higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

29 JULY 2019

This marking guideline consists of 20 pages.

NOTE: This paper is marked out of 200 and divided by 2 to get a mark out of 100.

QUESTION 1

1.1

$$z = x^2 + 2xy + y^2$$

$$\frac{\partial z}{\partial x} = 2x + 2y \checkmark \qquad \frac{\partial z}{\partial y} = 2x + 2y \checkmark$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x(2x+2y) + y(2x+2y) \checkmark$$

$$= (2x+2y)(x+y) \checkmark$$

$$= 2(x+y)(x+y) \checkmark$$

$$= 2(x+y)^2$$

$$= 2(x^2 + 2xy + y^2) \checkmark$$

$$= 2z$$

Alternative

$$z = x^2 + 2xy + y^2$$

$$= (x+y)^2 \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x.2(x+y).1 + y.2(x+y).1$$

$$= 2(x+y)(x+y) \checkmark$$

$$= 2(x+y) \checkmark$$

$$= 2z$$

(6)

1.2

$$x = \sqrt{t} = t^{\frac{1}{2}} \qquad y = \frac{1}{\sqrt{t}} = t^{-\frac{1}{2}}$$

$$\checkmark \qquad \checkmark$$

$$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}} \qquad \frac{dy}{dt} = -\frac{1}{2}t^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}t^{-\frac{3}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}} = -t^{-1} \quad \checkmark \quad \checkmark$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = t^{-2}\,\frac{1}{\frac{1}{2}t^{-\frac{1}{2}}} = t^{-2}\,2t^{\frac{1}{2}} = 2t^{-\frac{3}{2}} \quad \checkmark \quad \checkmark$$

$$x = \sqrt{t} \qquad y = \frac{1}{\sqrt{t}}$$

$$y = \frac{1}{x} \checkmark$$

$$\frac{dy}{dx} = -\frac{1}{x^2} = -x^{-2} \quad \checkmark \checkmark$$

$$\frac{d^2y}{dx^2} = 2x^{-3} \quad \checkmark$$

$$= 2\left(t^{\frac{1}{2}}\right)^{-3} = 2t^{-\frac{3}{2}} \quad \checkmark \quad \checkmark$$

(6)

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QUESTION 2

2.1 $\int y\,dx = \int x^2 e^{3x}\,dx \qquad f(x) = x^2 \qquad g'(x) = e^{3x}$

$$= x^2\frac{e^{3x}}{3} - \int 2x\frac{e^{3x}}{3}\,dx$$

$$= x^2\frac{e^{3x}}{3} - \left[2x\frac{e^{3x}}{9} - \int 2\frac{e^{3x}}{9}\,dx\right] \qquad ✓✓✓✓✓✓$$

$$= x^2\frac{e^{3x}}{3} - 2x\frac{e^{3x}}{9} + \frac{2}{27}e^{3x} + c$$

Alternative

$f(x)$	$g'(x)$	
x^2	e^{3x}	
$2x$	$\dfrac{e^{3x}}{3}$	✓
2	$\dfrac{e^{3x}}{9}$	✓
0	$\dfrac{e^{3x}}{27}$	✓

$$\boxed{x^2\frac{e^{3x}}{3} - 2x\frac{e^{3x}}{9} + 2\frac{e^{3x}}{27} + c} \quad ✓✓✓$$

(6)

2.2 $\int \cos^5\frac{x}{5}\,dx = \int \cos^4\frac{x}{5}\cos\frac{x}{5}\,dx$ ✓

$$= \int\left(\cos^2\frac{x}{5}\right)\cos\frac{x}{5}dx \quad ✓$$

$$= \int\left(1-\sin^2\frac{x}{5}\right)^2\cos\frac{x}{5}dx \quad ✓ \qquad u = \sin\frac{x}{5} \qquad \frac{du}{dx} = \frac{1}{5}\cos\frac{x}{5}$$

$$= \int\left(1-u^2\right)^2\cos\frac{x}{5}\,\frac{5du}{\cos\frac{x}{5}} \quad ✓ \qquad dx = \frac{5du}{\cos\frac{x}{5}}$$

$$= 5\int\left(1-u^2\right)^2 du \quad ✓$$

$$= 5\int\left(1-2u^2+u^4\right)du \quad ✓$$

$$= 5\left[u - 2\frac{u^3}{3} + \frac{u^5}{5}\right] + c \quad ✓$$

$$= 5\left[\sin\frac{x}{5} - 2\frac{\sin^3\frac{x}{5}}{3} + \frac{\sin^5\frac{x}{5}}{5}\right] + c$$ ✓

$$= 5\sin\frac{x}{5} - \frac{10}{3}\sin^3\frac{x}{5} + \sin^5\frac{x}{5} + c$$ (8)

2.3

$$\int \tan^3 x \sec x dx$$

$$= \int \tan^2 x \sec x \tan x dx$$ ✓

$$= \int (\sec^2 x - 1)\sec x \tan x dx$$ ✓

$$= \int (u^2 - 1) du$$

$$= \frac{u^3}{3} - u + c$$ ✓

$$= \frac{\sec^3 x}{3} - \sec x + c$$ ✓ ✓ ✓

$u = \sec x$	$\frac{du}{dx} = \sec x \tan x$	kkk
	$dx = \frac{du}{\sec x \tan x}$	

Alternative

$$\int \tan^3 x \sec x dx \qquad u = \tan x$$ ✓

$$= \int u^3 \sec x \frac{du}{\sec^2 x} \qquad \frac{du}{dx} = \sec^2 x \qquad dx = \frac{du}{\sec^2 x}$$

$$= \int u^3 \frac{du}{\sec x}$$ ✓

$$= \int u^3 \frac{du}{\sqrt{1+u^2}} \qquad v = 1 + u^2 \qquad \Rightarrow \frac{dv}{du} = 2u \Rightarrow du = \frac{dv}{2u}$$

$$= \int \frac{u^3}{v^{\frac{1}{2}}}\frac{dv}{2u} = \frac{1}{2}\int u^2 v^{-\frac{1}{2}} dv$$ ✓

$$= \frac{1}{2}\int (v-1)v^{-\frac{1}{2}} dv \qquad = \frac{1}{2}\int v^{\frac{1}{2}} - v^{-\frac{1}{2}} dv$$ ✓

$$= \frac{1}{2}\left[\frac{v^{\frac{3}{2}}}{\frac{3}{2}} - \frac{v^{\frac{1}{2}}}{\frac{1}{2}}\right] \quad = \frac{1}{3}v^{\frac{3}{2}} - v^{\frac{1}{2}}$$ ✓

$$= \frac{1}{3}\sec^3 x - \sec x + c$$ ✓

Alternative

$$\int \tan^3 x \sec x dx$$

$$\int \frac{\sin^3 x}{\cos^3 x}\frac{1}{\cos x}dx$$

$$\int \frac{\sin^3 x}{\cos^4 x}dx \quad \checkmark$$

$$= \int \frac{\sin^2 x \sin x}{\cos^4 x}dx$$

$$= \int \frac{1-\cos^2 x \sin x}{\cos^4 x}dx \quad \checkmark \qquad u = \cos x \qquad \frac{du}{dx} = -\sin x$$

$$= \int \frac{1-u^2 \sin x}{u^4}\frac{du}{-\sin x}$$

$$dx = \frac{du}{-\sin x}$$

$$= -\int \frac{1-u^2}{u^4}du \quad \checkmark$$

$$= -\int u^{-4} - u^{-2}du \quad \checkmark$$

$$= -\left[\frac{u^{-3}}{-3} - \frac{u^{-1}}{-1}\right] + c \quad \checkmark$$

$$= -\left[\frac{(\cos x)^{-3}}{-3} - \frac{(\cos x)^{-1}}{-1}\right] + c = \frac{\sec^3 x}{3} - \sec x + c \quad \checkmark$$

(6)

2.4

$$\int \frac{1}{9-4x-x^2}dx$$

$$= \int \frac{1}{13-(x+2)^2}dx \quad \checkmark$$

$$= \frac{1}{2\sqrt{13}}\ln\frac{\sqrt{13}+(x+2)}{\sqrt{13}-(x+2)} + c \quad \checkmark \checkmark \checkmark$$

$$9-4x-x^2$$

$$= -\left[x^2+4x+4-9-4\right] \quad \checkmark$$

$$= -\left[\left(x+2\right)^2 - 13\right] \quad \checkmark \checkmark$$

$$= 13 - \left(x+2\right)^2 \quad \checkmark$$

$$\text{or} \frac{1}{7,211} or 0,139 \ \ln\frac{\sqrt{13}+(x+2)}{\sqrt{13}-(x+2)} + c$$

$$\text{or } -\int \frac{1}{\left(x+2\right)^2 - 13}dx$$

$$= -\frac{1}{2\sqrt{13}}\ln\frac{\left(x+2\right)-\sqrt{13}}{\left(x+2\right)+\sqrt{13}} + c$$

$$ax^2+bx+c$$

$$= \frac{4ac-b^2}{4a} + a\left(x+\frac{b}{2a}\right)^2$$

$$9-4x-x^2$$

$$= \frac{4(-1)9-(4)^2}{4(-1)} - \left(x+\frac{-4}{2(-1)}\right)^2$$

$$= \frac{-36-16}{-4} - (x+2)^2$$

$$= 13-(x+2)^2$$

(8)

2.5 $\int \frac{1}{ab}\tan^{-1}\frac{bx}{a}\,dx \qquad f(x)=\tan^{-1}\frac{bx}{a}$ and $g'(x)=\frac{1}{ab}$ ✓ ✓

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{1}{ab}x\frac{\frac{b}{a}}{1+\frac{b^2x^2}{a^2}}\,dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{1}{ab}x\frac{ba}{a^2+b^2x^2}\,dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{x}{a^2+b^2x^2}\,dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\frac{1}{2b^2}\int\frac{2b^2x}{a^2+b^2x^2}\,dx \quad ✓$$

$$==\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\frac{1}{2b^2}\ln\left(a^2+b^2x^2\right)+c \quad ✓ \quad ✓$$

$$\boxed{\begin{aligned} u&=a^2+b^2x^2\\ \frac{du}{dx}&=2b^2x\\ dx&=\frac{du}{2b^2x}\end{aligned}}$$

Alternative

$$\int \frac{1}{ab}\tan^{-1}\frac{bx}{a}\,dx$$

$$u = \tan^{-1}\frac{bx}{a} \qquad \frac{bx}{a} = \tan u$$

$$\frac{b}{a} = \sec^2 u\frac{du}{dx} \qquad dx = \frac{a}{b}\sec^2 u du$$

$$\therefore \int \frac{1}{ab}\tan^{-1}\frac{bx}{a}\,dx$$

$$= \int \frac{1}{ab}u\frac{a}{b}\sec^2 u du$$

$$= \frac{1}{b^2}\int u\sec^2 u du \qquad f(u) = u \qquad g'(u) = \sec^2 u$$

$$= \frac{1}{b^2}\left[u\tan u - \int \tan u du\right]$$

$$\frac{1}{b^2}\left[u\tan u - \ln(\sec u)\right]$$

$$\frac{1}{b^2}\left[u\tan u - \ln(1+\tan^2 u)^{\frac{1}{2}}\right]$$

$$\frac{1}{b^2}\left[u\tan u - \frac{1}{2}\ln(1+\tan^2 u)\right]$$

$$\frac{1}{b^2}\left[\tan^{-1}\frac{bx}{a}\left(\frac{bx}{a}\right) - \frac{1}{2}\ln(1+\frac{b^2x^2}{a^2})\right]+c$$

$$= \frac{1}{b^2}\left[\left(\frac{bx}{a}\right)\tan^{-1}\frac{bx}{a} - \frac{1}{2}\ln\frac{a^2+b^2x^2}{a^2}\right]+c$$

$$= \frac{1}{b^2}\left[\left(\frac{bx}{a}\right)\tan^{-1}\frac{bx}{a} - \frac{1}{2}\ln\left(a^2+b^2x^2\right) + \frac{1}{2}\ln a^2\right]+c$$

$$= \frac{1}{ab}x\tan^{-1}\frac{bx}{a} - \frac{1}{2b^2}\ln\left(a^2+b^2x^2\right) + \frac{1}{2b^2}\ln a^2 + c$$

$$= \frac{1}{ab}x\tan^{-1}\frac{bx}{a} - \frac{1}{2b^2}\ln\left(a^2+b^2x^2\right) + K$$

where K is a constant $= \frac{1}{2b^2}\ln a^2 + c$

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QUESTION 3

3.1

$$\int \frac{8x^2-2x+3}{2x^3-2x^2+x-1}dx$$

$2x^3-2x^2+x-1 = 2x^2(x-1)+(x-1)$ ✓

$= (x-1)(2x^2+1)$ ✓

$$\frac{8x^2-2x+3}{2x^3-2x^2+x-1} = \frac{8x^2-2x+3}{(x-1)(2x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{2x^2+1}$$ ✓ ✓

$8x^2-2x+3 = A(2x^2+1)+(Bx+C)(x-1)$ ✓

$x=1 \quad 8-2+3 = A(2+1) \quad \therefore A=3$ ✓

$8x^2-2x+3 = (2Ax^2+A)+(Bx^2+Cx-Bx-C)$ ✓

Consider using other values of x to calculate B and C.

$2A+B=8 \quad \therefore B=2$ ✓

$C-B=-2 \quad \therefore C=0$ ✓

$$\int \frac{8x^2-2x+3}{2x^3-2x^2+x-1}dx = \int \frac{3}{x-1}dx + \int \frac{2x}{2x^2+1}dx$$ ✓

$$= 3\ln(x-1)+\frac{1}{2}\ln(2x^2+1)+c$$ ✓ ✓

Alternative

$$\int \frac{8x^2-2x+3}{2x^3-2x^2+x-1}dx$$ ✓

$2x^3-2x^2+x-1 = 2x^2(x-1)+(x-1)$ ✓ and $8x^2-2x+3 = 6x^2+3+2x^2-2x$

$= (x-1)(2x^2+1)$ ✓ $= 3\left(2x^2+1\right)+2x(x-1)$ ✓

$$\therefore \frac{8x^2-2x+3}{2x^3-2x^2+x-1} = \frac{3\left(2x^2+1\right)+2x(x-1)}{(x-1)(2x^2+1)}$$ ✓ ✓

$$= \frac{3\left(2x^2+1\right)}{(x-1)(2x^2+1)}+\frac{2x(x-1)}{(x-1)(2x^2+1)}$$ ✓

$$= \frac{3}{x-1}+\frac{2x}{2x^2+1}$$ ✓ ✓

$$\therefore \int \frac{8x^2-2x+3}{2x^3-2x^2+x-1}dx = \int \frac{3}{x-1}dx + \int \frac{2x}{2x^2+1}dx$$ ✓

$$= 3\ln(x-1)+\frac{1}{2}\ln(2x^2+1)+c$$ ✓ ✓

(12)

3.2 $\frac{(3x+2)(2x-3)}{(3x+2)^2-(2x-3)^2} = \frac{6x^2-5x-6}{5x^2+24x-5}$ ✓✓

$$= \frac{6}{5} + \frac{-\frac{169}{5}x}{(5x-1)(x+5)} \; using\ long\ division\ method$$ ✓✓

$$= 1{,}2 + \frac{-33{,}8x}{(5x-1)(x+5)} \; where$$

$\frac{-\frac{169}{5}x}{(5x-1)(x+5)} = \frac{A}{(5x-1)} + \frac{B}{x+5}$ ✓

$\Rightarrow -\frac{169}{5}x = A(x+5) + B(5x-1)$ ✓

$x = -5 \Rightarrow B = -6{,}5 = -\frac{13}{2}$ ✓

$x = \frac{1}{5} \Rightarrow A = -1{,}3 = -\frac{13}{10}$ ✓

$\Rightarrow \int \frac{(3x+2)(2x-3)}{(3x+2)^2-(2x-3)^2}dx = \int \frac{6}{5}dx - \int \frac{13}{10(5x-1)}dx - \int \frac{13}{2(x+5)}dx$ ✓

$$= \frac{6}{5}x - \frac{13}{50}\ln(5x-1) - \frac{13}{2}\ln(x+5) + C$$ ✓✓✓

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QUESTION 4

4.1

$$\frac{dy}{dx} = \tan x - y\cot x$$

$$\frac{dy}{dx} + y\cot x = \tan x \quad \checkmark$$

$$e^{\int pdx} = e^{\int \cot x dx} \quad \checkmark$$

$$= e^{\ln(\sin x)} \quad \checkmark$$

$$= \sin x \quad \checkmark$$

$$\int Qe^{\int pdx} = \int \tan x \sin x dx \quad \checkmark$$

$$= \int \frac{\sin x}{\cos x}\sin x dx$$

$$= \int \frac{\sin^2 x}{\cos x} dx \quad \checkmark$$

$$= \int \frac{1-\cos^2 x}{\cos x} dx \quad \checkmark$$

$$= \int \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} dx \quad \checkmark$$

$$= \int \sec x - \cos x dx \quad \checkmark$$

$$= \ln(\sec x + \tan x) - \sin x + c \quad \checkmark \quad \checkmark$$

$$\therefore y\sin x = \ln(\sec x + \tan x) - \sin x + c \quad \checkmark$$

(12)

4.2

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 2e^{2x}$$

$$r^2 - 2r + 2 = 0 \quad \checkmark$$

$$r = \frac{2 \pm \sqrt{4 - 4.1.2}}{2} \quad \checkmark$$

$$= \frac{2 \pm \sqrt{-4}}{2} \quad \checkmark$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i \quad \checkmark$$

$$y_c = e^x\left[A\cos x + B\sin x\right] \quad \checkmark$$

$y = Ce^{2x}$ ✓

$\frac{dy}{dx} = 2Ce^{2x}$ ✓

$\frac{d^2y}{dx^2} = 4Ce^{2x}$ ✓

$\therefore 4Ce^{2x} - 2\left(2Ce^{2x}\right) + 2Ce^{2x} = 2e^{2x}$ ✓

$C = 1$ ✓

$y_p = e^{2x}$

$y = y_c + y_p$ ✓

$y = e^x\left[A\cos x + B\sin x\right] + e^{2x}$ ✓

OR

$y = 2Ce^{2x}$

$\frac{dy}{dx} = 4Ce^{2x}$

$\frac{d^2y}{dx^2} = 8Ce^{2x}$

$\therefore 8Ce^{2x} - 2\left(4Ce^{2x}\right) + 2(2Ce^{2x}) = 2e^{2x}$

$4Ce^{2x} = 2e^{2x} \Rightarrow 4C = 2 \therefore C = \frac{1}{2}$

$y_p = 2Ce^{2x}$

$y_p = 2\left(\frac{1}{2}\right)e^{2x} = e^{2x}$

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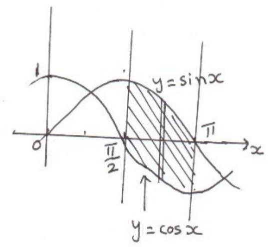
QUESTION 5

5.1 5.1.1



✓ shape of each graph (2 x 1)
✓ x intercepts $\frac{\pi}{2}$ and π (2 x 1)
✓ strip
✓ area

(6)

5.1.2

$Area = \int_a^b y_1 - y_2 dx$ ✓

$= \int_{\frac{\pi}{2}}^{\pi} \sin x - \cos x dx$ ✓ ✓

$= \left[-\cos x - \sin x\right]_{\frac{\pi}{2}}^{\pi}$ ✓ ✓

$= \left[-\cos\pi - \sin\pi - \{-\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\}\right]$ ✓

or $-\left[\cos\pi + \sin\pi - \{\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\}\right]$

$= 2\text{units}^2$ ✓ ✓

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5.1.3

$$A_{m-y} = \int r dA$$

✓

$$= \int_{\frac{\pi}{2}}^{\pi} x(\sin x - \cos x)dx \quad ✓ \quad ✓ \quad ✓\ f(x) = x \qquad g'(x) = \sin x - \cos x$$

$$= \left[x(-\cos x - \sin x) - \int (-\cos x - \sin x)dx \right]_{\frac{\pi}{2}}^{\pi} \quad ✓ \quad ✓$$

$$= -\left[x(\cos x + \sin x) - \int (\cos x + \sin x)dx \right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\left[x(\cos x + \sin x) - (\sin x - \cos x) \right]_{\frac{\pi}{2}}^{\pi} \quad ✓$$

$$= -\left[\pi(\cos \pi + \sin \pi) - (\sin \pi - \cos \pi) - \left\{ \frac{\pi}{2}(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) - (\sin \frac{\pi}{2} - \cos \frac{\pi}{2} \right\} \right] \quad ✓✓$$

$$= -\left[\pi(-1+0) - (0 - -1) - \left\{ \frac{\pi}{2}(0+1) - (1-0) \right\} \right]$$

$$= -\left[-\pi - 1 - \frac{\pi}{2} + 1 \right]$$

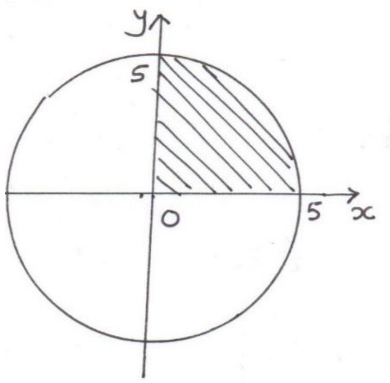
$$= \frac{3\pi}{2} \quad \text{or} \quad 4{,}712 \ \text{units}^3 \quad ✓$$

✓

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{4{,}712}{2} = 2{,}356 \ units \quad ✓$$

(12)

5.2 5.2.1



✓ strip (vertical or horizontal used in calculation)
✓ area
✓ graph (shape)
✓ intercept

(4)

5.2.2

$$V_x = \pi\int_a^b y_1^2 - y_2^2 dx \checkmark$$

$$= \pi\int_0^5 25 - x^2 dx \checkmark \checkmark$$

$$= \pi\left[25x - \frac{x^3}{3}\right]_0^5 \checkmark$$

$$= \pi\left[25(5) - \frac{(5)^3}{3}\right] \checkmark$$

$$= \frac{250}{3}\pi \quad \text{or} \quad 261{,}800 \ \text{units}^3 \checkmark$$

(6)

5.2.3

$$V_{m-y} = \pi\int_a^b x\left(y_1^2 - y_2^2\right)dx \checkmark$$

$$= \pi\int_0^5 x\left(25 - x^2\right)dx \checkmark \checkmark$$

$$= \pi\int_0^5 25x - x^3 dx \checkmark$$

$$= \pi\left[\frac{25}{2}x^2 - \frac{x^4}{4}\right]_0^5 \checkmark$$

$$= \frac{625}{4}\pi = 156{,}25\pi = 490{,}874\text{units}^4 \checkmark$$

$$\bar{x} = \frac{261{,}800}{490{,}874} = \frac{15}{8} = 1{,}875 \ \text{units} \checkmark \checkmark$$

Using horizontal strip

$$V_x = 2\pi\int_a^b y\left(x_1 - x_2\right)dy \checkmark$$

$$= 2\pi\int_0^5 y\sqrt{25 - y^2}\,dy \checkmark \checkmark$$

$$= -\pi\int_0^5 -2y\left(25 - y^2\right)^{\frac{1}{2}}dy$$

$$= -\frac{2}{3}\pi\left[\left(25 - y^2\right)^{\frac{3}{2}}\right]_0^5 \checkmark \checkmark$$

$$= -\frac{2}{3}\pi\left[0 - 25^{\frac{3}{2}}\right] = \frac{250}{3}\pi \checkmark$$

$$= 83{,}333\pi = 261{,}800\text{units}^3$$

$$V_{m-y} = \int_a^b r dv$$

$$= \int_a^b \frac{x_1 + x_2}{2} 2\pi y\left(x_1 - x_2\right)dy$$

$$= \pi\int_a^b y\left(x_1^2 - x_2^2\right)dy \checkmark$$

$$= \pi\int_0^5 y\left(25 - y^2\right)dy \checkmark$$

$$= \pi\int_a^5 \left(25y - y^3\right)dy \checkmark$$

$$= \pi\left[\frac{25y^2}{2} - \frac{y^4}{4}\right]_0^5 \checkmark$$

$$= \pi\left[\frac{25(5)^2}{2} - \frac{y(5)^4}{4} - \{0\}\right] \checkmark$$

$$= \frac{625}{4}\pi \quad \text{or} \quad 490{,}874 \ \text{units}^4 \checkmark$$

$$\bar{x} = \frac{261{,}800}{490{,}874} = \frac{15}{8} = 1{,}875 \ \text{units} \checkmark \checkmark$$

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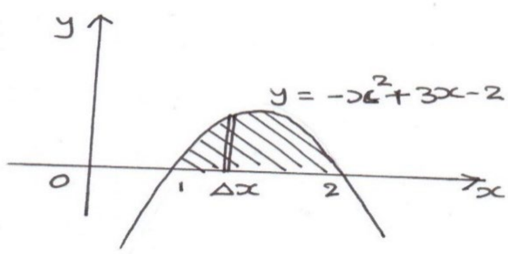
5.3 5.3.1



✓ graph
✓ x-intercepts
✓ area
✓ strip (4)

5.3.2

$$A = \int_a^b y_1 - y_2 dx \quad ✓$$

$$= \int_1^2 -x^2 + 3x - 2\, dx \quad ✓$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2 \quad ✓ \quad ✓$$

$$= \left[-\frac{(2)^3}{3} + \frac{3(2)^2}{2} - 2(2) - \left\{ -\frac{1}{3} + \frac{3}{2} - 2 \right\} \right] \quad ✓$$

$$= \frac{1}{6} \text{ or } 0{,}167 \text{ units}^2 \quad ✓$$

(6)

5.3.3

$$I_y = \int_a^b r^2 dA \quad ✓$$

$$= \int_1^2 x^2(-x^2 + 3x - 2)\, dx \quad ✓$$

$$\int_1^2 (-x^4 + 3x^3 - 2x^2)\, dx \quad ✓$$

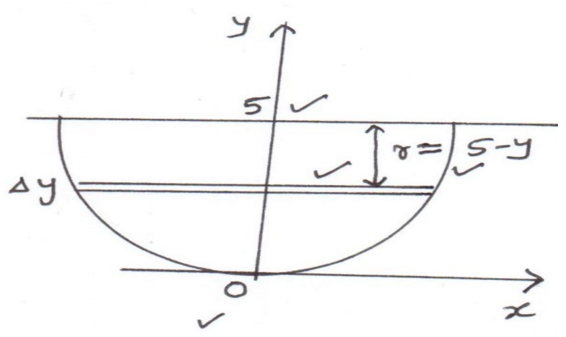
$$= \left[-\frac{x^5}{5} + \frac{3x^4}{4} - \frac{2x^3}{3} \right]_1^2 \quad ✓$$

$$\left[-\frac{(2)^5}{5} + \frac{3(2)^4}{4} - \frac{2(2)^3}{3} - \left\{ -\frac{1}{5} + \frac{3}{4} - \frac{2}{3} \right\} \right] \quad ✓$$

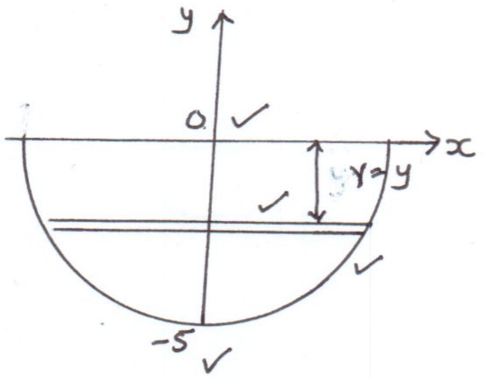
$$= \frac{23}{60} \quad or 0{,}383 \overset{✓}{=} \frac{0{,}383}{0{,}167} A = 2{,}295A \quad ✓ \quad ✓$$

(8)

5.4 5.4.1



Alternative (with x-axis at the water level)



(4)

5.4.2

$$x^2 + (y-5)^2 = 25$$

$$x = \sqrt{25-(y-5)^2} \quad ✓$$

$$\text{first moment} = \int_a^b r\,dA \quad ✓$$

$$= \int_0^5 (5-y)2\sqrt{25-(y-5)^2}\,dy \quad ✓ ✓ ✓ \qquad u = 5-y \quad dy = -du$$

$$= -2\int_5^0 u\sqrt{25-u^2}\,du \quad ✓ \qquad y = 0 \quad u = 5$$

$$= \frac{2}{3}\left[\left(25-u^2\right)^{\frac{3}{2}}\right]_5^0 \quad ✓ \qquad y = 5 \quad u = 0$$

$$= \frac{2}{3}\left[25^{\frac{3}{2}} - 0\right] ✓ = \frac{250}{3} = 83,333\ m^3 \quad ✓$$

$$\boxed{\begin{aligned} &= \left[\frac{\left(25-u^2\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_5^0 \\ &= \frac{2}{3}\left[\left(25-u^2\right)^{\frac{3}{2}}\right]_5^0 \\ &= \frac{2}{3}\left[(25)^{\frac{3}{2}} - \{0\}\right] \\ &= \frac{250}{3} \quad \text{or} \quad 83,333m^3 \end{aligned}}$$

5.4.3

$$\bar{y} = \frac{245,437}{83,333} ✓ = 2,945m \quad ✓$$

Alternative

5.4.2 First moment of area $= \int_a^b r dA$ ✓

$= \int_a^b y2x dy$ ✓✓

But $x^2 + y^2 = 25 \Rightarrow x = \sqrt{25 - y^2}$ ✓✓

$= \int_{-5}^{0} y2\sqrt{25 - y^2}\, dy$ ✓✓

$= -\int_{-5}^{0} -y2\left(25 - y^2\right)^{\frac{1}{2}} dy$ ✓

$= -\left[\dfrac{\left(25 - y^2\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-5}^{0}$ ✓

$= -\dfrac{2}{3}\left[\left(25 - y^2\right)^{\frac{3}{2}}\right]_{-5}^{0}$

$= -\dfrac{2}{3}\left[(25)^{\frac{3}{2}} - \left\{25 - (-5)^2\right\}^{\frac{3}{2}}\right]$ ✓ ✓

$= -\dfrac{2}{3}(25)^{\frac{3}{2}} = -83\dfrac{1}{3}$ or $-\dfrac{250}{3} m^3$ ✓ (12)

5.4.3 $\bar{\bar{y}} = \dfrac{245,437 m^4}{-83,333 m^3} = -2,945 m$ ✓ ✓ (2)

[80]

QUESTION 6

6.1

$$2y = x^2$$

$$\frac{dy}{dx} = x \quad \checkmark$$

$$\left[\frac{dy}{dx}\right]^2 = x^2 \quad \checkmark$$

$$1 + \left[\frac{dy}{dx}\right]^2 = 1 + x^2 \quad \checkmark$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dy \quad \checkmark$$

$$= \int_2^4 \sqrt{1 + x^2}\, dy \quad \checkmark \quad \checkmark$$

$$= \left[\frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2}\ln\left(x + \sqrt{1 + x^2}\right)\right]_2^4 \quad \checkmark \quad \checkmark$$

$$\left[\frac{4}{2}\sqrt{1 + 4^2} + \frac{1}{2}\ln\left(4 + \sqrt{1 + 4^2}\right) - \left\{\frac{2}{2}\sqrt{1 + 2^2} + \frac{1}{2}\ln\left(2 + \sqrt{1 + 2^2}\right)\right\}\right] \quad \checkmark \quad \checkmark$$

$$= \left[2\sqrt{17} + \frac{1}{2}\ln\left(4 + \sqrt{17}\right) - \left\{\sqrt{5} + \frac{1}{2}\ln\left(2 + \sqrt{5}\right)\right\}\right]$$

$$= 6{,}336 \text{ units} \quad \checkmark \quad \checkmark$$

Alternative

$$2y = x^2$$

$$x = \sqrt{2}y^{\frac{1}{2}} \qquad or\sqrt{2}\sqrt{y}$$

$$\frac{dx}{dy} = \sqrt{2}\frac{1}{2}y^{-\frac{1}{2}} \quad \checkmark$$

$$\left[\frac{dx}{dy}\right]^2 = \frac{1}{2y} \quad \checkmark$$

$$1 + \left[\frac{dx}{dy}\right]^2 = 1 + \frac{1}{2y} \quad \checkmark$$

$$= \frac{2y+1}{2y}$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \quad \checkmark$$

$$= \int_2^8 \sqrt{\frac{2y+1}{2y}}\, dy$$

$$\int_2^8 \frac{\sqrt{2y+1}}{\sqrt{2y}} dy \quad \checkmark$$

$$= \int_2^4 \frac{\sqrt{1+u^2}}{u} udu \quad \checkmark$$

$$= \int_2^4 \sqrt{1+u^2}\, du \quad \checkmark$$

$$= \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u + \sqrt{1+u^2}\right)\right]_2^4 \quad \checkmark \quad \checkmark$$

$$\left[\frac{4}{2}\sqrt{1+4^2} + \frac{1}{2}\ln\left(4 + \sqrt{1+4^2}\right) - \left\{\frac{2}{2}\sqrt{1+2^2} + \frac{1}{2}\ln\left(2 + \sqrt{1+2^2}\right)\right\}\right] \quad \checkmark$$

$$= \left[2\sqrt{17} + \frac{1}{2}\ln\left(4 + \sqrt{17}\right) - \left\{\sqrt{5} + \frac{1}{2}\ln\left(2 + \sqrt{5}\right)\right\}\right]$$

$$= 6{,}336 \text{ units} \quad \checkmark \quad \checkmark$$

(12)

6.2

$$x = \frac{1}{9}y^2$$

$$\frac{dx}{dy} = \frac{2}{9}y \quad ✓$$

$$\left[\frac{dx}{dy}\right]^2 = \left(\frac{2}{9}y\right)^2 \quad ✓$$

$$1+\left[\frac{dx}{dy}\right]^2 = 1+\left(\frac{2}{9}y\right)^2 = 1+\frac{4y^2}{81} = \frac{81+4y^2}{81} \quad ✓ \quad ✓$$

$$A_x = 2\pi\int_0^6 y\frac{\sqrt{81+4y^2}}{9}dy \quad ✓ \quad ✓$$

$$= \frac{2}{9}\pi\int_{81}^{225} y u^{\frac{1}{2}}\frac{du}{8y} \quad ✓$$

$$= \frac{2}{9}\frac{1}{8}\pi\int_{81}^{225} u^{\frac{1}{2}}du$$

$$= \frac{1}{36}\pi\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{81}^{225} \quad ✓ \quad ✓$$

$$= \frac{1}{36}\frac{2}{3}\pi\left[u^{\frac{3}{2}}\right]_{81}^{225}$$

$$= \frac{1}{54}\pi\left[225^{\frac{3}{2}} - 81^{\frac{3}{2}}\right] \quad ✓$$

$$= 49\pi \text{ or } 153{,}938 \text{ units} \quad ✓ \quad ✓$$

Alternative

$$x = \frac{1}{9}y^2$$

$$y^2 = 9x \quad \checkmark \Rightarrow y = 3\sqrt{x}$$

$$2y\frac{dy}{dx} = 9$$

$$\frac{dy}{dx} = \frac{9}{2y} = \frac{9}{2\left(3\sqrt{x}\right)} = \frac{9}{6\sqrt{x}} \quad = \frac{3}{2\sqrt{x}} \quad \checkmark \quad \checkmark$$

$$\left[\frac{dy}{dx}\right]^2 = \left(\frac{3}{2\sqrt{x}}\right)^2 \quad \checkmark$$

$$1+\left[\frac{dy}{dx}\right]^2 = 1+\left(\frac{3}{2\sqrt{x}}\right)^2 = 1+\frac{9}{4x} = \frac{4x+9}{4x} \quad \checkmark$$

$$A_x = 2\pi\int_0^4 y\sqrt{1+\left[\frac{dy}{dx}\right]^2}dx \quad \checkmark \quad y = 0 \quad x = 0$$

$$= 2\pi\int_0^4 y\sqrt{\frac{4x+9}{4x}}dx \quad \checkmark \qquad y = 6 \quad x = \frac{1}{9}y^2 = \frac{1}{9}(6)^2 = 4 =$$

$$2\pi\int_0^4 3\sqrt{x}\,\frac{\sqrt{4x+9}}{2\sqrt{x}}dx \quad \checkmark$$

$$= 3\pi\int_0^4 \sqrt{4x+9}dx$$

$$= 3\pi\frac{1}{4}\left[\frac{(4x+9)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 \quad \checkmark$$

$$\frac{1}{2}\pi\left[(4x+9)^{\frac{3}{2}}\right]_0^4$$

$$= \frac{1}{2}\pi\left[(4(4)+9)^{\frac{3}{2}} - (0+9)^{\frac{3}{2}}\right] \quad \checkmark$$

$$\frac{1}{2}\pi\left[(25)^{\frac{3}{2}} - (9)^{\frac{3}{2}}\right]$$

$$= 49\pi \quad \text{or} \quad 153{,}938\text{units}^2 \quad \checkmark \quad \checkmark$$

(12)
[24]

TOTAL = 200 ÷ 2: 100