higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

1 April 2020 (X-paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

050Q1A2001

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.

2. Read all the questions carefully.

3. Number the answers according to the numbering system used in this question paper.

4. Start each section on a new page.

5. Use only black or blue pen.

6. Write neatly and legibly.

QUESTION 1

1.1 Given: $z = \ln\left(\sqrt{x} + \sqrt{y}\right)$

Prove that $\sqrt{x}\dfrac{\partial z}{\partial x} + \sqrt{y}\dfrac{\partial z}{\partial y} = \dfrac{1}{\sqrt{x} + \sqrt{y}}$ (3)

1.2 The radius (r) of a right circular cylinder increases from 4 cm to 4,1 cm and its height (h) increases from 20 cm to 20,5 cm.

Calculate its approximate change in volume.

$V = \pi r^2 h$ (3)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \dfrac{1}{(x+3)^2 - 8x}$ (4)

2.2 $y = \ln 2x \ln x$ (4)

2.3 $y = \dfrac{1 + \tan^2 x}{\tan^3 x}$ (2)

2.4 $y = \sin^3 x + \cos^3 x$ (5)

2.5 $y = 3\tan^{-1}\dfrac{x}{3}$ (3)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int x + \dfrac{5 - 5x}{6x^2 + x - 1}\,dx$ (5)

3.2 $\int \dfrac{2x^3 + 6x^2 - 12}{x(x+3)(x^2 + 3x + 4)}\,dx$ (7)

[12]

QUESTION 4

4.1 Determine the particular solution of $\frac{dx}{dy} - 3y = 2x$ at (1;0) (5)

4.2 Determine the particular solution of
$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 18e^{-3x}$ when $y = 1$; $x = 0$ *and* $\frac{dy}{dx} = 2$; $x = 0$ (7)
[12]

QUESTION 5

5.1 5.1.1 Sketch the graphs of $y = 2\ln x$ and $y = 2x$. Show the area bounded by the graphs, the x-axis and the line y = 2. Show the representative strip that you will use to calculate the area. (2)

5.1.2 Calculate the area described in QUESTION 5.1.1 (4)

5.1.3 Calculate the area moment about the y-axis as well as the x-co-ordinate of the centroid of the area described in QUESTION 5.1.1 (6)

5.2 5.2.1 Sketch the graph of $y = \tan x$ for $0 \le x \le \frac{\pi}{2}$. The area enclosed by the graph, the x-axis and the line $x = \frac{\pi}{4}$ rotates about the x-axis. Show the area and the representative strip that you will use to calculate the volume. (2)

5.2.2 Calculate the volume generated when the area described in QUESTION 5.2.1 rotates about the x-axis. (3)

5.2.3 Calculate the moment of inertia about the x-axis of the solid obtained when the area in QUESTION 5.2.1 rotates about the x-axis. (5)

5.3 5.3.1 Sketch the graph of $y = e^{-2x}$.

Show the area bounded by the graph, the x-axis, the y-axis and the line $x = 2$. Show the representative strip that you will use to calculate the area and the second moment of area. (2)

5.3.2 Calculate the area described in QUESTION 5.3.1 (3)

5.3.3 Calculate the second moment of area about the y-axis of the area described in QUESTION 5.3.1 (5)

5.4 5.4.1 A triangular plate of sides 5 m, 5 m and 6 m is placed vertically in a canal which is 5 m deep. The longest side of the plate is horizontal and is 1 m below the water level.

Sketch the plate and show the representative strip that you will use to calculate the area moment of the plate about the water level.

Calculate the relation between the variables x and y. (3)

5.4.2 Calculate the second moment of area of the plate about the water level as well as the depth of the centre of pressure on the plate if the area moment is given as numerically equal to 28 m^3. (5)

[40]

QUESTION 6

6.1 Determine the length of the curve $y = 9 - x^2$ from $x = 0$ *to* $x = 3$ (6)

6.2 Calculate the surface area generated when the curve $x = y^3$ for $0 \le y \le 1$ is rotated about the y-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan\left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$

$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$

$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$

APPLICATIONS OF INTEGRATION

AREAS

$A_x = \int_a^b y\,dx\,;\; A_x = \int_a^b (y_1 - y_2)\,dx$

$A_y = \int_a^b x\,dy\,;\; A_y = \int_a^b (x_1 - x_2)\,dy$

VOLUMES

$$V_x = \pi\int_a^b y^2 dx \,;\, V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right) dx \,;\, V_x = 2\pi\int_a^b xydy$$

$$V_y = \pi\int_a^b x^2 dy \,;\, V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right) dy \,;\, V_y = 2\pi\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\overline{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \overline{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\overline{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \overline{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 \ (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \square\ ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$