higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

6 APRIL 2021

This marking guideline consists of 23 pages.

QUESTION 1

1.1 1.1.1

$$z = a^{3x-2y}$$
$$\frac{\partial z}{\partial x} = a^{3x-2y}.3\ln a \checkmark$$ (1)

1.1.2

$$\frac{\partial^2 z}{\partial y \partial x} = 3\ln a.a^{3x-2y}.-2\ln a \checkmark\checkmark$$
$$= -6(\ln a)^2 a^{3x-2y}$$
OR $= -6\ln^2 a.a^{3x-2y}$ (2)

1.2 1.2.1

$$x = \frac{1}{1+2t} = (1+2t)^{-1}$$
$$\frac{dx}{dt} = -(1+2t)^{-2}.2 = -\frac{2}{(1+2t)^2} \checkmark$$
$$y = 1+2t$$
$$\frac{dy}{dt} = 2$$
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad or \quad \frac{dy}{dt}.\frac{dt}{dx}$$
$$= 2.\frac{-(1+2t)^2}{2} = -(1+2t)^2 \checkmark$$

OR

$$x = \frac{1}{1+2t} = \frac{1}{y} \quad \therefore y = \frac{1}{x} \checkmark$$
$$\frac{dy}{dx} = -\frac{1}{x^2}$$
$$= -\left(\frac{1}{x}\right)^2$$
$$= -y^2 \qquad \text{because } y = \frac{1}{x}$$
$$= -(1+2t)^2 \checkmark$$ (2)

1.2.2

$$\frac{d^2 y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$
$$= -2(1+2t)2.\frac{-(1+2t)^2}{2} \checkmark$$
$$= 2(1+2t)^3 \checkmark$$

OR

$$\frac{dy}{dx} = -\frac{1}{x^2} = -x^{-2}$$
$$\frac{d^2 y}{dx^2} = 2x^{-3} \checkmark \quad = 2\left(\frac{1}{x}\right)^3 = 2y^3 = 2(1+2t)^3 \checkmark$$ (1)

[6]

QUESTION 2

2.1

$$\int y\,dx$$
$$=\int \sin^3 2x\left(1-\sin^2 2x\right)dx$$
$$=\int \sin^3 2x\cos^2 2x\,dx \quad ✓$$
$$=\int \sin^2 2x\sin 2x\cos^2 2x\,dx \quad ✓$$
$$=\int (1-\cos^2 2x)\sin 2x\cos^2 2x\,dx \quad ✓$$
$$=\int \sin 2x\cos^2 2x-\sin 2x\cos^4 2x\;dx \quad ✓$$
$$=-\frac{1}{2}\frac{\cos^3 2x}{3}+\frac{1}{2}\frac{\cos^5 2x}{5}+C \quad ✓$$

Alternative

$$\int y\,dx$$
$$=\int \sin^3 2x\left(1-\sin^2 2x\right)dx$$
$$=\int \sin^3 2x-\sin^5 2xdx$$
$$=\int \sin^2 2x\sin 2x-\left(\sin^2 2x\right)^2\sin 2x\;dx$$
$$=\int \left(1-\cos^2 2x\right)\sin 2x-\left(1-\cos^2 2x\right)^2\sin 2x\;dx \quad ✓$$
$$=\int \left(1-\cos^2 2x\right)\sin 2x-\left(1-2\cos^2 2x+\cos^4 2x\right)\sin 2x\;dx \quad ✓$$
$$=\int \sin 2x-\cos^2 2x\sin 2x-\sin 2x+2\cos^2 2x\sin 2x-\cos^4 2x\sin 2x\;dx \quad ✓$$
$$=\int \cos^2 2x\sin 2x-\cos^4 2x\sin 2x\;dx \quad ✓$$
$$=-\frac{1}{2}\frac{\cos^3 2x}{3}+\frac{1}{2}\frac{\cos^5 2x}{5}+C \quad ✓$$

Or using
$u=\cos 2x$
$dx=\dfrac{du}{-2\sin 2x}$

(5)

2.2

$$\int y\,dx=\int \tan^3 3x\sec^4 3x\,dx$$
$$=\int \tan^3 3x\sec^2 3x\sec^2 3x\,dx$$
$$=\int \tan^3 3x\left(1+\tan^2 3x\right)\sec^2 3x\,dx \qquad u=\tan 3x \qquad dx=\frac{du}{3\sec^2 3x} \quad ✓$$
$$=\int u^3\left(1+u^2\right)\sec^2 3x\frac{du}{3\sec^2 3x} \quad ✓$$
$$=\frac{1}{3}\int u^3+u^5du$$
$$=\frac{1}{3}\left(\frac{u^4}{4}+\frac{u^6}{6}\right) \quad ✓$$
$$=\frac{1}{3}\left(\frac{\tan^4 3x}{4}+\frac{\tan^6 3x}{6}\right)+C \quad ✓ \quad or \quad \frac{\tan^4 3x}{12}+\frac{\tan^6 3x}{18}+C$$

Alternative 1

$$\int y\,dx = \int \tan^3 3x \sec^4 3x\,dx$$

$$= \int \tan^3 3x \sec^2 3x \sec^2 3x\,dx \quad ✓ \quad .n^3 3x\left(1+\tan^2 3x\right)\sec^2 3x\,dx \quad ✓$$

$$= \int \tan^3 3x \sec^2 3x + \tan^5 3x \sec^2 3x dx \quad ✓ \quad \frac{\tan^4 3x}{12} + \frac{\tan^6 3x}{18} + C \quad ✓$$

by inspection

Alternative 2

$$\int y\,dx = \int \tan^3 3x \sec^4 3x\,dx$$

$$= \int \frac{\sin^3 3x}{\cos^3 3x}\frac{1}{\cos^4 3x}dx \quad ✓$$

$$= \int \frac{\sin^2 3x \sin 3x}{\cos^7 3x}dx$$

$$= \int \frac{(1-\cos^2 3x)\sin 3x}{\cos^7 3x}dx \qquad u = \cos 3x \qquad \frac{du}{dx} = -3\sin 3x \qquad dx = \frac{du}{-3\sin 3x} \quad ✓$$

$$= \int \frac{(1-u^2)\sin 3x}{u^7}\frac{du}{-3\sin 3x}$$

$$= -\frac{1}{3}\int u^{-7} - u^{-5}du \quad ✓$$

$$= -\frac{1}{3}\left[\frac{u^{-6}}{-6} - \frac{u^{-4}}{-4}\right] + C$$

$$= -\frac{1}{3}\left[-\frac{1}{6}\sec^6 3x + \frac{1}{4}\sec^4 3x\right] + C \quad ✓$$

Alternative 3

$$\int y\,dx = \int \tan^3 3x \sec^4 3x\,dx$$

$$= \int \tan 3x \tan^2 3x \sec^4 3x\,dx \quad ✓$$

$$= \int \tan 3x(\sec^2 3x - 1)\sec^4 3x\,dx$$

$$= \int \sec^6 3x \tan 3x - \sec^4 3x \tan 3x dx \quad ✓$$

$$= \int \sec^5 3x \sec 3x \tan 3x - \sec^3 3x \sec 3x \tan 3x\,dx \quad ✓$$

$$= \int \frac{1}{3}\sec^5 3x(3\sec 3x \tan 3x) - \frac{1}{3}\sec^3 3x(3\sec 3x \tan 3x)dx$$

$$= \frac{1}{3}\frac{\sec^6 3x}{6} - \frac{1}{3}\frac{\sec^4 3x}{4} + C \quad ✓$$

$$\frac{1}{18}\sec^6 3x - \frac{1}{12}\sec^4 3x + C = \frac{1}{18}\left(\sec^2 3x\right)^3 - \frac{1}{12}\left(\sec^2 3x\right)^2 + C$$

$$= \frac{1}{18}\left(1+\tan^2 3x\right)^3 - \frac{1}{12}\left(1+\tan^2 3x\right)^2 + C$$

$$= \frac{1}{18}\left(1+3\tan^2 3x+3\tan^2 3x+\tan^6 3x\right) - \frac{1}{12}\left(1+2\tan^2 3x+\tan^4 3x\right) + C$$

$$= \frac{1}{18} + \frac{1}{6}\tan^2 3x + \frac{1}{6}\tan^4 3x + \frac{1}{18}\tan^6 3x - \frac{1}{12} - \frac{1}{6}\tan^2 3x - \frac{1}{12}\tan^4 3x + C$$

$$= \frac{1}{12}\tan^4 3x + \frac{1}{18}\tan^6 3x + \frac{1}{18} - \frac{1}{12} + C \quad = \frac{1}{12}\tan^4 3x + \frac{1}{18}\tan^6 3x + C$$

(4)

2.3

$$\int y\,dx$$

$$= \int (x+1)^2 \ln(x+1)^2 dx \qquad f(x) = \ln(x+1)^2 \qquad g'(x) = (x+1)^2$$

$$f'(x) = \frac{2}{x+1} \qquad g(x) = \frac{(x+1)^3}{3} \quad ✓$$

$$= \ln(x+1)^2 \frac{(x+1)^3}{3} - \int \frac{2}{x+1}\frac{(x+1)^3}{3}dx \quad ✓$$

$$= \ln(x+1)^2 \frac{(x+1)^3}{3} - \frac{2}{3}\int (x+1)^2 dx \quad ✓$$

$$= \ln(x+1)^2 \frac{(x+1)^3}{3} - \frac{2}{3}\frac{(x+1)^3}{3} + C \quad ✓$$

or $= \frac{2}{3}(x+1)^3 \ln(x+1) - \frac{2}{9}(x+1)^3 + C$ or $= \frac{(x+1)^3}{3}\left[2\ln(x+1) - \frac{2}{3}\right] + C$

OR $\int y dx =$ $\int (x+1)^2 \ln(x+1)^2 dx = 2\int (x+1)^2 \ln(x+1) dx$ follow up

Alternative

$$\int y dx = \int (x+1)^2 \, ln(x+1)^2 dx$$

$$= \int (x+1)^2 . 2\ln(x+1)\, dx \checkmark$$

$f(x) = \ln(x+1) \Rightarrow f'(x) = \frac{1}{x+1}$
$g'(x) = (x+1)^2 \;\Rightarrow g(x) = \dfrac{(x+1)^3}{3}$

$$= 2\left[\frac{(x+1)^3}{3}.\ln(x+1) - \int \frac{1}{x+1}.\frac{(x+1)^3}{3} dx\right] \checkmark$$

$$= 2\left[\frac{(x+1)^3}{3}.\ln(x+1) - \int \frac{(x+1)^2}{3} dx\right] \checkmark$$

$$= 2\left[\frac{(x+1)^3}{3}.\ln(x+1) - \frac{(x+1)^3}{9}\right] + C\checkmark$$

Note also

$$-\int \frac{(x+1)^2}{3} dx = -\frac{1}{3}\int (x^2+2x+1)dx$$

$$= -\frac{1}{3}\left[\frac{x^3}{3} + x^2 + x\right] + C$$

(4)

2.4

$$\frac{9}{4} - 3x - x^2 = -\left[x^2 + 3x - \frac{9}{4}\right]$$

$$= -\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} - \left(\frac{3}{2}\right)^2\right]$$

$$= -\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{2}\right] \quad \checkmark$$

$$= \frac{9}{2} - \left(x + \frac{3}{2}\right)^2 \quad \checkmark$$

$$\int y\,dx$$

$$= \int \frac{1}{\sqrt{\frac{9}{4} - 3x - x^2}}\,dx = \int \frac{1}{\sqrt{\frac{9}{2} - \left(x + \frac{3}{2}\right)^2}}\,dx$$

$$= \frac{1}{1}\sin^{-1}\frac{x + \frac{3}{2}}{\frac{3}{\sqrt{2}}} + C \quad \checkmark$$

$$= \sin^{-1}\frac{2x+3}{2\frac{3}{\sqrt{2}}} + C \quad or = \sin^{-1}\frac{2x+3}{3\sqrt{2}} + C \quad \checkmark$$

Alternative

$$ax^2 + bx + c = a\left(x + \frac{b}{2b}\right)^2 + \frac{4ac - b^2}{4a}$$

$$\Rightarrow -x^2 - 3x + \frac{9}{4} = -1\left(x + \frac{3}{2}\right)^2 + \frac{4(-1)\left(\frac{9}{4}\right) - (-3)^2}{4(-1)} \quad \checkmark$$

$$= \frac{9}{2} - 1\left(x + \frac{3}{2}\right)^2 \quad \checkmark$$

$$\int \frac{1}{\sqrt{-x^2 - 3x + \frac{9}{4}}}\,dx = \int \frac{1}{\sqrt{\frac{9}{2} - 1\left(x + \frac{3}{2}\right)^2}}\,dx \quad \checkmark$$

$$= \frac{1}{1}\sin^{-1}\left[\frac{x + \frac{3}{2}}{\frac{3}{\sqrt{2}}}\right] + C \quad \checkmark$$

(4)

2.5 $\int y\,dx = \int \sin^2 2x - \cos^2 2x\,dx$

$$= -\int \cos^2 2x - \sin^2 2x\,dx$$

$$= -\int \cos 4x\,dx$$

$$= -\frac{\sin 4x}{4} + C \quad \checkmark$$

Alternative 1

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$\frac{x}{2} - \frac{\sin 4x}{8} - \left(\frac{x}{2} + \frac{\sin 4x}{8}\right) + C$$

$$\frac{x}{2} - \frac{\sin 4x}{8} - \frac{x}{2} - \frac{\sin 4x}{8} + C$$

$$-\frac{\sin 4x}{4} + C \quad \checkmark$$

Use the formula sheet.

Alternative 2

$$\int y \, dx = \int \sin^2 2x - \cos^2 2x \, dx$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos 4x - \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)$$

$$= -\int \cos 4x \, dx = -\frac{\sin 4x}{4} + C \quad \checkmark$$

Alternative 3

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$\int \sin^2 2x - (1 - \sin^2 2x) \, dx$$

$$\int 2\sin^2 2x - 1 \, dx$$

$$2\left(\frac{x}{2} - \frac{\sin 4x}{8}\right) - x + C$$

$$x - \frac{\sin 4x}{4} - x + C$$

$$-\frac{\sin 4x}{4} + C \quad \checkmark$$

Alternative 4

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$= \int 1 - \cos^2 2x - \cos^2 2x \, dx$$

$$= \int 1 - 2\cos^2 2x \, dx$$

$$= x - 2\left(\frac{x}{2} + \frac{\sin 4x}{8}\right) + C$$

$$= x - x - \frac{\sin 4x}{4} + C$$

$$= -\frac{\sin 4x}{4} + C \quad \checkmark$$

(1)

[18]

QUESTION 3

3.1 $$\int \frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}dx$$

$$\frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)} = \frac{Ax+B}{x^2+1}+\frac{C}{x+1}+\frac{D}{x-1} \quad \checkmark$$

$$3x^2-6x-1 = (Ax+B)(x+1)(x-1)+C(x^2+1)(x-1)+D(x^2+1)(x+1)............(1) \quad \checkmark$$

$$3x^2-6x-1 = (Ax+B)(x^2-1)+C(x^3+x-x^2-1)+D(x^3+x+x^2+1)$$

$$3x^2-6x-1 = Ax^3+Bx^2-Ax-B+Cx^3+Cx-Cx^2-C+Dx^3+Dx+Dx^2+D$$

Equating x^3 $A+C+D=0$.............(2)

Equating x^2 $B-C+D=3$.............(3)

Equating x $-A+C+D=-6$.............(4)

Equating constants: $-B-C+D=-1$.............(5)

$(3)-(5) \Rightarrow 2B=4 \quad \therefore B=2 \quad \checkmark$

$(2)-(4) \Rightarrow 2A=6 \quad \therefore A=3$

$(2)+(4) \Rightarrow 2C+2D=-6 \quad \therefore D=-3-C$

Substituting B and D in (5) $\quad -2-C-3-C=-1 \quad \therefore C=-2 \quad \checkmark$

$\therefore D=-3+2$

$D=-1 \quad \checkmark$

$$\int \frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}dx = \int\frac{3x+2}{x^2+1}dx+\int\frac{-2}{x+1}dx+\int\frac{-1}{x-1}dx$$

$$= \int\frac{3x}{x^2+1}dx+\int\frac{2}{x^2+1}dx-\int\frac{2}{x+1}dx-\int\frac{1}{x-1}dx$$

$$= \frac{3}{2}\ln(x^2+1)+2\tan^{-1}x-2\ln(x+1)-\ln(x-1)+C \quad \checkmark$$

Alternative 1

$$\frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x+1}+\frac{D}{x-1} \quad ✓$$

$3x^2-6x-1=(Ax+B)(x+1)(x-1)+C(x^2+1)(x-1)+D(x^2+1)(x+1)$............(1) ✓

When $x=1$ $\quad -4=4D \quad \therefore D=-1$

When $x=-1$ $\quad 8=-4C \quad \therefore C=-2$

When $x=0$ $\quad -1=-B-C+D \Rightarrow -1=-B+2-1 \quad \therefore B=2$

Equating x^3 $\quad A+C+D=0 \Rightarrow A-2-1=0 \quad \therefore A=3$ ✓

$A=3$	$B=2$	$C=-2$	$D=-1$

$$\int\frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}dx=\int\frac{3x+2}{x^2+1}dx+\int\frac{-2}{x+1}dx+\int\frac{-1}{x-1}dx \quad ✓$$

$$=\int\frac{3x}{x^2+1}dx+\int\frac{2}{x^2+1}dx-\int\frac{2}{x+1}dx-\int\frac{1}{x-1}dx \quad ✓$$

$$=\frac{3}{2}\ln(x^2+1)+2\tan^{-1}x-2\ln(x+1)-\ln(x-1)+C \quad ✓$$

Alternative 2

$$\frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2-1} \quad ✓$$

$3x^2-6x-1=(Ax+B)(x^2-1)+(Cx+D)(x^2+1)$............(1) ✓

When $x=1$ $\quad -4=(C+D)2 \quad \therefore C+D=-2$...........(2)

When $x=-1$ $\quad 8=(-C+D) \quad \therefore -C+D=4$...........(3)

(2) + (3) $\quad 2D=2 \quad \therefore D=1$ ✓

Put $D=1$ in (2) $\quad C=-3$ ✓

Note that the values of C and D have changed.

When $x=0$ $\quad -1=-B+D \quad \therefore B=2$ ✓

Equating x^3 $\quad A+C=0 \Rightarrow A-3=0 \quad \therefore A=3$ ✓

$$\int \frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}dx = \int \frac{3x+2}{x^2+1}dx + \int \frac{-2}{x+1}dx + \int \frac{-1}{x-1}dx$$

$$\int \frac{3x^2-6x-1}{(x^2+1)(x+1)(x-1)}dx = \int \frac{3x+2}{x^2+1}dx + \int \frac{-3x+1}{x^2-1}dx \quad ✓$$

$$= \int \frac{3x}{x^2+1}dx + \int \frac{2}{x^2+1}dx + \int \frac{-3x}{x^2-1}dx + \int \frac{1}{x^2-1}dx \quad ✓$$

$$= \frac{3}{2}\ln(x^2+1) + 2\tan^{-1}x - \frac{3}{2}\ln(x^2-1) + \frac{1}{2}\ln\frac{x-1}{x+1} + C \quad ✓ \; ✓ \; ✓ \; ✓$$

$$= \frac{3}{2}\ln(x^2+1) + 2\tan^{-1}x - \frac{3}{2}\ln(x+1) - \frac{3}{2}\ln(x-1) + \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + C$$

$$= \frac{3}{2}\ln(x^2+1) + 2\tan^{-1}x - 2\ln(x+1) - \ln(x-1) + C$$

(6)

3.2 $\int \frac{x^3-10x-15}{x^2-9}dx$

$x^2-9)x^3-10x-15(x$

x^3-9x

$-x-15$ ✓

$$\frac{x^3-10x-15}{x^2-9} = x + \frac{-x-15}{x^2-9} \quad ✓$$

$$\frac{-x-15}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3} \quad ✓$$

$$-x-15 = A(x-3) + B(x+3) \quad ✓$$

when $x=3$ $\quad -18 = B.6 \quad \therefore B=-3$

when $x=-3$ $\quad 3-15 = A(-6) \quad \therefore A=2$

$$\int \frac{x^3-10x-15}{x^2-9}dx = \int x + \frac{2}{x+3} + \frac{-3}{x-3}dx \quad ✓$$

$$= \frac{x^2}{2} + 2\ln(x+3) - 3\ln(x-3) + C \quad ✓$$

Alternative

$$\frac{x^3-10x-15}{x^2-9}=x+A+\frac{B}{x+3}+\frac{C}{x-3}\quad ✓$$

$$x^3-10x-15=(x+A)(x+3)(x-3)+B(x-3)+C(x+3)\quad ✓$$

$x=3$ $\quad 27-30-15=C(3+3)$ $\quad C=-3$

$x=-3$ $\quad -27+30-15=B(-3-3)$ $\quad B=2$ ✓

$x=0$ $\quad -15=A(3)(-3)-3B+3C$

$\quad -15=-9A-3(2)+3(-3)$ $\quad \therefore A=0$ ✓

$$\int\frac{x^3-10x-15}{x^2-9}dx=\int x+0+\frac{2}{x+3}+\frac{-3}{x-3}\,dx\quad ✓$$

$$=\frac{x^2}{2}+2\ln(x+3)-3\ln(x-3)+C\quad ✓$$

(6)

[12]

QUESTION 4

4.1 $\frac{dy}{dx}+y\sec x=x\cos x$

$p=\sec x$ $\quad Q=x\cos x$

$e^{\int pdx}=e^{\int \sec x\,dx}$

$=e^{\ln(\sec x+\tan x)}$ ✓

$=\sec x+\tan x$

$\int Qe^{\int pdx}dx=\int x\cos x(\sec x+\tan x)dx$ ✓

$\int x\cos x(\frac{1}{\cos x}+\frac{\sin x}{\cos x})dx$

$=\int x+x\sin x\,dx$

$=\int x\,dx+\int x\sin x\,dx$ using by parts for the second integral

$=\frac{x^2}{2}+\left\{x(-\cos x)-\int 1.-\cos x\,dx\right\}$ ✓

$=\frac{x^2}{2}+\left\{-x\cos x+\int\cos x\,dx\right\}$ ✓

$=\frac{x^2}{2}-x\cos x+\sin x$ ✓

$y(\sec x+\tan x)=\frac{x^2}{2}-x\cos x+\sin x+C$ ✓

(6)

4.2 $r^2 - 6r + 12 = 0$

$r = \dfrac{6 \pm \sqrt{36-48}}{2}$

$= \dfrac{6 \pm \sqrt{-12}}{2} = \dfrac{6 \pm 2\sqrt{3}i}{2}$

$= 3 + \sqrt{3}i$ ✓

$\therefore y_c = e^{3x}\left[A\cos\sqrt{3}x + B\sin\sqrt{3}x\right]$ ✓

$y = y_c + y_p$ But $y_p = 0$

$y = e^{3x}\left[A\cos\sqrt{3}x + B\sin\sqrt{3}x\right]$

$x = 0$; $y = 2$ $\quad 2 = e^0\left[A\cos 0 + B\sin 0\right]$ ✓

$\therefore A = 2$

$\dfrac{dy}{dx} = e^{3x}\left[-\sqrt{3}A\sin\sqrt{3}x + \sqrt{3}B\cos\sqrt{3}x\right] + 3e^{3x}\left[A\cos\sqrt{3}x + B\sin\sqrt{3}x\right]$ ✓

$x = 0;\ \dfrac{dy}{dx} = 6 + 4\sqrt{3}$ $\quad 6 + 4\sqrt{3} = e^0\left[-\sqrt{3}A\sin 0 + \sqrt{3}B\cos 0\right] + 3e^0\left[A\cos 0 + B\sin 0\right]$

$6 + 4\sqrt{3} = \sqrt{3}B + 3A$

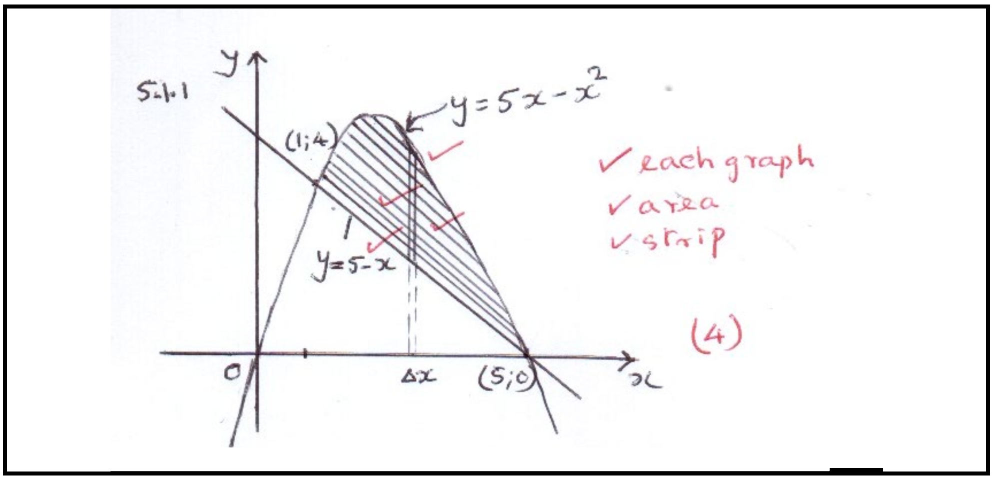
$put A = 2$ $\quad 6 + 4\sqrt{3} = \sqrt{3}B + 6 \quad \therefore B = 4$ ✓

$\therefore\ y = e^{3x}\left[2\cos\sqrt{3}x + 4\sin\sqrt{3}x\right]$ ✓ (6)

[12]

QUESTION 5

5.1 5.1.1



(4)

$$5x - x^2 = 5 - x \quad ✓$$
$$x^2 - 6x + 5 = 0$$
$$(x-5)(x-1) = 0 \quad ✓$$
$$x = 5 \quad x = 1$$
$$y = 0 \quad y = 4$$
$$(5;0) \quad (1;4) \quad ✓$$

OR

$$5x - x^2 = 5 - x$$
$$x(5 - x) = 5 - x$$
$$x(5 - x) - (5 - x) = 0$$
$$(5 - x)(x - 1) = 0$$
$$x = 5 \quad x = 1$$
$$y = 0 \quad y = 4$$

(3)

5.1.2

$$A = \int_1^5 5x - x^2 - (5 - x)\,dx \quad ✓$$
$$= \int_1^5 (6x - x^2 - 5)\,dx$$
$$= \left[\frac{6x^2}{2} - \frac{x^3}{3} - 5x\right]_1^5 \quad ✓$$
$$= \left[3(5)^2 - \frac{(5)^3}{3} - 5(5) - \left\{3(1)^2 - \frac{(1)^3}{3} - 5(1)\right\}\right]$$
$$= 10\frac{2}{3} \quad or \quad \frac{32}{3} \quad or \quad 10{,}667\text{units}^2 \quad ✓$$

(3)

5.1.3

$$V_y = 2\pi\int_a^b x(y_1 - y_2)\,dx \quad ✓$$

$$= 2\pi\int_1^5 x(6x - x^2 - 5)\,dx$$

$$= 2\pi\int_1^5 (6x^2 - x^3 - 5x)\,dx$$

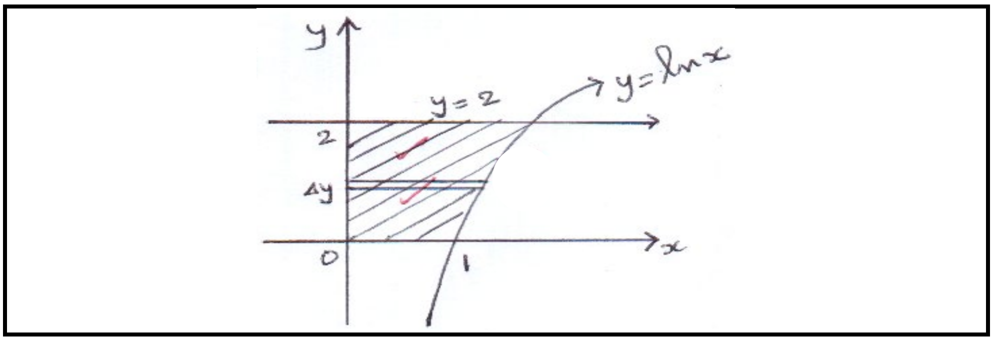
$$= 2\pi\left[6\frac{x^3}{3} - \frac{x^4}{4} - 5\frac{x^2}{2}\right]_1^5 \quad ✓$$

$$= 2\pi\left[6\frac{5^3}{3} - \frac{5^4}{4} - 5\frac{5^2}{2} - \left\{6\frac{1^3}{3} - \frac{1^4}{4} - 5\frac{1^2}{2}\right\}\right] \quad ✓$$

$$= 64\pi \quad = 201,062\text{units}^3 \quad ✓$$

(4)

5.2 5.2.1



(2)

5.2.2

$$A = \int_a^b (x_1 - x_2)dy \quad ✓ \qquad y = \ln x \Rightarrow x = e^y \quad ✓$$

$$= \int_0^2 e^y dy \quad ✓$$

$$= \left[e^y\right]_0^2 \quad ✓$$

$$= \left[e^2 - e^0\right] \quad ✓$$

$$= 6,389\text{units}^2 \quad ✓$$

$$I_x = \int_a^b r^2 dA$$

$$= \int_{-4}^{-2} y^2 \frac{3}{2}(y+4)^{\frac{1}{2}} dy \qquad u = y+4 \quad dy = du \qquad y = u-4$$

$$y = -4 \quad u = 0 \text{ and } y = -2 \quad u = 2$$

$$= \frac{3}{2}\int_0^2 (u-4)^2 u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{3}{2}\int_0^2 (u^2 - 8u + 16)u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{3}{2}\int_0^2 (u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}}) du \quad \checkmark$$

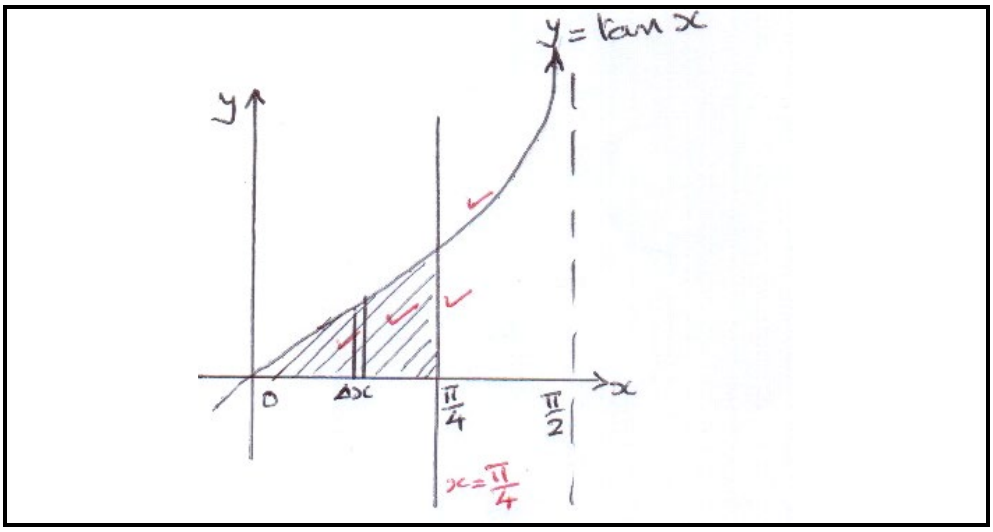
$$= \frac{3}{2}\left[\frac{2}{7}u^{\frac{7}{2}} - 8.\frac{2}{5}u^{\frac{5}{2}} + 16.\frac{2}{3}u^{\frac{3}{2}}\right]_0^2 \quad \checkmark$$

$$= \frac{3}{2}\left[\frac{2}{7}2^{\frac{7}{2}} - 8.\frac{2}{5}2^{\frac{5}{2}} + 16.\frac{2}{3}2^{\frac{3}{2}} - (0)\right] \quad \checkmark$$

$$= 22{,}951 \text{units}^4 \quad \checkmark$$

(6)

5.3 5.3.1



(2)

5.3.2 $I_x = \frac{1}{2}\pi\rho\int_a^b y^4\,dx$

$\frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^4 x\,dx$

$\frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^2 x\tan^2 x\,dx$ ✓

$\frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} \tan^2 x(\sec^2 x - 1)\,dx$

$\frac{1}{2}\pi\rho\int_0^{\frac{\pi}{4}} (\tan^2 x\sec^2 x - \tan^2 x)\,dx$ ✓

$\frac{1}{2}\pi\rho\left[\frac{\tan^3 x}{3} - (\tan x - x)\right]_0^{\frac{\pi}{4}}$

$\frac{1}{2}\pi\rho\left[\frac{\tan^3 \frac{\pi}{4}}{3} - \tan\frac{\pi}{4} + \frac{\pi}{4} - (0)\right]$ ✓

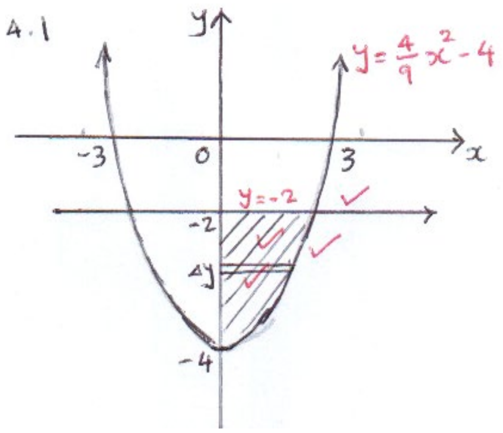
$\frac{1}{2}\pi\rho\left[\frac{1}{3} - 1 + \frac{\pi}{4}\right]$

$= 0{,}187\rho$ ✓

For x intercepts $\frac{4}{9}x^2 - 4 = 0$ $x^2 = 9$ $\therefore x = -3$ $x = 3$ ✓

y intercept $y = -4$ ✓ (6)

5.4 5.4.1



$$y=\frac{4}{9}x^2-4$$

$$\frac{4}{9}x^2=y+4$$

$$x^2=\frac{9}{4}(y+4)$$

$$x=\frac{3}{2}(y+4)^{\frac{1}{2}} \quad ✓$$

(4)

5.4.2

$$I_x=\int_a^b r^2 dA$$

$$=\int_{-4}^{-2} y^2\,\frac{3}{2}(y+4)^{\frac{1}{2}}dy \qquad u=y+4 \quad dy=du \qquad y=u-4$$

$$y=-4 \quad u=0 \text{ and } y=-2 \quad u=2$$

$$=\frac{3}{2}\int_0^2 (u-4)^2 u^{\frac{1}{2}}du \quad ✓$$

$$=\frac{3}{2}\int_0^2 (u^2-8u+16)u^{\frac{1}{2}}du \quad ✓$$

$$=\frac{3}{2}\int_0^2 (u^{\frac{5}{2}}-8u^{\frac{3}{2}}+16u^{\frac{1}{2}})du \quad ✓$$

$$=\frac{3}{2}\left[\frac{2}{7}u^{\frac{7}{2}}-8.\frac{2}{5}u^{\frac{5}{2}}+16.\frac{2}{3}u^{\frac{3}{2}}\right]_0^2 \quad ✓$$

$$=\frac{3}{2}\left[\frac{2}{7}2^{\frac{7}{2}}-8.\frac{2}{5}2^{\frac{5}{2}}+16.\frac{2}{3}2^{\frac{3}{2}}-(0)\right] \quad ✓$$

$$=22,951\text{units}^4 \quad ✓$$

(6)

[40]

QUESTION 6

6.1

$$y = \left(3 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}\left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} . -\frac{2}{3}x^{-\frac{1}{3}} \qquad = -\left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} x^{-\frac{1}{3}}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(3 - x^{\frac{2}{3}}\right)x^{-\frac{2}{3}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{\left(3 - x^{\frac{2}{3}}\right)}{x^{\frac{2}{3}}} \qquad = \frac{x^{\frac{2}{3}} + \left(3 - x^{\frac{2}{3}}\right)}{x^{\frac{2}{3}}} = \frac{3}{x^{\frac{2}{3}}} \quad ✓$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{\sqrt{3}}{x^{\frac{1}{3}}} = \sqrt{3}x^{-\frac{1}{3}} \quad ✓$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{3}x^{-\frac{1}{3}} dx \quad ✓$$ For limits

$$= \sqrt{3}\left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right]_0^2 \quad ✓$$ For integration

$$= \sqrt{3}\,\frac{3}{2}\left[2^{\frac{2}{3}} - 0\right] \quad ✓$$ For substitution of limits

$$= 4{,}124 \text{ units} \quad ✓$$

Alternative

$x = 0 \Rightarrow y = 5{,}196$
$x = 2 \Rightarrow y = 1{,}679$

$y = (3 - x^{2/3})^{3/2}$

$y^{2/3} = 3 - x^{2/3}$

$x = (3 - y^{2/3})^{3/2}$

$\Rightarrow \frac{dx}{dy} = \frac{3}{2}\left(3 - y^{2/3}\right)^{1/2} \times -\frac{2}{3}y^{-1/3}$

$\Rightarrow \left(\frac{dx}{dy}\right)^2 = (3 - y^{2/3}) \times y^{-2/3}$

$\Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{(3-y^{2/3})}{y^{2/3}} = \frac{y^{2/3}+(3-y^{2/3})}{y^{2/3}} = \frac{3}{y^{2/3}}$ ✓

$\Rightarrow S = \int_{1.679}^{5.196} \sqrt{\frac{3}{y^{2/3}}}\, dy$ ✓

$= \sqrt{3}\int_{1.679}^{5.196} y^{-1/3} dy$ ✓

$= \sqrt{3}\left[\frac{y^{2/3}}{2/3}\right]_{1.679}^{5.196}$ ✓

$= \frac{3\sqrt{3}}{2}\left[(5.196)^{2/3} - (1.679)^{2/3}\right]$ ✓

$= 4.124$ ✓ (6)

6.2 $x = \sqrt{r^2 - y^2}$

$$\Rightarrow y = \sqrt{r^2 - x^2} = (r^2 - x^2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \times -2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} 2\pi x \sqrt{\frac{r^2}{r^2 - x^2}}\, dx \checkmark$$

$$= 2\pi r \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi r \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} -2x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi r \left[\frac{(r^2 - x^2)^{\frac{1}{2}}}{1/2}\right]_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} \checkmark$$

$$= -2\pi r[a - (a + h)] \checkmark$$

$$= 2\pi r h \checkmark$$

Alternative 1

$x = \sqrt{r^2 - y^2}$

$\Rightarrow x^2 = r^2 - y^2$

$\Rightarrow 2x\frac{dx}{dy} = -2y$

$\Rightarrow \frac{dx}{dy} = -\frac{y}{x}$

$\Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{y^2}{x^2}$

$\Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{x^2} = \frac{x^2+y^2}{x^2} = \frac{r^2}{x^2}$

$A_y = \int_a^b 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$ ✓

$= \int_a^{a+h} 2\pi x\sqrt{\frac{r^2}{x^2}}\, dy$ ✓

$= 2\pi r\int_a^{a+h} dy$ ✓

$= 2\pi r[y]_a^{a+h}$ ✓

$= 2\pi r[a + h - a]$ ✓

$= 2\pi rh$ ✓

Alternative 2

$x = \sqrt{r^2 - y^2}$

$y = a \Rightarrow\ x = \sqrt{r^2 - a^2}$
$y = a + h \Rightarrow\ x = \sqrt{r^2 - (a+h)^2}$

$\Rightarrow y = \sqrt{r^2 - x^2} = (r^2 - x^2)^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \times -2x$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2 - x^2}$

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} 2\pi x \sqrt{\frac{r^2}{r^2 - x^2}}\, dx \checkmark$$

$$= 2\pi r \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi r \int_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} -2x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi r \left[\frac{(r^2 - x^2)^{\frac{1}{2}}}{1/2}\right]_{\sqrt{r^2-(a+h)^2}}^{\sqrt{r^2-a^2}} \quad \checkmark$$

$= -2\pi r[a - (a+h)] \checkmark$

$= 2\pi r h \checkmark$ (6)

[12]

TOTAL: 100