



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

27 July 2021

This marking guideline consists of 16 pages.

QUESTION 1

1.1

$$z = \sqrt{x^2 - y^2}$$

$$= (x^2 - y^2)^{\frac{1}{2}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x^2 - y^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(x^2 - y^2)^{-\frac{1}{2}} \cdot -2y = \frac{-y}{\sqrt{x^2 - y^2}} \quad \checkmark \checkmark$$

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{x^2}{\sqrt{x^2 - y^2}} + \frac{-y^2}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$= \frac{x^2 - y^2}{\sqrt{x^2 - y^2}}$$

$$= \sqrt{x^2 - y^2} \quad \checkmark$$

$$Z = \sqrt{x^2 - y^2} \Rightarrow Z^2 = x^2 - y^2$$

$$\Rightarrow 2Z \frac{\partial Z}{\partial x} = 2x \quad \checkmark$$

$$\Rightarrow \frac{\partial Z}{\partial x} = \frac{x}{Z} = \frac{x}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$2Z \frac{\partial Z}{\partial y} = -2y \quad \checkmark$$

$$\Rightarrow \frac{\partial Z}{\partial y} = -\frac{y}{Z} = \frac{-y}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$\therefore x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = \frac{x^2}{\sqrt{x^2 - y^2}} - \frac{y^2}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$= \frac{x^2 - y^2}{\sqrt{x^2 - y^2}} \quad \checkmark$$

$$= \sqrt{x^2 - y^2}$$

(3)

1.2

1.2.1

$$x = 6 \cos \theta$$

$$y = 6 \sin \theta$$

$$\frac{dx}{d\theta} = -6 \sin \theta \quad \checkmark \quad \frac{dy}{d\theta} = 6 \cos \theta \quad \checkmark$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \frac{d\theta}{dx} = \frac{6 \cos \theta}{-6 \sin \theta} \quad \checkmark = -\cot \theta \quad \checkmark$$

$$x^2 + y^2 = 36 \cos^2 \theta +$$

$$36 \sin^2 \theta = 36 \quad \checkmark$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} = 0 \quad \checkmark$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y} = -\frac{\cos \theta}{\sin \theta} = -\cot \theta \quad \checkmark \checkmark$$

(2)

1.2.2

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx} = \operatorname{cosec}^2 \theta \frac{1}{-6 \sin \theta} \quad \checkmark$$

$$= -\frac{1}{6} \operatorname{cosec}^3 \theta \quad \text{or} \quad = \frac{1}{-6 \sin^3 \theta} \quad \checkmark$$

$$\frac{d^2 y}{dx^2} = \frac{-y + x \frac{dy}{dx}}{y^2} \quad \checkmark$$

$$= \frac{-6 \sin \theta + 6 \cos \theta (-\cot \theta)}{36 \sin^2 \theta}$$

$$= -6 \left(\sin \theta + \frac{\cos^2 \theta}{\sin \theta} \right) \div 36 \sin^2 \theta$$

$$= \frac{1}{6 \sin^3 \theta} \quad \checkmark$$

(1)
[6]

QUESTION 2

$$\begin{aligned}
 2.1 \quad \int y dx &= \int (1 + \tan^2 3x)(\sec^2 3x - 1) dx \\
 &= \int \sec^2 3x \tan^2 3x dx \quad \checkmark \quad \checkmark \\
 &= \frac{1}{3} \int 3 \sec^2 3x \tan^2 3x dx \quad \checkmark \quad \text{Or using } u = \tan 3x \\
 &= \frac{1}{3} \frac{\tan^3 3x}{3} + C \quad \checkmark
 \end{aligned}$$

Alternative

$$\begin{aligned}
 \int y dx &= \int (1 + \tan^2 3x)(\sec^2 3x - 1) dx \\
 &= \int (\sec^2 3x + \tan^2 3x \sec^2 3x - 1 - \tan^2 3x) dx \quad \checkmark \\
 &= \frac{1}{3} \tan 3x + \frac{1}{3} \frac{\tan^3 3x}{3} - x - \left(\frac{1}{3} \tan 3x - x \right) \quad \checkmark \quad \checkmark \\
 &= \frac{1}{3} \frac{\tan^3 3x}{3} + C \quad \checkmark
 \end{aligned}$$

(2)

$$\begin{aligned}
 2.2 \quad \int y dx &= \int \sin^{-1} bx dx & f(x) &= \sin^{-1} bx & g'(x) &= 1 \quad \checkmark \\
 & & f'(x) &= \frac{b}{\sqrt{1-b^2x^2}} & g(x) &= x \quad \checkmark \\
 & \checkmark & & & & \\
 & = x \sin^{-1} bx - \int \frac{b}{\sqrt{1-b^2x^2}} x dx & & & u = 1 - b^2x^2 & dx = \frac{du}{-2b^2x} \quad \checkmark \\
 & = x \sin^{-1} bx - \int \frac{b}{\sqrt{u}} x \frac{du}{-2b^2x} & \checkmark & \checkmark & & \\
 & = x \sin^{-1} bx + \frac{1}{2b} \int u^{-\frac{1}{2}} du & & \checkmark & & \\
 & = x \sin^{-1} bx + \frac{1}{2b} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C & & \checkmark & & \\
 & = x \sin^{-1} bx + \frac{1}{b} \sqrt{1-b^2x^2} + C & & \checkmark & &
 \end{aligned}$$

(5)

2.3 $6x - x^2 = -[x^2 - 6x] \quad \checkmark$
 $= -[x^2 - 6x + 9 - 9] \quad \checkmark$
 $= -[(x-3)^2 - 9] \quad \checkmark = 9 - (x-3)^2 \quad \checkmark$
 $\int y dx = \int \sqrt{9 - (x-3)^2} dx \quad \checkmark$
 $= \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2 x^2} + C \quad (\text{formula used}) \quad \checkmark \quad \checkmark \quad \checkmark$
 $= \frac{9}{2} \sin^{-1} \frac{x-3}{3} + \frac{x-3}{2} \sqrt{9 - (x-3)^2} + C \quad \text{or} \quad \frac{9}{2} \sin^{-1} \frac{x-3}{3} + \frac{x-3}{2} \sqrt{6x - x^2} + C$
Alternative: $\frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a} \right)^2 = \frac{0 - 36}{-4} - \left(x + \frac{6}{-2} \right)^2 = 9 - (x-3)^2$ (4)

2.4 $\int y dx = \int \cos^4 \left(\frac{3}{2} x \right) dx = \int \left[\cos^2 \left(\frac{3}{2} x \right) \right]^2 dx \quad \checkmark$
 $= \int \left[\frac{1}{2} + \frac{1}{2} \cos 3x \right]^2 dx \quad \text{or} \quad \int \frac{1}{4} (1 + \cos 3x)^2 dx \quad \checkmark$
 $= \int \frac{1}{4} + \frac{1}{2} \cos 3x + \frac{1}{4} \cos^2 3x dx \quad \checkmark \quad \checkmark$
 $= \frac{1}{4} x + \frac{1}{2} \cdot \frac{1}{3} \sin 3x + \frac{1}{4} \left\{ \frac{x}{2} + \frac{\sin 6x}{12} \right\} \quad \checkmark \quad \checkmark \quad \checkmark$
 $= \frac{1}{4} x + \frac{1}{6} \sin 3x + \frac{x}{8} + \frac{\sin 6x}{48} + C \quad \checkmark \quad \text{or} \quad = \frac{3}{8} x + \frac{1}{6} \sin 3x + \frac{\sin 6x}{48} + C$ (4)

2.5 $\int y dx = \int x \tan x + \ln \sec x dx$ Use integration by parts for the first integral only
 $= \int x \tan x dx + \int \ln \sec x dx \quad \checkmark$ using $f(x) = x$ and $g'(x) = \tan x$
 $f'(x) = 1 \quad \checkmark \quad g(x) = \ln \sec x \quad \checkmark$
 $= \left\{ x \ln \sec x - \int \ln \sec x dx \right\} + \int \ln \sec x dx \quad \checkmark \quad \checkmark$
 $= x \ln \sec x + C \quad \checkmark$

(3)
[18]

QUESTION 3

$$3.1 \quad 2x^3 - x^2 - 6x = x(2x^2 - x - 6) \quad \checkmark$$

$$= x(2x+3)(x-2) \quad \checkmark$$

$$\frac{(x+3)(x-4)}{2x^3 - x^2 - 6x} = \frac{A}{x} + \frac{B}{2x+3} + \frac{C}{x-2} \quad \checkmark \quad \checkmark$$

$$(x+3)(x-4) = A(2x+3)(x-2) + Bx(x-2) + Cx(2x+3) \quad \checkmark$$

$$\text{when } x = -\frac{3}{2} \quad \left(-\frac{3}{2}+3\right)\left(-\frac{3}{2}-4\right) = B \cdot -\frac{3}{2}\left(-\frac{3}{2}-2\right)$$

$$-\frac{33}{4} = \frac{21}{4}B \quad \therefore B = -\frac{33}{21} \quad \checkmark = -\frac{11}{7}$$

$$\text{when } x = 2 \quad 5 \cdot -2 = C \cdot 2 \cdot (4+3) \quad -10 = 14C \quad \therefore C = -\frac{5}{7} \quad \checkmark$$

$$\text{when } x = 0 \quad -12 = A \cdot 3 \cdot (-2) \quad \therefore A = 2 \quad \checkmark$$

$$\int \frac{(x+3)(x-4)}{2x^3 - x^2 - 6x} dx = \int \frac{2}{x} dx + \int \frac{-\frac{11}{7}}{2x+3} dx + \int \frac{-\frac{5}{7}}{x-2} dx \quad \checkmark$$

$$= 2 \ln x - \frac{11}{7} \cdot \frac{1}{2} \ln(2x+3) - \frac{5}{7} \ln(x-2) + C \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= 2 \ln x - \frac{11}{14} \ln(2x+3) - \frac{5}{7} \ln(x-2) + C$$

(6)

$$3.2 \quad \frac{3x^2 + 16x + 26}{(x-2)(x^2 + 3x + 4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2 + 3x + 4} \quad \checkmark \quad \checkmark$$

$$3x^2 + 16x + 26 = A(x^2 + 3x + 4) + (Bx + C)(x - 2) \quad \checkmark$$

$$\text{when } x = 2 \quad 12 + 32 + 26 = A(4 + 6 + 4) \quad 70 = 14A \quad \therefore A = 5 \quad \checkmark$$

$$\text{when } x = 0 \quad 26 = A(4) + C(-2)$$

$$26 = 20 - 2C \quad \therefore C = -3 \quad \checkmark$$

$$\text{when } x = 1 \quad 3 + 16 + 26 = A(1 + 3 + 4) + (B + C)(-1) \quad \checkmark$$

$$45 = 8A - B - C$$

$$45 = 40 - B + 3 \quad \therefore B = -2 \quad \checkmark$$

$$\int \frac{3x^2 + 16x + 26}{(x-2)(x^2 + 3x + 4)} dx = \int \frac{5}{x-2} dx + \int \frac{-2x-3}{x^2 + 3x + 4} dx \quad \checkmark \quad \checkmark$$

$$= \int \frac{5}{x-2} dx - \int \frac{2x+3}{x^2 + 3x + 4} dx \quad \checkmark$$

$$= 5 \ln(x-2) - \ln(x^2 + 3x + 4) + C \quad \checkmark \quad \checkmark$$

Alternative

$$\frac{3x^2 + 16x + 26}{(x-2)(x^2 + 3x + 4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2 + 3x + 4} \quad \checkmark \quad \checkmark$$

$$3x^2 + 16x + 26 = A(x^2 + 3x + 4) + (Bx + C)(x - 2) \quad \checkmark$$

$$\text{when } x = 2 \quad 12 + 32 + 26 = A(4 + 6 + 4) \quad 70 = 14A \quad \therefore A = 5 \quad \checkmark$$

$$\text{Expanding} \quad 3x^2 + 16x + 26 = Ax^2 + 3Ax + 4A + Bx^2 + Cx - 2Bx - 2C \quad \checkmark$$

$$\text{Equating } x^2 \quad A + B = 3 \quad \Rightarrow 5 + B = 3 \quad \therefore B = -2 \quad \checkmark$$

$$\text{Equating } x \quad 3A + C - 2B = 16 \quad \Rightarrow 3(5) + C - 2(-2) = 16 \quad C = -3 \quad \checkmark$$

$$\int \frac{3x^2 + 16x + 26}{(x-2)(x^2 + 3x + 4)} dx = \int \frac{5}{x-2} dx + \int \frac{-2x-3}{x^2 + 3x + 4} dx \quad \checkmark \quad \checkmark$$

$$= \int \frac{5}{x-2} dx - \int \frac{2x+3}{x^2 + 3x + 4} dx \quad \checkmark$$

$$= 5 \ln(x-2) - \ln(x^2 + 3x + 4) + C \quad \checkmark \quad \checkmark$$

(6)
[12]

QUESTION 4

4.1

$$e^{\int p dx} = e^{-\int \frac{1}{x \ln x} dx} \quad \checkmark$$

$$= e^{-\int \frac{1}{x \ln x} dx} \quad \checkmark$$

or using $u = \ln x$

$$= e^{-\ln(\ln x)} \quad \checkmark$$

$$= (\ln x)^{-1} \quad \checkmark$$

$$= \frac{1}{\ln x} \quad \checkmark$$

$$\int Q e^{\int p dx} dx = \int \frac{1}{x \ln x} dx \quad \checkmark \quad \checkmark$$

$$= \int \frac{1}{\ln x} dx \quad \text{or using } u = \ln x \text{ as before}$$

$$= \ln(\ln x) \quad \checkmark$$

$$\therefore y \frac{1}{\ln x} = \ln(\ln x) + C \quad \checkmark$$

$$x = 2 \quad y = 0 \quad 0 = \ln(\ln 2) + C \quad \checkmark \quad \therefore C = 0,367 \quad \checkmark$$

$$\therefore y \frac{1}{\ln x} = \ln(\ln x) + 0,367 \quad \checkmark$$

(6)

4.2

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 2e^{-3x}$$

$$r^2 + r - 6 = 0 \quad \checkmark$$

$$(r+3)(r-2) = 0 \quad \checkmark$$

$$r = -3 \quad r = 2 \quad \checkmark$$

$$y_c = Ae^{-3x} + Be^{2x} \quad \checkmark$$

$$\text{For } y_p \quad y = Cxe^{-3x} \dots\dots\dots(1) \quad \checkmark$$

If used $y = 2Cxe^{-3x}$ follow up to get $C = -\frac{1}{5}$

$$\frac{dy}{dx} = -3Cxe^{-3x} + Ce^{-3x} \quad \checkmark$$

$$\frac{d^2y}{dx^2} = -3 \cdot -3Cxe^{-3x} - 3Ce^{-3x} - 3Ce^{-3x} \quad \checkmark$$

$$= 9Cxe^{-3x} - 6Ce^{-3x} \quad \checkmark$$

Substituting $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y in (1)

$$9Cxe^{-3x} - 6Ce^{-3x} + (-3Cxe^{-3x} + Ce^{-3x}) - 6Cxe^{-3x} = 2e^{-3x} \quad \checkmark$$

$$-5Ce^{-3x} = 2e^{-3x} \quad \therefore C = -\frac{2}{5} \quad \checkmark$$

$$\therefore y_p = -\frac{2}{5}xe^{-3x} \quad \checkmark$$

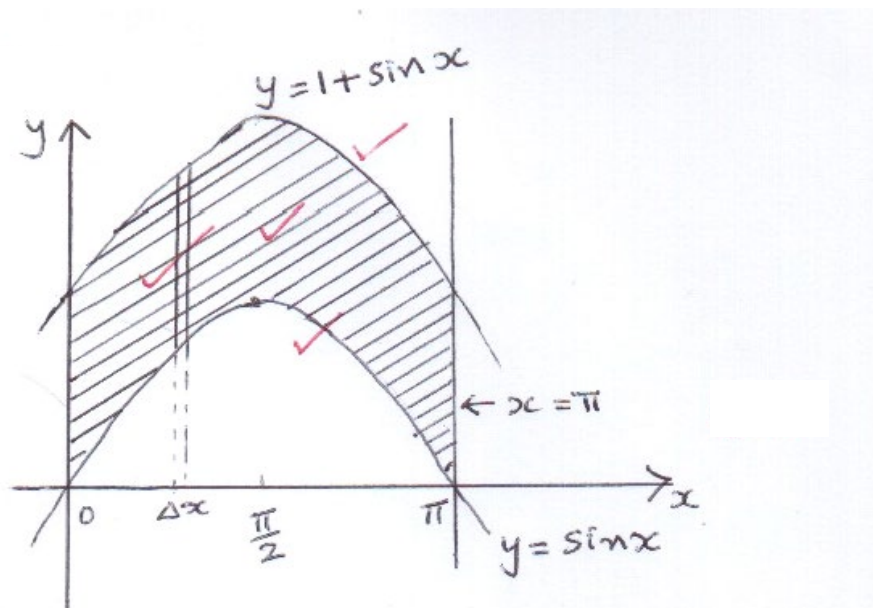
$$y = y_c + y_p$$

$$y = Ae^{-3x} + Be^{2x} - \frac{2}{5}xe^{-3x} \quad \checkmark$$

(6)
[12]

QUESTION 5

5.1 5.1.1



(2)

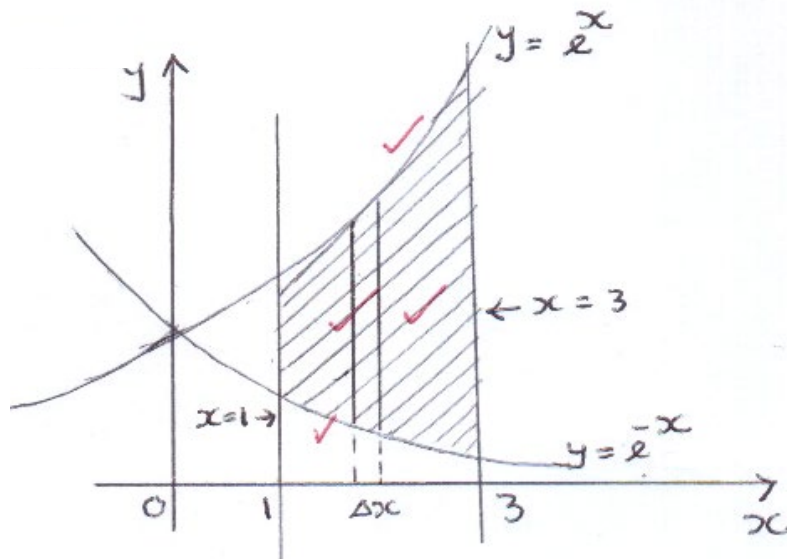
5.1.2

$$\begin{aligned}
V_x &= \pi \int_a^b (y_1^2 - y_2^2) dx \quad \checkmark \\
&= \pi \int_0^\pi \left\{ (1 + \sin x)^2 - \sin^2 x \right\} dx \quad \checkmark \quad \checkmark \quad \checkmark \\
&= \pi \int_0^\pi \left\{ (1 + 2 \sin x + \sin^2 x) - \sin^2 x \right\} dx \quad \checkmark \\
&= \pi \int_0^\pi \{1 + 2 \sin x\} dx \quad \checkmark \\
&= \pi [x - 2 \cos x]_0^\pi \quad \checkmark \quad \checkmark \\
&= \pi [\pi - 2 \cos \pi - (0 - 2 \cos 0)] \quad \checkmark \quad = \pi [\pi - 2(-1) + 2] = \pi [\pi + 4] \\
&= 22,4360 \text{units}^3 \quad \checkmark
\end{aligned} \tag{5}$$

5.1.3

$$\begin{aligned}
V_{m-y} &= \int_a^b r dV \quad \checkmark \\
&= \int_a^b \frac{y_1 + y_2}{2} 2\pi x (y_1 - y_2) dx \\
&= \pi \int_a^b x (y_1^2 - y_2^2) dx \quad \checkmark \\
&= \pi \int_0^\pi x \left\{ (1 + \sin x)^2 - \sin^2 x \right\} dx \quad \checkmark \\
&= \pi \int_0^\pi x \left\{ (1 + 2 \sin x + \sin^2 x) - \sin^2 x \right\} dx \quad \checkmark \\
&= \pi \int_0^\pi x \{1 + 2 \sin x\} dx \quad \checkmark \quad \text{using integration by parts: } f(x) = x \quad g'(x) = 1 + 2 \sin x \\
&\hspace{25em} f'(x) = 1 \quad g(x) = x - 2 \cos x \\
&= \pi \left[x(x - 2 \cos x) - \int 1 \cdot (x - 2 \cos x) dx \right]_0^\pi \quad \checkmark \quad \checkmark \\
&= \pi \left[(x^2 - 2x \cos x) - \left(\frac{x^2}{2} - 2 \sin x \right) \right]_0^\pi \quad \checkmark \\
&= \pi \left[(\pi^2 - 2\pi \cos \pi) - \left(\frac{\pi^2}{2} - 2 \sin \pi \right) - (0) \right] \quad \checkmark \quad = \pi \left[\frac{\pi^2}{2} + 2\pi \right] \\
&= 35,2423 \text{units}^4 \quad \checkmark \\
\bar{x} &= \frac{V_{m-y}}{V_x} = \frac{35,2423 \text{units}^4}{22,4360 \text{units}^3} = 1,571 \text{units} \quad \checkmark
\end{aligned} \tag{6}$$

5.2 5.2.1



(2)

5.2.2

$$\begin{aligned}
 A &= \int_a^b (y_1 - y_2) dx \\
 &= \int_1^3 e^x - e^{-x} dx \quad \checkmark \\
 &= [e^x + e^{-x}]_1^3 \quad \checkmark \\
 &= [e^3 + e^{-3} - (e^1 + e^{-1})] \quad \checkmark \\
 &= 17,0492 \text{ units}^2 \quad \checkmark
 \end{aligned}$$

(2)

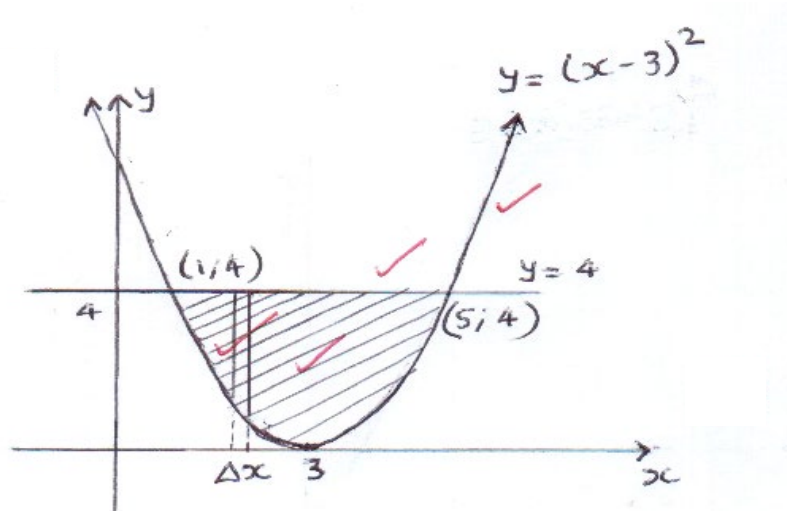
5.2.3

$$\begin{aligned}
 A_{m-y} &= \int_a^b r dA \quad \checkmark \\
 &= \int_a^b x(y_1 - y_2) dx \quad \checkmark \\
 &= \int_1^3 x(e^x - e^{-x}) dx \quad \checkmark \quad \checkmark \text{ using integration by parts: } f(x) = x \quad g'(x) = e^x - e^{-x} \\
 & \qquad \qquad \qquad f'(x) = 1 \quad g(x) = e^x + e^{-x} \\
 &= \left[x(e^x + e^{-x}) - \int (e^x + e^{-x}) dx \right]_1^3 \quad \checkmark \quad \checkmark \\
 &= \left[x(e^x + e^{-x}) - (e^x - e^{-x}) \right]_1^3 = \left[x(e^x + e^{-x}) - e^x + e^{-x} \right]_1^3 \quad \checkmark \\
 &= \left[3e^3 + 3e^{-3} - e^3 + e^{-3} - \{e^1 + e^{-1} - e^1 + e^{-1}\} \right] = \left[2e^3 + 4e^{-3} - 2e^{-1} \right] \quad \checkmark \\
 &= 39,6345 \quad \checkmark \quad \checkmark \\
 \bar{x} &= \frac{A_{m-y}}{A_x} \\
 &= \frac{39,6345}{17,0492} = 2,325 \text{ units} \quad \checkmark
 \end{aligned}$$

(6)

5.3

5.3.1



(2)

5.3.2

$$\begin{aligned}
 A &= \int_1^5 4 - (x-3)^2 dx \quad \checkmark \quad \checkmark \\
 &= \int_1^5 4 - x^2 + 6x - 9 dx \\
 &= \int_1^5 -5 - x^2 + 6x dx \quad \checkmark \\
 &= \left[-5x - \frac{x^3}{3} + 3x^2 \right]_1^5 \quad \checkmark \\
 &= \left[-5(5) - \frac{5^3}{3} + 3(5^2) - \left\{ -5 - \frac{1}{3} + 3 \right\} \right] \quad \checkmark \\
 &= 10,667 \text{units}^2 \quad \checkmark
 \end{aligned}$$

Alternative

$$\begin{aligned}
 A &= \int_1^5 4 - (x-3)^2 dx \quad \checkmark \quad \checkmark \\
 &= \left[4x - \frac{(x-3)^3}{3} \right]_1^5 \quad \checkmark \quad \checkmark \\
 &= \left[20 - \frac{8}{3} - \left\{ 4 - \frac{-8}{3} \right\} \right] \quad \checkmark \\
 &= 10,667 \quad \checkmark
 \end{aligned}$$

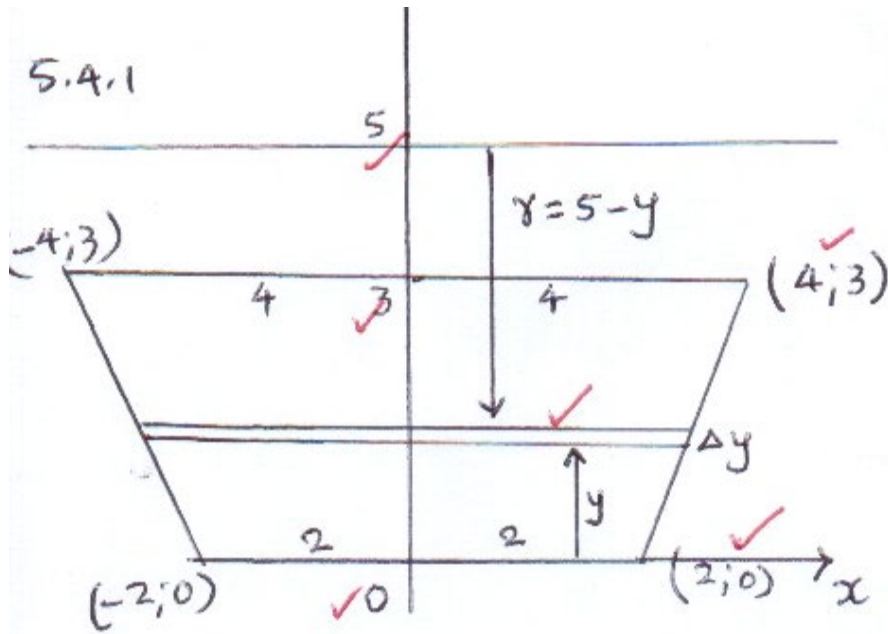
(3)

5.3.3

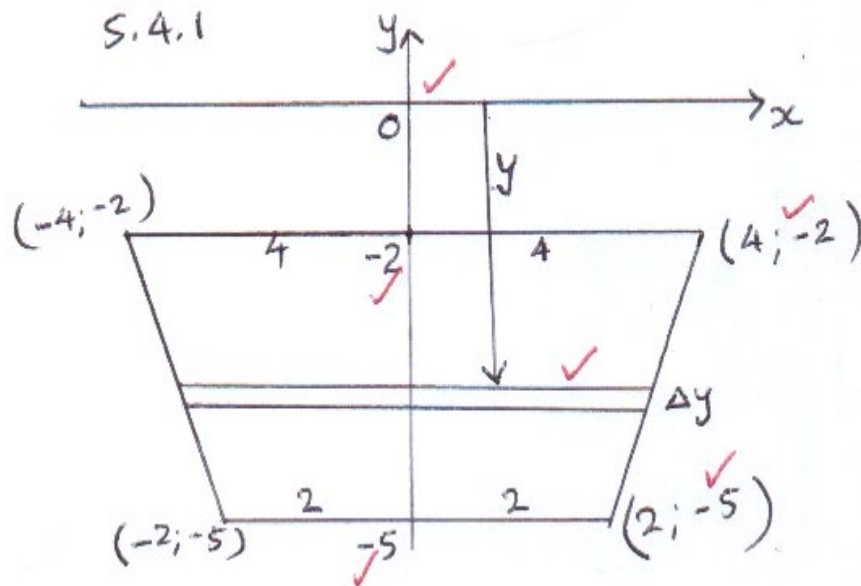
$$\begin{aligned}
 I_y &= \int_a^b r^2 dA \quad \checkmark \\
 &= \int_1^5 x^2 \{ 4 - (x-3)^2 \} dx \quad \checkmark \quad \checkmark \\
 &= \int_1^5 x^2 \{ -5 - x^2 + 6x \} dx \quad \checkmark \\
 &= \int_1^5 \{ -5x^2 - x^4 + 6x^3 \} dx \quad \checkmark \\
 &= \left[-\frac{5}{3}x^3 - \frac{x^5}{5} + \frac{6}{4}x^4 \right]_1^5 \quad \checkmark \\
 &= \left[-\frac{5}{3} \cdot 5^3 - \frac{5^5}{5} + \frac{3}{2}5^4 - \left\{ -\frac{5}{3} - \frac{1}{5} + \frac{3}{2} \right\} \right] \quad \checkmark \\
 &= 104,533 \text{units}^4 \quad \checkmark
 \end{aligned}$$

(4)

5.4 5.4.1



Alternative (with x-axis drawn at the water level)



(3)

5.4.2

$$\text{slope of the right edge } m = \frac{3-0}{4-2} = \frac{3}{2} \quad \checkmark$$

$$y - y_1 = m(x - x_1) \text{ using } (2;0)$$

$$y = \frac{3}{2}(x - 2)$$

$$y = \frac{3}{2}x - 3$$

$$x = \frac{2}{3}(y + 3) \quad \checkmark \cdot \text{length of strip} = 2x = \frac{4}{3}(y + 3)$$

$$\text{first moment of area} = \int_a^b r dA$$

$$= \int_0^3 (5 - y) \frac{4}{3}(y + 3) dy \quad \checkmark \quad \checkmark$$

$$= \frac{4}{3} \int_0^3 2y - y^2 + 15y dy \quad \checkmark$$

$$= \frac{4}{3} \left[y^2 - \frac{y^3}{3} + 15y \right]_0^3 \quad \checkmark = \frac{4}{3} \left[3^2 - \frac{3^3}{3} + 15 \cdot 3 - \{0\} \right] \quad \checkmark$$

$$= 60 \text{units}^3 \quad \checkmark$$

$$y = \frac{213}{60} = 3,55 \text{units} \quad \checkmark$$

Alternative:

$$\text{slope of the right edge } m = \frac{-2+5}{4-2} = \frac{3}{2} \quad \checkmark$$

$$y - y_1 = m(x - x_1) \text{ using } (2;-5)$$

$$y + 5 = \frac{3}{2}(x - 2)$$

$$y + 5 = \frac{3}{2}x - 3$$

$$x = \frac{2}{3}(y + 8) \quad \checkmark$$

$$\text{length of strip} = 2x = \frac{4}{3}(y + 8)$$

(4)

$$\begin{aligned}
 \text{first moment of area} &= \int_a^b r dA \\
 &= \int_{-5}^{-2} y \frac{4}{3} (y+8) dy \quad \checkmark \quad \checkmark \\
 &= \frac{4}{3} \int_{-5}^{-2} y^2 + 8y dy \quad \checkmark \\
 &= \frac{4}{3} \left[\frac{y^3}{3} + 4y^2 \right]_{-5}^{-2} \quad \checkmark \\
 &= \frac{4}{3} \left[\frac{(-2)^3}{3} + 4(-2)^2 - \left\{ \frac{(-5)^3}{3} + 4(-5)^2 \right\} \right] \quad \checkmark \\
 &= -60 \text{units}^3 \quad \checkmark \\
 &= \frac{213}{-60} \quad \checkmark = -3,55 \text{units}
 \end{aligned}$$

$$5.4.3 \quad Y = \frac{213}{-60} = -3,55 \text{units} \quad \checkmark \checkmark \quad (1)$$

[40]**QUESTION 6**

6.1

$$x = \frac{y^2}{4}$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy \quad \checkmark$$

$$\frac{dx}{dy} = \frac{2y}{4} = \frac{1}{2}y \quad \checkmark$$

$$\left(\frac{dx}{dy} \right)^2 = \left(\frac{1}{2}y \right)^2 = \frac{y^2}{4} \quad \checkmark$$

$$1 + \left(\frac{dx}{dy} \right)^2 = 1 + \frac{y^2}{4} = \frac{4 + y^2}{4} \quad \checkmark \quad \checkmark$$

$$\sqrt{1 + \left(\frac{dx}{dy} \right)^2} dy = \int_0^4 \frac{\sqrt{y^2 + 4}}{2} dy \quad \checkmark \quad \checkmark$$

$$= \frac{1}{2} \left[\frac{y}{2} \sqrt{y^2 + 4} + \frac{4}{2} \ln \left\{ y + \sqrt{y^2 + 4} \right\} \right]_0^4 \quad \checkmark \quad \checkmark$$

$$= \frac{1}{2} \left[\frac{4}{2} \sqrt{4^2 + 4} + 2 \ln \left\{ 4 + \sqrt{4^2 + 4} \right\} - \left(0 + 2 \ln \left\{ 0 + \sqrt{0^2 + 4} \right\} \right) \right] \quad \checkmark \quad \checkmark$$

$$= \frac{1}{2} \left[2\sqrt{20} + 2 \ln \left\{ 4 + \sqrt{20} \right\} - 2 \ln 2 \right]$$

$$= 5,916 \text{units} \quad \checkmark$$

(6)

6.2 $y = 2 \sin x$

$$A_x = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\frac{dy}{dx} = 2 \cos x \quad \checkmark$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4 \cos^2 x \quad \checkmark \quad \checkmark$$

$$A_x = 2\pi \int_{\frac{\pi}{2}}^{\pi} 2 \sin x \sqrt{1 + 4 \cos^2 x} dx \quad \checkmark \quad u = 2 \cos x \quad \frac{du}{dx} = -2 \sin x \quad \checkmark$$

$$dx = \frac{du}{-2 \sin x} \quad \text{follow up if used } u = \cos x$$

$$= 2\pi \int_{\frac{\pi}{2}}^{\pi} 2 \sin x \sqrt{1 + 4 \cos^2 x} \frac{du}{-2 \sin x} \quad \checkmark \quad x = \frac{\pi}{2} \quad u = 2 \cos \frac{\pi}{2} = 0$$

$$x = \pi \quad u = 2 \cos \pi = -2$$

$$= -2\pi \int_0^{-2} \sqrt{1 + u^2} du \quad \checkmark \quad \checkmark$$

$$= -2\pi \left[\frac{u}{2} \sqrt{1 + u^2} + \frac{1}{2} \ln \left\{ u + \sqrt{1 + u^2} \right\} \right]_0^{-2} \quad \checkmark \quad \checkmark$$

$$= -2\pi \left[\frac{-2}{2} \sqrt{1 + (-2)^2} + \frac{1}{2} \ln \left\{ -2 + \sqrt{1 + (-2)^2} \right\} - \left(0 + \frac{1}{2} \ln 1 \right) \right] \quad \checkmark$$

$$= -2\pi \left[-\sqrt{5} + \frac{1}{2} \ln \left\{ -2 + \sqrt{5} \right\} \right]$$

$$= 18,585 \text{ units}^2 \quad \checkmark$$

(6)
[12]

200 ÷ 2 100

TOTAL: 100