higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

27 July 2021 (X-paper)
09:00–12:00

Drawing instruments and scientific calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

052Q1G2127

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show all intermediate steps.
6. Write down all the formulae used.
7. Only use a black or blue pen.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z=\sqrt{x^2-y^2}$ prove that $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=\sqrt{x^2-y^2}$ (3)

1.2 Given $x=6\cos\theta$ and $y=6\sin\theta$ determine, in terms of θ

1.2.1 $\dfrac{dy}{dx}$ (2)

1.2.2 $\dfrac{d^2y}{dx^2}$ (1)

[6]

QUESTION 2

Determine $\int y\,dx$ if.

2.1 $y=\left(1+\tan^2 3x\right)\left(\sec^2 3x-1\right)$ (2)

2.2 $y=\sin^{-1}bx$ (5)

2.3 $y=\sqrt{6x-x^2}$ (4)

2.4 $y=\cos^4\left(\dfrac{3}{2}x\right)$ (4)

2.5 $y=x\tan x+\ln\sec x$ (3)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int\frac{(x+3)(x-4)}{2x^3-x^2-6x}\,dx$ (6)

3.2 $\displaystyle\int\frac{3x^2+16x+26}{(x-2)(x^2+3x+4)}\,dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of $\frac{dy}{dx} - \frac{y}{x \ln x} = \frac{1}{x}$ at (2; 0) (6)

4.2 Determine the general solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 2e^{-3x}$ (6)

[12]

QUESTION 5

5.1 5.1.1 Make a neat sketch of the graphs of

$y = \sin x$ and $y = 1 + \sin x$ for $0 \le x \le \pi$

Show the area bounded by the graphs, the y-axis and the line $x = \pi$. Show the representative strip/element that you will use to calculate the volume of the solid generated when this area rotates about the x-axis. (2)

5.1.2 Calculate the volume described in QUESTION 5.1.1 (5)

5.1.3 Calculate the volume moment about the y-axis as well as the distance of the centre of gravity from the y-axis. (6)

5.2 5.2.1 Make a neat sketch of the graphs of $y = e^x$ and $y = e^{-x}$.

Show the area bounded by the graphs and the lines $x = 1$ and $x = 3$. Show the representative strip/element that you will use to calculate the area. (2)

5.2.2 Calculate the area described in QUESTION 5.2.1 (2)

5.2.3 Calculate the area moment about the y-axis of the area described in QUESTION 5.2.1 as well as the distance of the centroid from the y-axis. (6)

5.3 5.3.1 The line $y = 4$ intersects the graph of $y = (x-3)^2$ at the points $(1;4)$ and $(5;4)$

Sketch the graph and show the area bounded by the graph and the line y = 4. Use a representative strip PERPENDICULAR to the x-axis. (2)

5.3.2 Calculate the area described in QUESTION 5.3.1 (3)

5.3.3 Calculate the second moment of the area about the y-axis of the area Described in QUESTION 5.3.1 (4)

5.4 5.4.1 The cross section of a water canal is in the shape of a trapezium. A sluice gate of height 3 m is vertically placed in the canal. The gate is 4 m wide at the bottom and 8 m wide at the top. The height of the water level in the canal is 5 m.

Make a neat sketch of the sluice gate and show the representative strip/element that you will use to calculate the first moment of area of the gate about the water level. (3)

5.4.2 Calculate the first moment of area of the gate about the water level. (4)

5.4.3 Calculate the depth of the centre of pressure on the sluice gate if the second moment of area of the gate about the water level is given as 213 units^4 (1)

[40]

QUESTION 6

6.1 Calculate the length of the curve $x = \frac{y^2}{4}$ from $y = 0$ to $y = 4$ (6)

6.2 Calculate the surface area generated when the curve $y = 2\sin x$ from $x = \frac{\pi}{2}$ to $x = \pi$ rotates about the x-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2 ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\, dx$
$\sin f(x)$	$\cos f(x)\ .\ f'(x)$	-
$\cos f(x)$	$-\sin f(x)\ .\ f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\ .\ f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x)\ .\ f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x)\ .\ f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x)\ .\ f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\ f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\ f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\ dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y dx \; ; \; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x dy \; ; \; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \qquad ; \qquad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\ (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore\ ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$