} \ \surd\surd = \frac{1}{2}\pi\rho\left[\frac{\tan^3 \frac{\pi}{4}}{3} - \tan\frac{\pi}{4} + \frac{\pi}{4} - (0 - 0 + 0)\right] \surd \ = 0{,}187\rho \ \surd$$

(10)

5.3 5.3.1



√√ graphs
√ strip
√ area (4)

5.3.2

$$A = \int_a^b y\,dx \qquad or \int_a^b y_1 - y_2\,dx \checkmark$$

$$= \int_0^2 e^{-2x}\,dx \checkmark\checkmark$$

$$= \left[\frac{e^{-2x}}{-2}\right]_0^2 \checkmark$$

$$= \frac{e^{-4}}{-2} - \frac{e^0}{-2} \checkmark$$

$$= 0,491\ units^2 \checkmark$$

$$I_y = \int_a^b r^2 dA \checkmark$$

$$= \int_0^2 x^2 e^{-2x} dx \checkmark\checkmark$$

(9)

5.3.3

$$= \left[x^2 \frac{e^{-2x}}{-2} - \int 2x.\frac{e^{-2x}}{-2} dx\right]_0^2 \checkmark\checkmark$$

$$\left[x^2 \frac{e^{-2x}}{-2} - \int x e^{-2x} dx\right]_0^2 \checkmark$$

$$\left[x^2 \frac{e^{-2x}}{-2} + x\frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} dx\right]_0^2 \checkmark$$

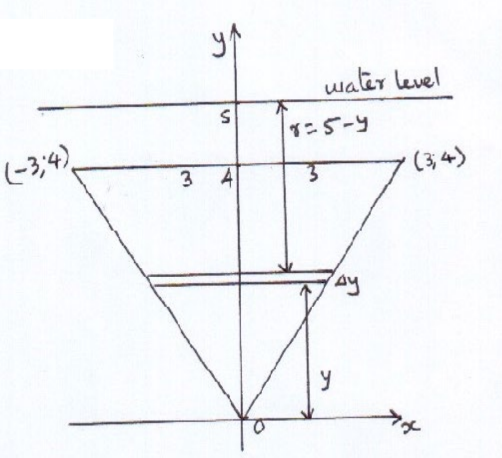
$$\left[x^2 \frac{e^{-2x}}{-2} + x\frac{e^{-2x}}{-2} + \frac{1}{2}\frac{e^{-2x}}{-2}\right]_0^2 \checkmark$$

$$\left[e^{-2x}\left(-\frac{x^2}{2} - \frac{1}{2}x - \frac{1}{4}\right)\right]_0^2 \quad = e^{-4}\left(-\frac{2^2}{2} - \frac{1}{2}(2) - \frac{1}{4}\right) - e^0\left(0 - 0 - \frac{1}{4}\right) \checkmark$$

$$= 0.190 units^4 \checkmark$$

(7)

5.4 5.4.1



✓ water level at 5
✓ height of plate at 4
✓ point (3;4)
✓ strip

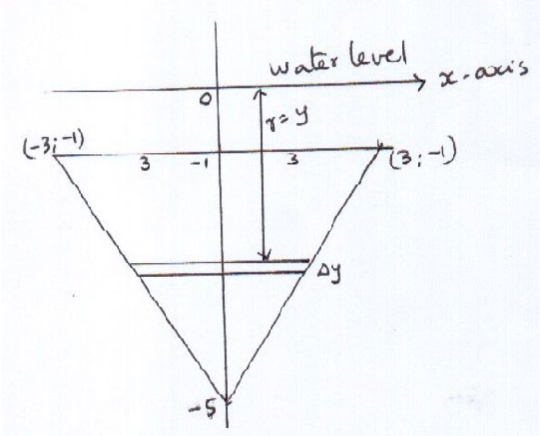
$m = \frac{4-0}{3-0} = \frac{4}{3}$ √

$y - y_1 = m(x - x_1)$

$y = \frac{4}{3}x$ √ $x = \frac{3}{4}y$

$\text{second moment} = \int_a^b r^2 dA$

Alternative:



✓ Water level at x axis
✓ point (3;-1)
✓ height of plate at -1
✓ strip

$(3;-1)$ $(0;-5)$ $m = \frac{-1+5}{3-0} = \frac{4}{3}$ √

$y + 5 = \frac{4}{3}x$ √ $x = \frac{3}{4}(y+5)$

$\text{second moment} = \int_a^b r^2 dA$

(6)

5.4.2

$\int_0^4 (5-y)^2 2.\frac{3}{4}y\,dy$ √√√

$\frac{3}{2}\int_0^4 (25 - 10y + y^2)y\,dy$ √

$\frac{3}{2}\int_0^4 (25y - 10y^2 + y^3)\,dy$ √

$\frac{3}{2}\left[25\frac{y^2}{2} - 10\frac{y^3}{3} + \frac{y^4}{4}\right]_0^4$ √

$\frac{3}{2}\left[25\frac{4^2}{2} - 10\frac{4^3}{3} + \frac{4^4}{4} - 0\right]$ √

$= 76m^4$ √

$\bar{y} = \frac{76}{28} = 2,714m$ √√

Alternative:

$= \int_{-5}^{-1} y^2 2.\frac{3}{4}(y+5)dy$ √√√

$\frac{3}{2}\int_{-5}^{-1} y^3 + 5y^2 dy$ √

$\frac{3}{2}\left[\frac{y^4}{4} + \frac{5y^3}{3}\right]_{-5}^{-1}$ √

$\frac{3}{2}\left[\frac{(-1)^4}{4} + \frac{5(-1)^3}{3} - \left\{\frac{(-5^4)}{4} + \frac{(-5)^3}{3}\right\}\right]$ √√

$76m^4$ √

$\bar{y} = \frac{76}{-28} = -2,714m$ √√

(10)
[80]

QUESTION 6

6.1

$y = 9 - x^2 \qquad \frac{dy}{dx} = -2x\ \surd \qquad \left(\frac{dy}{dx}\right)^2 = 4x^2\ \surd$

$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx\ \surd$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^2\ \surd$

$= \int_0^3 \sqrt{1 + 4x^2}\, dx\ \surd\surd \qquad = \int_0^3 \sqrt{4(\frac{1}{4} + x^2)}dx$

$= 2\int_0^3 \sqrt{\frac{1}{4} + x^2}\, dx\ \surd$

$= 2\left[\frac{x}{2}\sqrt{\frac{1}{4} + x^2} + \frac{\frac{1}{4}}{2}\ln\left(x + \sqrt{\frac{1}{4} + x^2}\right)\right]_0^3 \quad \surd\surd$

$2\left[\frac{3}{2}\sqrt{\frac{1}{4} + 3^2} + \frac{\frac{1}{4}}{2}\ln\left(3 + \sqrt{\frac{1}{4} + 3^2}\right) - \left(0 + \frac{1}{8}\ln(0 + \frac{1}{4}\right)\right]\ \surd$

$9,747\text{units}\ \surd\surd$ (12)

6.2

$A_y = 2\pi\int_a^b x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy\ \surd$

$x = y^3 \qquad \frac{dx}{dy} = 3y^2\ \surd \qquad \left(\frac{dx}{dy}\right)^2 = 9y^4\ \surd$

$1 + \left(\frac{dx}{dy}\right)^2 = 1 + 9y^4\ \surd$

$A_y = 2\pi\int_0^1 y^3\sqrt{1 + 9y^4}\, dy \quad \surd\surd\surd$

$A_y = \frac{2\pi}{36}\int_0^1 36y^3\sqrt{1 + 9y^4}\, dy\ \surd$ or using $u = 1 + 9y^4$

$= \frac{2\pi}{36}\frac{2}{3}\left[\left(1 + 9y^4\right)^{\frac{3}{2}}\right]_0^1 \quad \surd\surd$

$= \frac{\pi}{27}\left[\left(1 + 9.1\right)^{\frac{3}{2}} - \left(1 + 0\right)^{\frac{3}{2}}\right]\ \surd$

$= 3,563\text{units}^2\ \surd$ (12)

ALTERNATIVE

$= y^3 \; between \; 0 \le y \le 1 \;\Rightarrow y = x^{\frac{1}{3}} \; between \; 0 \le x \le 1$ √

$\Rightarrow A_y = \int_a^b 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ √

$y = x^{\frac{1}{3}} \Rightarrow \frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$ √

$= \int_0^1 2\pi x\sqrt{1+\frac{1}{9x^{\frac{4}{3}}}}\,dx$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{9x^{\frac{4}{3}}}$

$= \int_0^1 2\pi x\sqrt{\frac{1+9x^{\frac{4}{3}}}{9x^{\frac{4}{3}}}}\,dx$ √

$\Rightarrow 1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{1}{9x^{\frac{4}{3}}} = \frac{1+9x^{\frac{4}{3}}}{9x^{\frac{4}{3}}}$ √

$= \frac{2}{3}\pi\int_0^1 x.\frac{1}{x^{\frac{2}{3}}}\sqrt{1+9x^{\frac{4}{3}}}\,dx$ √

$let \; u = 1+9x^{\frac{4}{3}} \Rightarrow \frac{du}{dx} = 12x^{\frac{1}{3}}$

$= \frac{2}{3}\pi\int_0^1 x^{\frac{1}{3}}\sqrt{1+9x^{\frac{4}{3}}}\,dx$ √

$\Rightarrow dx = \frac{du}{12x^{\frac{1}{3}}}$ √

$= \frac{2}{3}\pi\int_0^1 x^{\frac{1}{3}}\sqrt{u}\,\frac{du}{12x^{\frac{1}{3}}}$ √

$= \frac{\pi}{18}\int_1^{10} u^{\frac{1}{2}}du$

$if \; x = 0 \Rightarrow u = 1$

$if \; x = 1 \Rightarrow u = 10$

$= \frac{\pi}{18}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{10}$ √

$= \frac{\pi}{27}\left[10^{\frac{3}{2}}-1\right]$ √

$= 3{,}563$ √

[24]

TOTAL: 200 ÷ 2 = 100

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