higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

# MARKING GUIDELINE

## NATIONAL CERTIFICATE

## MATHEMATICS N6

**25 JULY 2018**

**This marking guideline consists of 17 pages.**

**NOTE: MARKING GUIDELINE = 200/2 = 100**

## QUESTION 1

1.1

$$z = \cos^2\left(x^2y^2\right)$$

$$\frac{\partial z}{\partial x} = 2\cos\left(x^2y^2\right)\left\{-\sin\left(x^2y^2\right)\right\}2xy^2 \quad ✓✓✓✓$$

(4)

1.2

$$z = \sin^{-1}\frac{y}{x}$$

$$\frac{\partial z}{\partial y} = \frac{\frac{1}{x}\ ✓}{\sqrt{1-\left(\frac{y}{x}\right)^2}\ ✓}$$

(2)

1.3

$$A = \frac{1}{2}xy$$

$$\Delta A = \frac{\partial A}{\partial x}\Delta x + \frac{\partial A}{\partial y}\Delta y \quad ✓$$

$$= \frac{1}{2}y\Delta x + \frac{1}{2}x\Delta y \quad ✓✓$$

$$= \frac{1}{2}(4)(0,2) + \frac{1}{2}(3)(-0,2) \quad ✓✓$$

$$= 0,1 units^2 \quad ✓$$

(6)
**[12]**

## QUESTION 2

2.1

$$y = \int x^2 \ln x^2 dx = \int x^2\, 2\ln x dx \quad ✓$$

$$= 2\ln x\frac{x^3}{3} - \int\frac{2}{x}\frac{x^3}{3}dx \quad ✓✓$$

$$= 2\ln x\frac{x^3}{3} - \int\frac{2}{3}x^2 dx \quad ✓$$

$$= 2\ln x\frac{x^3}{3} - \frac{2}{3}\frac{x^3}{3} + C \quad ✓✓$$

$$= \frac{2}{3}x^3\ln x - \frac{2}{9}x^3 + C$$

(6)

2.2 $y = x\tan^2 x$

$\int x\tan^2 x dx$

$= \int x(\sec^2 x - 1)dx$ ✓

✓ ✓ ✓

$= x(\tan x - x) - \int 1(\tan x - x)dx$

✓ ✓

$= x(\tan x - x) - \ln(\sec x) + \frac{x^2}{2} + C$ (6)

2.3 $x(x-1)+1 = x^2 - x + 1$ ✓

✓ ✓

$= \left\{x^2 - x + \left(\frac{1}{2}\right)^2\right\} - \left(\frac{1}{2}\right)^2 + 1$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$ ✓

Or using $\frac{4ac - b^2}{4a} + a\left(x + \frac{b}{2a}\right)^2$

$= \int \frac{1}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}} dx$ ✓

✓ ✓ ✓

$= \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} + C = \frac{2}{\sqrt{3}} \tan^{-1} \frac{2\left(x - \frac{1}{2}\right)}{\sqrt{3}} + C$

$= \frac{2}{\sqrt{3}} \tan^{-1} \frac{2x - 1}{\sqrt{3}} + C$ (8)

2.4 $\int \frac{1}{\tan^4 \frac{x}{4}} dx$

$= \int \cot^4 \frac{x}{4} dx$ ✓

$= \int \cot^2 \frac{x}{4} \cot^2 \frac{x}{4} dx$ ✓

$= \int \cot^2 \frac{x}{4}\left(\cos ec^2 \frac{x}{4} - 1\right) dx$ ✓

$= \int \cot^2 \frac{x}{4} \cos ec^2 \frac{x}{4} dx - \int \cot^2 \frac{x}{4} dx$ ✓

$= -4\left(\frac{1}{3}\right)\cot^3 \frac{x}{4} - \left\{-4\cot \frac{x}{4} - x\right\} + C$ ✓ ✓

$= -\frac{4}{3}\cot^3 \frac{x}{4} + 4\cot \frac{x}{4} + x + C$ ✓✓ (8)

2.5 $\int \sin^5 3x \cos^3 3x dx$

$= \int \sin^5 3x \cos^2 3x \cos 3x dx$ ✓

$= \int \sin^5 3x(1 - \sin^2 3x) \cos 3x dx$ ✓

$= \int \sin^5 3x \cos 3x dx - \int \sin^7 3x \cos 3x dx$ ✓ ✓

$= \frac{1}{3}\left(\frac{\sin^6 3x}{6}\right) - \frac{1}{3}\left(\frac{\sin^8 3x}{8}\right) + C$ ✓ ✓ ✓ ✓

(Consider the u substitution also) (8)

Alternative 2.5 (breaking up $\sin^5 3x$ )

$\int \sin^5 3x \cos^3 3x dx$

$= \int \sin^4 3x \sin 3x \cos^3 3x dx$ ✓

$= \int \left(\sin^2 3x\right)^2 \sin 3x \cos^3 3x dx$ ✓

$= \int \left(1 - \cos^2 3x\right)^2 \sin 3x \cos^3 3x dx$ ✓

$u = \cos 3x$

$\frac{du}{dx} = -3\sin 3x$

$dx = -\frac{du}{3\sin 3x}$

$= \int \left(1 - u^2\right)^2 \sin 3x u^3 \left(-\frac{du}{3\sin 3x}\right)$ ✓

$= -\frac{1}{3}\int \left(1 - 2u^2 + u^4\right) u^3 du$ ✓

$= -\frac{1}{3}\int \left(u^3 - 2u^5 + u^7\right) du$ ✓

$= -\frac{1}{3}\left[\frac{u^4}{4} - \frac{2u^6}{6} + \frac{u^8}{8}\right] + C$ ✓

$= -\frac{1}{3}\left[\frac{\cos^4 3x}{4} - \frac{\cos^6 3x}{3} + \frac{\cos^8 3x}{8}\right] +$ ✓

$= -\frac{\cos^4 3x}{12} + \frac{\cos^6 3x}{9} - \frac{\cos^8 3x}{24} + C$

**[36]**

## QUESTION 3

3.1 $\int \frac{3x^2-3x+7}{(3x+1)(x^2-2x+2)}dx$

$\frac{3x^2-3x+7}{(3x+1)(x^2-2x+2)} = \frac{A}{3x+1} + \frac{Bx+C}{x^2-2x+2}$ ✓

$3x^2-3x+7 = A(x^2-2x+2)+(Bx+C)(3x+1)$ ✓

$x = -\frac{1}{3}$ $\quad 3\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right)+7 = A\left\{\left(-\frac{1}{3}\right)^2 - 2\left(-\frac{1}{3}\right)+2\right\} \therefore A = 3$ ✓

$3x^2-3x+7 = Ax^2-2Ax+2A+3Bx^2+3Cx+Bx+C$ ✓

$x^2$ $\quad A+3B=3 \quad \therefore B=0$ ✓

$cons\tan ts$ $\quad 2A+C=7 \quad \therefore C=1$ ✓

$\int \frac{3x^2-3x+7}{(3x+1)(x^2-2x+2)}dx = \int \frac{3}{3x+1} + \frac{1}{x^2-2x+2}dx$ ✓

$= \int \frac{3}{3x+1}dx + \int \frac{1}{x^2-2x+2}dx$ ✓

$= \int \frac{3}{3x+1}dx + \int \frac{1}{(x-1)^2+1}dx$ ✓ ✓

$= \ln(3x+1) + \tan^{-1}(x-1) + C$
✓ ✓

(12)

3.2 $\int \frac{x^2-7x+15}{\left(x^3-6x^2+9x\right)}dx$

$$\frac{x^2-7x+15}{\left(x^3-6x^2+9x\right)}=\frac{x^2-7x+15}{x\left(x^2-6x+9\right)}\checkmark=\frac{x^2-7x+15}{x\left(x-3\right)^2}\checkmark=\frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}\checkmark$$

$$x^2-7x+15=A(x-3)^2+Bx(x-3)+C\checkmark$$

$Let\quad x=3\qquad C=1$ ✓

$Let\quad x=0\qquad 15=9A\qquad \therefore A=\frac{5}{3}$ ✓

$Let\quad x=1\qquad 9=4A-2B+C\qquad \therefore B=-\frac{2}{3}$ ✓

$$\int \frac{x^2-7x+15}{\left(x^3-6x^2+9x\right)}dx=\int \frac{A}{x}dx+\int \frac{B}{x-3}dx+\int \frac{C}{(x-3)^2}dx\checkmark$$

$$=\int \frac{\frac{5}{3}}{x}dx+\int \frac{-\frac{2}{3}}{x-3}dx+\int \frac{1}{(x-3)^2}dx\checkmark$$

$$=\frac{5}{3}\ln x\checkmark-\frac{2}{3}\ln(x-3)\checkmark-\frac{1}{x-3}\checkmark+C$$

**NOTE**: Credit the method of expanding and equating coefficients to find the values of A, B and C. (12)

**[24]**

## QUESTION 4

4.1

$$\ln x\frac{dy}{dx}+\frac{y}{x}-\cos x=0$$

$$x\frac{dy}{dx}+\frac{y}{x\ln x}=\frac{\cos x}{\ln x} \quad ✓$$

$$e^{\int\frac{1}{x\ln x}dx}=e^{\ln(\ln x)}=\ln x \quad ✓✓✓$$

$$\int Qe^{\int pdx}dx=\int\frac{\cos x}{\ln x}(\ln x)dx=\int\cos xdx=\sin x \quad ✓✓✓✓$$

$$y\ln x=\sin x+C \quad ✓$$

$$2\ln\frac{\pi}{3}=\sin\frac{\pi}{3}+C \quad ✓ \quad C=-0,774 \quad ✓$$

$$y\ln x=\sin x-0,774 \quad ✓$$

(12)

4.2

$$3\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=x^2-x+5$$

$$3r^2+r-2=0 \quad ✓$$

$$(r+1)(3r-2)=0$$

$$r=-1 \quad r=\frac{2}{3} \quad ✓$$

$$y_c=Ae^{-x}+Be^{\frac{2}{3}x} \quad ✓$$

$$y=Cx^2+Dx+E \quad ✓$$

$$\frac{dy}{dx}=2Cx+D \quad ✓$$

$$\frac{d^2y}{dx^2}=2C \quad ✓$$

$$\therefore 3(2C)+2Cx+D-2(Cx^2+Dx+E)=x^2-x+5$$

$$6C+2Cx+D-2Cx^2-2Dx-2E=x^2-x+5 \quad ✓$$

$$-2C=-1 \qquad \therefore C=-\frac{1}{2} \quad ✓$$

$$2C-2D=-1 \qquad D=0$$

$$6C+D-2E=5 \qquad E=-4 \quad ✓$$

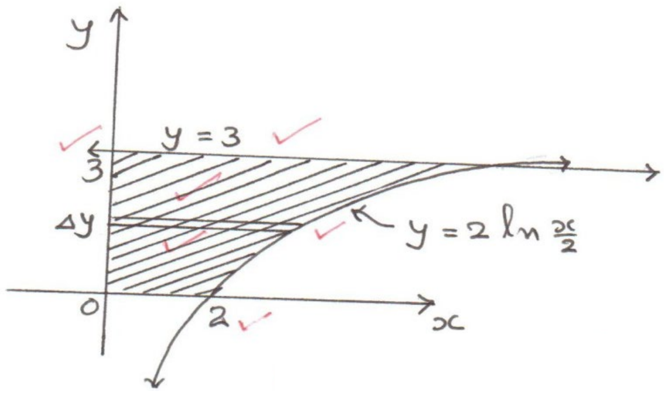
$$y_p=-\frac{1}{2}x^2-4 \quad ✓$$

$$y=Ae^{-x}+Be^{\frac{2}{3}x}-\frac{1}{2}x^2-4 \quad ✓✓ \quad r \qquad y=Ae^{\frac{2}{3}x}+Be^{-x}+-\frac{1}{2}x^2-4$$

(12)
**[24]**

## QUESTION 5

5.1 5.1.1




✓✓ $x$ intercepts
✓✓ each grapgh
✓ area ✓ strip (6)

5.1.2

$y = 2\ln\frac{x}{2}$ $\quad \therefore \frac{x}{2} = {}^{\frac{y}{}}$ ✓ $\quad x = 2e^{\frac{y}{2}}$

$V_x = 2\pi\int_a^b y\left(x_1 - x_2\right)dy$ ✓

$= 2\pi\int_0^3 y(2e^{\frac{y}{2}})dy$ ✓ ✓ ✓

$= 4\pi\left[ye^{\frac{y}{2}}2 - \int e^{\frac{y}{2}}2\right]_0^3$ ✓ ✓

$= 4\pi\left[ye^{\frac{y}{2}}2 - 4e^{\frac{y}{2}}\right]_0^3$

$= 4\pi\left[3e^{\frac{3}{2}}2 - 4e^{\frac{3}{2}} - \left(0-4\right)\right]$

$= 4\pi\left(2e^{\frac{3}{2}} + 4\right) = 51,854\pi = 162,903 units$ ✓ (8)

5.1.3

$$V_{m-y} = \int_a^b r\,dV$$

$$= \int_a^b \frac{x_1 + x_2}{2} 2\pi y\left(x_1 - x_2\right)dy \quad ✓$$

$$= \pi\int_a^b y\left(x_1^2 - x_2^2\right)dy \quad ✓$$

$$= \pi\int_0^3 y\left(2e^{\frac{y}{2}}\right)^2 dy \quad ✓$$

$$= \pi\int_0^3 y4e^y dy \quad ✓$$

$$= 4\pi\left[ye^y - e^y\right]_0^3 \quad ✓ \quad ✓$$

$$= 4\pi\left[3e^3 - e^3 - (0 - e^0)\right] \quad ✓$$

$$= 164,684\pi = 517,371 units^4 \quad ✓$$

$$\bar{x} = \frac{164,684\pi}{51,854\pi} = 3,7159 = 3,716 units \quad ✓ \quad ✓$$

(10)

5.2 5.2.1

$$y = \frac{1}{4}x^2 \qquad y = \frac{1}{4}(x+2)$$

$$\frac{1}{4}x^2 = \frac{1}{4}(x+2)$$

$$x^2 = x + 2$$

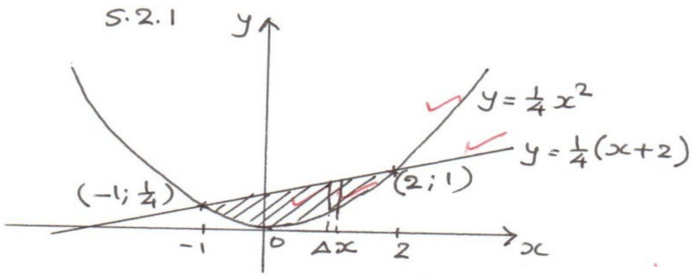
$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \qquad x = 2$$

$$y = \frac{1}{4} \qquad y = 1$$

$$\left(-1;\frac{1}{4}\right) ✓ \qquad (2;1) ✓$$




✓ each graph
✓ area
✓ strip (6)

5.2.2

$$A = \int_a^b y_1 - y_2 dx \quad ✓$$

$$= \int_{-1}^{2} \frac{1}{4}(x+2) - \frac{1}{4}x^2 dx \quad ✓✓✓$$

$$= \frac{1}{4}\int_{-1}^{2} x + 2 - x^2 dx \quad ✓$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} \quad ✓$$

$$= \frac{1}{4}\left[\frac{2^2}{2} + 2(2) - \frac{2^3}{3} - \left\{\frac{1}{2} - 2 + \frac{1}{3}\right\}\right] \quad ✓$$

$$= 1{,}125 units^2 \quad ✓$$

(8)

5.2.3

$$A_{m-x} = \int_a^b r dA$$

$$= \int_{-1}^{2} \frac{1}{2}(y_1 + y_2)(y_1 - y_2) dx$$

$$= \frac{1}{2}\int_{-1}^{2} (y_1^2 - y_2^2) dx \quad ✓$$

$$= \frac{1}{2}\int_{-1}^{2} \left\{\frac{1}{4}(x+2)\right\}^2 - \left\{\frac{1}{4}x^2\right\}^2 dx \quad ✓✓✓$$

$$= \frac{1}{32}\int_{-1}^{2} (x+2)^2 - x^4 dx$$

$$= \frac{1}{32}\int_{-1}^{2}(x^2+4x+4)-x^4dx \quad ✓$$

Or $\frac{1}{32}\left[\frac{(x+2)^3}{3}-\frac{x^5}{5}\right]_{-1}^{2}$

$$= \frac{1}{32}\left[\frac{x^3}{3}+\frac{4x^2}{2}+4x-\frac{x^5}{5}\right]_{-1}^{2} \quad ✓$$

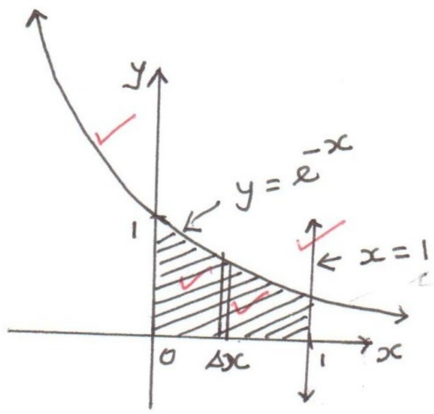
$$\frac{1}{32}\left[\frac{2^3}{3}+2^2+4(2)-\frac{2^5}{5}-\left\{\frac{(-1)^3}{3}+2(-1)^2+4(-1)-\frac{(-1)^5}{5}\right\}\right] \quad ✓$$

$$= \frac{9}{20} = 0,45units^3 \quad ✓$$

$$\bar{y} = \frac{1,125}{0,45} ✓ = 0,4units \quad ✓$$

(10)

5.3 5.3.1




✓ each graph
✓ area
✓ strip

(4)

5.3.2

$$I_x = \frac{1}{2}\pi\rho\int_a^b y^4dx \quad ✓$$

$$= \frac{1}{2}\pi\rho\int_0^1 \left(e^{-x}\right)^4 dx \quad ✓ ✓$$

$$= \frac{1}{2}\pi\rho\int_0^1 e^{-4x}dx \quad ✓$$

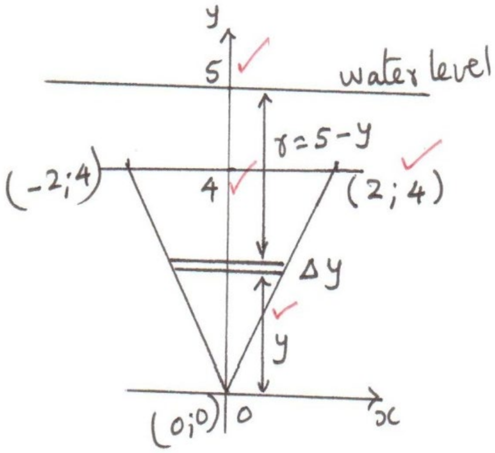
$$= \frac{1}{2}\pi\rho\left[\frac{e^{-4x}}{-4}\right]_0^1 \quad ✓$$

$$= -\frac{1}{8}\pi\rho\left[e^{-4}-e^0\right] \quad ✓ ✓$$

$$= 0,1227\pi\rho = 0,386\rho \quad ✓$$

(8)

5.4 5.4.1




✓ for shape (4)

5.4.2

$m = \frac{4-0}{2-0} = 2$ ✓

$\therefore y = 2x$ ✓

First moment

$= \int_a^b r dA$

| |
|---|
| $r = 5 - y$ |
| $dA = x_1 - x_2 = 2x = y$ |

$= \int_0^4 (5-y) y dy$ ✓ ✓

$= \int_0^4 (5y - y^2) dy$ ✓

$= \left[ \frac{5y^2}{2} - \frac{y^3}{3} \right]_0^4$ ✓

$= \left[ \frac{5(4)^2}{2} - \frac{(4)^3}{3} - 0 \right]$ ✓

$= \frac{56}{3} = 18,6667 m^3$ ✓

(8)

5.4.3

$$\int_a^b r^2 dA$$

$$= \int_0^4 (5-y)^2 \, y dy \quad ✓$$

$$= \int_0^4 \left(25 - 10y + y^2\right) y dy \quad ✓$$

$$= \int_0^4 \left(25y - 10y^2 + y^3\right) dy \quad ✓$$

$$= \left[\frac{25y^2}{2} - \frac{10y^3}{3} + \frac{y^4}{4}\right]_0^4 \quad ✓$$

$$= \left[\frac{25(4)^2}{2} - \frac{10(4)^3}{3} + \frac{(4)^4}{4} - 0\right] \quad ✓$$

$$= \frac{152}{3} = 50{,}6667 m^4 \quad ✓ \quad ✓$$

$$\bar{y} = \left(\frac{152}{3}\right) \div \left(\frac{56}{3}\right) = \frac{56{,}6667 m^4}{18{,}6667 m^3} = 2{,}714 m \quad ✓$$

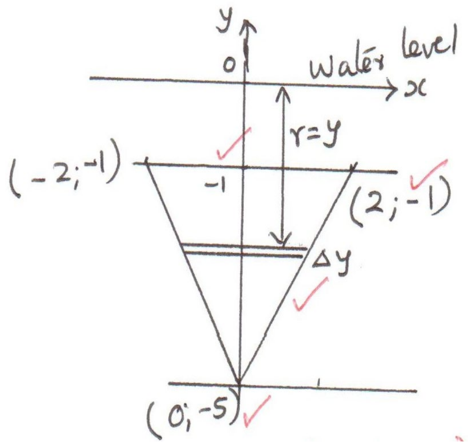
(8)

Note:

$$\int_0^4 (5-y)^2 \, y dy$$

can be integrated by parts

Alternative 5.4.1




✓ for shape

Alternative 5.4.2

$$m = \frac{-5+1}{0-2} = 2 \quad ✓$$

$$y + 5 = 2(x - 0)$$

$$\therefore 2x = y + 5 \quad ✓$$

First moment

$$= \int_a^b r\,dA = \int_{-5}^{-1} y(y+5)\,dy \quad ✓ \quad ✓$$

$$= \int_{-5}^{-1} (y^2 + 5y)\,dy \quad ✓$$

$$= \left[ \frac{y^3}{3} + \frac{5y^2}{2} \right]_{-5}^{-1} \quad ✓$$

$$= \frac{(-1)^3}{3} + \frac{5(-1)^2}{2} - \left\{ \frac{(-5)^3}{3} + \frac{5(-5)^2}{2} \right\} \quad ✓$$

$$= -18,6667 m^3 \quad ✓$$

Please put a tick for zero at the water level and -5 at the bottom of the gate

5.4.3 Second moment =

$$=\int_a^b r^2 dA$$

$$=\int_{-5}^{-1} y^2(y+5)dy \quad ✓ \quad ✓$$

$$=\int_{-5}^{-1} (y^3+5y^2)dy \quad ✓$$

$$=\left[\frac{y^4}{4}+\frac{5y^3}{3}\right]_{-5}^{-1} \quad ✓$$

$$=\left[\frac{(-1)^4}{4}+\frac{5(-1)^3}{3}-\frac{(-5)^4}{4}-\frac{5(-5)^3}{3}\right] \quad ✓$$

$$=\frac{152}{3}=50,6667m^4 \quad ✓$$

$$\bar{y}=\frac{50,6667}{-18,6667}=-2,714m \quad ✓ \quad ✓$$

**[80]**

## QUESTION 6

6.1

$$y=\frac{x^3}{6}+\frac{1}{2x}$$

✓ ✓

$$\frac{dy}{dx}=\frac{x^2}{2}-\frac{1}{2x^2}$$

$$\left(\frac{dy}{dx}\right)^2=\frac{x^4}{4}-\frac{1}{2}+\frac{1}{4x^4} \quad ✓$$

$$1+\left(\frac{dy}{dx}\right)^2=1+\frac{x^4}{4}-\frac{1}{2}+\frac{1}{4x^4} \quad ✓$$

$$=\frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4}$$

$$=\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2 \quad ✓$$

$$S=\int_1^2\sqrt{\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2}dx \quad ✓ \quad ✓$$

$$=\int_1^2 \frac{x^2}{2}+\frac{1}{2x^2}dx \quad ✓$$

$$=\left[\frac{x^3}{6}+\frac{x^{-1}}{-2}\right]_1^2 \quad ✓ \quad ✓$$

$$=\left[\frac{x^3}{6}-\frac{1}{2x}\right]_1^2$$

$$=\frac{2^3}{6}-\frac{1}{2(2)}-\left\{\frac{1}{6}-\frac{1}{2}\right\} \quad ✓$$

$$=1,417 units \quad ✓$$

(12)

6.2

$$y=\cos x$$

$$\frac{dy}{dx}=-\sin x \quad ✓$$

$$\left(\frac{dy}{dx}\right)^2=\sin^2 x \quad ✓$$

✓ ✓ ✓

$$A_x=2\pi\int_0^{\frac{\pi}{2}}\cos x\sqrt{1+\sin^2 x}\,dx$$

$$u=\sin x \qquad \frac{du}{dx}=\cos x \qquad dx=\frac{du}{\cos x}$$

$$x=0 \quad u=\sin 0=0 \quad x=\frac{\pi}{2} \quad u=\sin\frac{\pi}{2}=1$$

$$=2\pi\int_0^1\cos x\sqrt{1+u^2}.\frac{du}{\cos x} \quad ✓ \quad ✓ \quad ✓$$

$$=2\pi\int_0^1\sqrt{1+u^2}\,du$$

$$=2\pi\left[\frac{u}{2}\sqrt{1+u^2}+\frac{1}{2}\ln\left\{u+\sqrt{1+u^2}\right\}\right]_0^1 \quad ✓$$

$$=2\pi\left[\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln\left(1+\sqrt{2}\right)-\left\{0+\frac{1}{2}\ln\left(0+\sqrt{1}\right)\right\}\right] \quad ✓$$

$$=2\pi\left[\frac{1}{\sqrt{2}}+\frac{1}{2}\ln\left(1+\sqrt{2}\right)-0\right] \quad ✓$$

$$=7,212 units^2 \quad ✓$$

(12)
**[24]**

**TOTAL: 200**