higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

# NATIONAL CERTIFICATE

# MATHEMATICS N6

(16030186)

**6 April 2021 (X-paper)**
**09:00–12:00**

**This question paper consists of 5 pages and a formula sheet of 7 pages.**

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# DEPARTMENT OF HIGHER EDUCATION AND TRAINING
# REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

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**INSTRUCTIONS AND INFORMATION**

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show all the intermediate steps.
6. All the formulae used must be written down.
7. Graphs must be large, neat and clear and may be done in pencil.
8. Use only a black or blue pen.
9. Write neatly and legibly.

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**QUESTION 1**

1.1 Given: $z = a^{3x-2y}$

Determine:

1.1.1 $\dfrac{\partial z}{\partial x}$ (1)

1.1.2 $\dfrac{\partial^2 z}{\partial y \partial x}$ (2)

1.2 Given: The parametric equations $x = \dfrac{1}{1+2t}$ and $y = 1+2t$

Calculate, in terms of t:

1.2.1 $\dfrac{dy}{dx}$ (2)

1.2.2 $\dfrac{d^2 y}{dx^2}$ (1)

**[6]**

**QUESTION 2**

Determine $\int y\,dx$:

2.1 $y = \sin^3 2x\left(1-\sin^2 2x\right)$ (5)

2.2 $y = \tan^3 3x \sec^4 3x$ (4)

2.3 $y = (x+1)^2 \ln(x+1)^2$ (4)

2.4 $y = \dfrac{1}{\sqrt{\dfrac{9}{4} - 3x - x^2}}$ (4)

2.5 $y = \sin^2 2x - \cos^2 2x$ (1)

**[18]**

 

## QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{3x^2 - 6x - 1}{(x^2+1)(x+1)(x-1)} dx$ (6)

3.2 $\int \frac{x^3 - 10x - 15}{x^2 - 9} dx$ (6)

**[12]**

## QUESTION 4

4.1 Determine the general solution of:

$$\frac{dy}{dx} + y \sec x = x \cos x$$ (6)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 12y = 0 \quad \text{if } y = 2 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 6 + 4\sqrt{3} \text{ when } x = 0$$ (6)

**[12]**

## QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the curve $y = 5x - x^2$ and the line $y = 5 - x$.

Make a neat sketch of the graphs and clearly indicate the area bounded by the graphs. Show the representative strip/element that you will use to calculate the area and the volume when this area rotates about the $y$-axis. (7)

5.1.2 Calculate the bounded area described in QUESTION 5.1.1. (3)

5.1.3 Calculate the volume generated when the bounded area, described in QUESTION 5.1.1, rotates about the $y$-axis. (4)

5.2 5.2.1 Make a neat sketch to show the area bounded by the curve $y = \ln x$, the $x$ and $y$ axes and the line $y = 2$. Show the representative strip/element that you will use to calculate the area. (2)

5.2.2 Calculate the area described in QUESTION 5.2.1. Calculate the area moment about the x-axis of the area described in QUESTION 5.2.1 as well as the y-coordinate of the centroid. (6)

5.3 5.3.1 Draw the graph of $y = \tan x$ for $0 \leq x \leq \frac{\pi}{2}$.

Show the area bounded by the graph, the x-axis and the line $x = \frac{\pi}{4}$.

Use a representative strip PERPENDICULAR to the x-axis. (2)

5.3.2 Calculate the moment of inertia about the *x*-axis when the area described in QUESTION 5.3.1 rotates about the *x*-axis. (6)

5.4 5.4.1 Determine the *x*-intercept and the y-intercept of the graph of $y = \frac{4}{9}x^2 - 4$.

Make a neat sketch of the graph of $y = \frac{4}{9}x^2 - 4$ for $-3 \leq x \leq 3$.

Show the area in the fourth quadrant bounded by the graph, the *y*-axis and the line *y* = -2. Use a representative strip PERPENDICULAR to the *y*-axis. (4)

5.4.2 Calculate the second moment of area about the x-axis of the area described in QUESTION 5.4.1. (6)

**[40]**

**QUESTION 6**

6.1 Calculate the length of the curve given by $y = \left(3 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$ from $x = 0$ to $x = 2$. (6)

6.2 The part of the curve $x = \sqrt{r^2 - y^2}$ between the parallel lines $y = a$ and $y = a + h$ is rotated about the *y*-axis.

Calculate the surface area generated. (6)

**[12]**

**TOTAL: 100**

**MATHEMATICS N6**

**FORMULA SHEET**

Any applicable formula may also be used.

**Trigonometry**

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$ ; $\sin x = \dfrac{1}{\operatorname{cosec} x}$ ; $\cos x = \dfrac{1}{\sec x}$

 

| $f(x)$ | $\frac{d}{dx}\, f(x)$ | $\int f(x)dx$ |
|---|---|---|
| $x^n$ | $nx^{n-1}$ | $\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ |
| $ax^n$ | $a\,\frac{d}{dx}\, x^n$ | $a \int x^n\, dx$ |
| $e^{ax+b}$ | $e^{ax+b}.\frac{d}{dx}\,(ax+b)$ | $\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$ |
| $a^{dx+e}$ | $a^{dx+e}.\ln a.\frac{d}{dx}\,(dx+e)$ | $\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)} + C$ |
| $\ln(ax)$ | $\frac{1}{ax}\cdot\frac{d}{dx}\,ax$ | $x\ln ax - x + C$ |
| $e^{f(x)}$ | $e^{f(x)}\,\frac{d}{dx}\,f(x)$ | - |
| $a^{f(x)}$ | $a^{f(x)}.\ln a.\frac{d}{dx}\,f(x)$ | - |
| $\ln f(x)$ | $\frac{1}{f(x)}\cdot\frac{d}{dx}\,f(x)$ | - |
| $\sin ax$ | $a\cos ax$ | $-\frac{\cos ax}{a} + C$ |
| $\cos ax$ | $-a\sin ax$ | $\frac{\sin ax}{a} + C$ |
| $\tan ax$ | $a\sec^2 ax$ | $\frac{1}{a}\ln[\sec(ax)] + C$ |
| $\cot ax$ | $-a\,\text{cosec}^2\, ax$ | $\frac{1}{a}\ln[\sin(ax)] + C$ |
| $\sec ax$ | $a\sec ax\tan ax$ | $\frac{1}{a}\ln[\sec ax + \tan ax] + C$ |
| $\text{cosec}\, ax$ | $-a\,\text{cosec}\, ax\cot ax$ | $\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right] + C$ |

| $f(x)$ | $\frac{d}{dx}\, f(x)$ | $\int f(x)\, dx$ |
|---|---|---|
| $\sin f(x)$ | $\cos f(x)\,.\; f'(x)$ | - |
| $\cos f(x)$ | $-\sin f(x)\,.\; f'(x)$ | - |
| $\tan f(x)$ | $\sec^2 f(x)\,.\; f'(x)$ | - |
| $\cot f(x)$ | $-\text{cosec}^2 f(x)\,.\; f'(x)$ | - |
| $\sec f(x)$ | $\sec f(x)\tan f(x)\,.\; f'(x)$ | - |
| $\text{cosec}\, f(x)$ | $-\text{cosec}\, f(x)\cot f(x)\,.\; f'(x)$ | - |
| $\sin^{-1} f(x)$ | $\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$ | - |
| $\cos^{-1} f(x)$ | $\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$ | - |
| $\tan^{-1} f(x)$ | $\frac{f'(x)}{[f(x)]^2+1}$ | - |
| $\cot^{-1} f(x)$ | $\frac{-f'(x)}{[f(x)]^2+1}$ | - |
| $\sec^{-1} f(x)$ | $\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$ | - |
| $\text{cosec}^{-1} f(x)$ | $\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$ | - |
| $\sin^2(ax)$ | - | $\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$ |
| $\cos^2(ax)$ | - | $\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$ |
| $\tan^2(ax)$ | - | $\frac{1}{a}\tan(ax)-x+C$ |

| $f(x)$ | $\frac{d}{dx}\,f(x)$ | $\int f(x)\,dx$ |
|---|---|---|

$\cot^2(ax)$ - $-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$

$$\int [f(x)]^n\ f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\ f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\ dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

**Applications of integration**

**AREAS**

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

 

**VOLUMES**

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

**AREA MOMENTS**

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

**CENTROID**

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

**SECOND MOMENT OF AREA**

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

**VOLUME MOMENTS**

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

**CENTRE OF GRAVITY**

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

**MOMENTS OF INERTIA**

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

**GENERAL**

$$I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$$

**CIRCULAR LAMINA**

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

**CENTRE OF FLUID PRESSURE**

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3 \ (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \ dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \ dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$