higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N6

24 NOVEMBER 2016

This marking guideline consists of 20 pages.

✓ full mark

TOTAL: $\frac{200}{2} = 100$

NOTE: Do NOT subtract marks for incorrect units or units omitted

QUESTION 1

1.1 1.1.1 $z = \tan(x^3y^2) + \text{cosec}(xy^2)$

$$\frac{\partial z}{\partial x} = \overset{\checkmark}{3x^2y^2} \overset{\checkmark}{\sec^2(x^3y^2)} - \overset{\checkmark}{y^2\text{cosec}(xy^2)} \overset{\checkmark}{\cot(xy^2)}$$

(4)

1.1.2 $$\frac{\partial z}{\partial y} = \overset{\checkmark}{2x^3y\sec^2(x^3y^2)} - \overset{\checkmark}{2xy\text{cosec}(xy^2)\cot(xy^2)}$$

(2)

1.2 $x = t^2$ $\quad$ $y = 2t^5$

$\frac{dx}{dt} = 2t$ ✓ $\quad$ $\frac{dy}{dt} = 10t^4$ ✓

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{10t^4}{2t} \quad \checkmark$$

$$= 5t^3$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(5t^3\right) \times \frac{1}{2t} \quad \checkmark$$

$$= \overset{\checkmark}{15t^2} \times \frac{1}{2t}$$

$$= \frac{15t}{2} \quad or \quad 7{,}5t \quad \checkmark$$

(6)

[12]

QUESTION 2

2.1 $y = \int \sin^{-1} 3x dx$

$$f(x) = \sin^{-1} 3x \qquad g'(x) = 1$$

$$f'(x) = \frac{3}{\sqrt{1-(3x)^2}} \qquad g(x) = x$$

$$= \overset{\checkmark}{x.\sin^{-1} 3x} - \overset{\checkmark}{\int x.\frac{3}{\sqrt{1-9x^2}}dx}$$

$$= x.\sin^{-1} 3x - 3\int x.(1-9x^2)^{-\frac{1}{2}}dx$$

$$= x.\sin^{-1} 3x + \overset{\checkmark}{\frac{3}{18}}.\overset{\checkmark}{\frac{(1-9x^2)^{\frac{1}{2}}}{\frac{1}{2}}}$$

$$= x.\sin^{-1} 3x + \frac{1}{3}\sqrt{1-9x^2} + c$$

(4)

2.2 $y = \int \frac{2}{\sec^4 2x}dx$

$$= \int 2\cos^4 2xdx \ \checkmark$$

$$= 2\int (\frac{1}{2} + \frac{1}{2}\cos 4x)^2 dx \ \checkmark$$

$$= 2\int (\frac{1}{2} + \frac{1}{2}\cos 4x)(\frac{1}{2} + \frac{1}{2}\cos 4x)dx$$

$$= 2\int (\frac{1}{4} + \frac{1}{2}\cos 4x + \frac{1}{4}\cos^2 4x)dx \ \checkmark$$

$$= \overset{\checkmark}{2}\left[\overset{\checkmark}{\frac{1}{4}x} + \overset{\checkmark}{\frac{1}{2}.\frac{\sin 4x}{4}} + \frac{1}{4}\left(\overset{\checkmark}{\frac{x}{2}} + \overset{\checkmark}{\frac{\sin 8x}{16}}\right)\right] + c$$

$$= 2\left[\frac{1}{4}x + \frac{\sin 4x}{8} + \frac{x}{8} + \frac{\sin 8x}{64}\right] + c$$

$$= \frac{1}{2}x + \frac{\sin 4x}{4} + \frac{x}{4} + \frac{\sin 8x}{32} + c$$

$$= \frac{3}{4}x + \frac{\sin 4x}{4} + \frac{\sin 8x}{32} + c$$

(8)

2.3

$$y = \int \frac{1}{4x^2+12x+24}dx$$

$$= 4x^2+12x+24$$

$$= 4(x^2+3x+6) \quad ✓$$

$$= 4\left[\overset{✓}{\left(x+\frac{3}{2}\right)^2} + \overset{✓}{6-\left(\frac{3}{2}\right)^2}\right]$$

$$= 4\left[\left(x+\frac{3}{2}\right)^2+\frac{15}{4}\right] \quad ✓$$

$$= 15+4\left(x+\frac{3}{2}\right)^2 \quad ✓$$

$$\therefore \int \frac{1}{4x^2+12x+24}dx$$

$$= \int \frac{1}{15+4\left(x+\frac{3}{2}\right)^2}dx \quad ✓$$

$$= \overset{✓}{\frac{1}{2\sqrt{15}}}\tan^{-1}\overset{✓}{\frac{2(x+\frac{3}{2})}{\sqrt{15}}}+c \text{ or } = 0{,}129\tan^{-1}\frac{2x+3}{3{,}873}+c$$

Or

$$y = \int \frac{1}{4x^2+12x+24}dx$$

$$= \int \frac{1}{4(x^2+3x+6)}dx ✓$$

$$= \overset{✓}{\frac{1}{4}}\int \frac{1}{x^2+3x+\underset{✓}{\left(\frac{3}{2}\right)^2}-\underset{✓}{\left(\frac{3}{2}\right)^2+6}}dx$$

$$= \frac{1}{4}\int \frac{1}{\underset{✓}{\left(x+\frac{3}{2}\right)^2}+\underset{✓}{\frac{15}{4}}}dx$$

$$= \frac{1}{4}\left[\overset{✓}{\frac{1}{\sqrt{\frac{15}{4}}}}\tan^{-1}\overset{✓}{\frac{(x+\frac{3}{2})}{\sqrt{\frac{15}{4}}}}\right]+c \qquad \text{or } = \frac{1}{4}\left[\frac{1}{\frac{\sqrt{15}}{2}}\tan^{-1}\frac{(x+\frac{3}{2})}{\frac{\sqrt{15}}{2}}\right]+c$$

(8)

2.4 $y = \int \text{cosec}^5 4x.\cos^3 4xdx$

✓

$= \int \text{cosec}^5 4x.\cos^2 4x.\cos 4xdx$

✓

$= \int \text{cosec}^5 4x.(1-\sin^2 4x).\cos 4xdx$

✓

$= \frac{1}{4}\int \frac{1}{u^5}.(1-u^2)du$ ✓

$u = \sin 4x$

$du = 4\cos 4xdx$

✓

$= \frac{1}{4}\int (\frac{1}{u^5}-\frac{1}{u})du$

$= \frac{1}{4}\int \ (u^{-5}-\frac{1}{u})du$

✓ ✓

$= \frac{1}{4}\left[\frac{u^{-4}}{-4}-\ln u\right]+c$

$= \frac{1}{4}\left[\frac{\sin^{-4}4x}{-4}-\ln(\sin 4x)\right]+c$ ✓

Or $= -\frac{1}{16\sin^4 4x}-\frac{1}{4}\ln(\sin 4x)+c$ or $-\frac{1}{16}\text{cosec}^4 4x-\frac{1}{4}\ln(\sin 4x)+c$ (8)

2.5 $y = \int \ x^3.\sin 2xdx$

$f(x) = x^3$ $g'(x) = \sin 2x$

$f'(x) = 3x^2$ $g(x) = -\frac{\cos 2x}{2}$

✓ ✓

$\int ydx = x^3(-\frac{\cos 2x}{2}) - \int 3x^2(-\frac{\cos 2x}{2})dx$

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{2}\int x^2.\cos 2xdx$

$f(x) = x^2$ $g'(x) = \cos 2x$

$f'(x) = 2x$ $g(x) = \frac{\sin 2x}{2}$

✓ ✓

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{2}\left[x^2.\frac{\sin 2x}{2} - \int 2x.\frac{\sin 2x}{2}dx\right]$

✓

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x - \frac{3}{2}\int x.\sin 2xdx$

$f(x) = x$ $g'(x) = \sin 2x$

$f'(x) = 1$ $g(x) = -\frac{\cos 2x}{2}$

✓ ✓

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x - \frac{3}{2}\left[x.-\frac{\cos 2x}{2} - \int -\frac{\cos 2x}{2}dx\right]$

✓

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x + \frac{3}{4}x.\cos 2x - \frac{3}{4}.\frac{\sin 2x}{2} + c$

$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2\sin 2x + \frac{3}{4}x\cos 2x - \frac{3}{8}\sin 2x + c$

(8)

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QUESTION 3

3.1 $$\int \frac{3x^2-3x+1}{(2x+1)^2(x-1)}\ dx$$

$$\frac{3x^2-3x+1}{(2x+1)^2(x-1)}=\frac{A}{(2x+1)^2}+\frac{B}{(2x+1)}+\frac{C}{(x-1)} \quad ✓$$

$$3x^2-3x+1=A(x-1)+B(2x+1)(x-1)+C(2x \quad ✓$$

$$Let\ \ x=-\frac{1}{2};\quad \therefore A=-\frac{13}{6}\quad (2{,}167)\quad ✓$$

$$Let\ \ x=1;\qquad \therefore C=\frac{1}{9}\quad (0{,}111)\quad ✓$$

$$3x^2-3x+1=Ax-A+2Bx^2-Bx-B+4Cx^2+4Cx+C \quad ✓$$

$$\text{Equate coeff of }\ x^2:\quad 3=2B+4C\quad \therefore B=\frac{23}{18}\quad (1{,}278)\quad ✓$$

$$=\int \frac{-\frac{13}{6}}{(2x+1)^2}dx+\int \frac{\frac{23}{18}}{(2x+1)}dx+\int \frac{\frac{1}{9}}{(x-1)}dx \quad ✓$$

$$=-\frac{13}{6}\int (2x+1)^{-2}dx+\frac{23}{18}\int \frac{1}{(2x+1)}dx+\frac{1}{9}\int \frac{1}{(x-1)}dx$$

$$=-\overset{✓}{\frac{13}{6}}\left(\frac{1}{2}\right)\overset{✓}{\left[\frac{(2x+1)^{-1}}{-1}\right]}+\overset{✓}{\frac{23}{18}}\left(\frac{1}{2}\right)\overset{✓}{\ln(2x+1)}+\frac{1}{9}\overset{✓}{\ln(x-1)}+c$$

$$=\frac{13}{12(2x+1)}+\frac{23}{36}\ln(2x+1)+\frac{1}{9}\ln(x-1)+c$$

$$=\frac{1{,}083}{(2x+1)}+0{,}639\ln(2x+1)+0{,}111\ln(x-1)+c$$

(12)

3.2 $\int \frac{x^4 + x^2 - 2}{x^3 + x}\ dx$

$$\rightarrow \qquad \begin{array}{r|l} & x \ \checkmark \\ \hline x^3 + x & x^4 + x^2 - 2 \\ & \underline{x^4 + x^2} \\ & \qquad\quad -2 \end{array}$$

$= \int \left(x - \frac{2}{x^3 + x} \right) dx$ ✓

$= \int x\ dx - \int \frac{2}{x^3 + x}\ dx$

$\therefore \frac{2}{x(x^2+1)} \overset{}{=} \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$ ✓ ✓

$2 \ = A(x^2 + 1) + (Bx + C)x$ ✓

$\therefore$ If $x = 0$; A = 2 ✓

$2 \ = Ax^2 + A + Bx^2 + C$ ✓

Equate coeff of x^2 : $\therefore B = -2$ ✓

Equate coeff of x : $\therefore C = 0$ ✓

$= \int x dx + \int \frac{2}{x} dx + \int \frac{-2x}{x^2 + 1} dx$

✓ ✓ ✓

$= \frac{x^2}{2} - 2 \ln x + \ln(x^2 + 1) + c$

Or

(1 mark for long division)

$= \int x\ dx + \int \frac{-2}{x^3 + x}\ dx$ ✓

$\therefore \frac{-2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$ ✓ ✓

$-2 \ = A(x^2 + 1) + (Bx + C)x$ ✓

$\therefore$ If $x = 0$; A $= -2$ ✓

$2 \ = Ax^2 + A + Bx^2 + Cx$ ✓

Equate coeff of x^2 : $\therefore B = 2$ ✓

Equate coeff of x : $\therefore C = 0$ ✓

$= \int x dx + \int \frac{-2}{x} dx + \int \frac{2x}{x^2 + 1} dx$

✓ ✓ ✓

$= \frac{x^2}{2} - 2 \ln x + \ln(x^2 + 1) + c$ (12)

[24]

QUESTION 4

4.1

$$t\frac{dy}{dt}-2y=t^2-t+1$$

$$\frac{dy}{dt}-\frac{2y}{t}=t-1+\frac{1}{t} \quad \checkmark$$

$$R=e^{\int -\frac{2}{t}dt} \quad \checkmark$$

$$=e^{-2\ln t} \quad \checkmark$$

$$=e^{\ln t^{-2}}$$

$$=t^{-2} \quad \checkmark$$

$$=\frac{1}{t^2}$$

$$\frac{1}{t^2}.y=\int \frac{1}{t^2}.(t-1+\frac{1}{t})dt \quad \checkmark$$

$$\frac{y}{t^2}=\int (\frac{1}{t}-t^{-2}+t^{-3})dt \quad \checkmark$$

$$\frac{y}{t^2}=\ln t-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}+c \quad \checkmark$$

$$\boxed{or \quad \frac{y}{t^2}=\ln t+\frac{1}{t}-\frac{1}{2t^2}+c}$$

$$\frac{-\frac{1}{2}}{(1)^2}=\ln(1)-\frac{(1)^{-1}}{-1}+\frac{(1)^{-2}}{-2}+c \quad \checkmark$$

$$\therefore \text{c}=-1 \quad \checkmark$$

$$\frac{y}{t^2}=\ln t-\frac{t^{-1}}{-1}+\frac{t^{-2}}{-2}-1 \quad \checkmark$$

$$or \quad \frac{y}{t^2}=\ln t+\frac{1}{t}-\frac{1}{2t^2}-1$$

(10)

4.2 $$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y = 6e^{\frac{x}{2}}$$

$y_c:\quad m^2+2m+2=0$ ✓

$$m=\frac{-2\pm\sqrt{4-4(2)}}{2}$$

$m=-1\pm j$ ✓

$y_c=e^{-x}(A\cos x+B\sin x)$ ✓

To find y$_p$ $\therefore y\ =Ce^{\frac{x}{2}}$ ✓

$$\frac{dy}{dx}=\frac{1}{2}Ce^{\frac{x}{2}}$$ ✓

$$\frac{d^2y}{dx^2}=\frac{1}{4}Ce^{\frac{x}{2}}$$ ✓

$$\frac{1}{4}Ce^{\frac{x}{2}}+2(\frac{1}{2}Ce^{\frac{x}{2}})+2(Ce^{\frac{x}{2}})=6e^{\frac{x}{2}}$$ ✓

$$\frac{13}{4}Ce^{\frac{x}{2}}=6e^{\frac{x}{2}}$$

$\therefore C=\frac{24}{13}$ (1,846) ✓

$\therefore y_p=\frac{24}{13}e^{\frac{x}{2}}$ ✓

$y=e^{-x}(A\cos x+B\sin x)+\frac{24}{13}e^{\frac{x}{2}}$ ✓

$$1=e^{-0}(A\cos 0+B\sin 0)+\frac{24}{13}e^{\frac{0}{2}}$$

$\therefore A=-\frac{11}{13}$ (–0,846) ✓

$$\frac{dy}{dx}=e^{-x}(-A\sin x+B\cos x)-e^{-x}(A\cos x+B\sin x)+\frac{1}{2}.\frac{24}{13}e^{\frac{x}{2}}$$ ✓

$$1=e^{-0}(-A\sin 0+B\cos 0)-e^{-0}(A\cos 0+B\sin 0)+\frac{12}{13}e^{\frac{0}{2}}$$

$\therefore B=-\frac{10}{13}$ $(-0{,}769)$ ✓

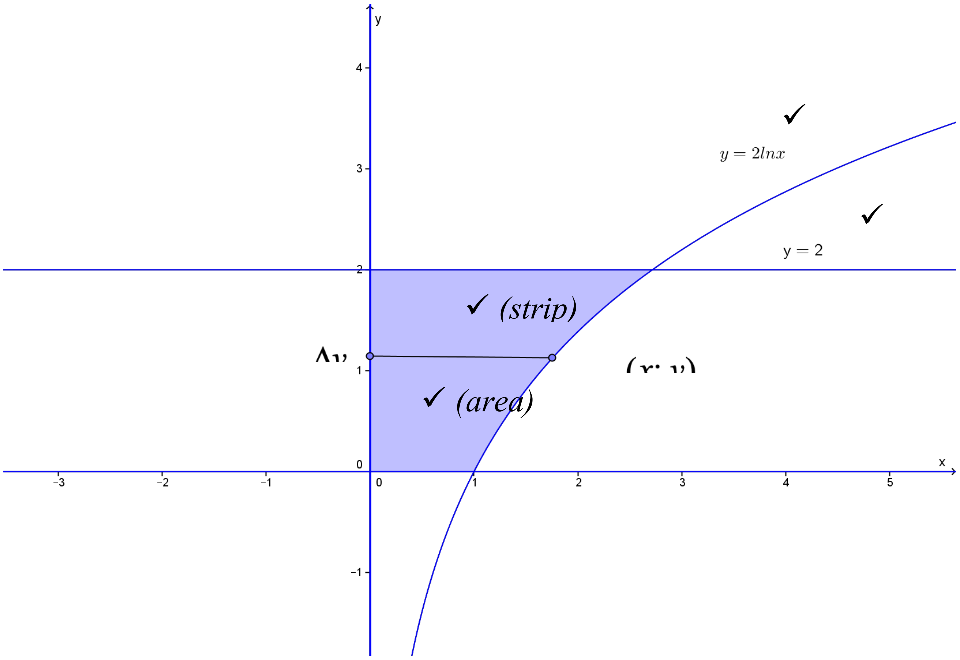
$\therefore y=e^{-x}(-\frac{11}{13}\cos x-\frac{10}{13}\sin x)+\frac{24}{13}e^{\frac{x}{2}}$ ✓

or $y=e^{-x}(-0{,}864\cos x-0{,}769\sin x)+\frac{24}{13}e^{\frac{x}{2}}$

(14)
[24]

QUESTION 5

5.1 5.1.1



(4)

5.1.2

✓ ✓

$$\Delta V_x = 2\pi yx \times \Delta y$$

✓

$$V_x = 2\pi\int_0^2 xydy$$

Incorrect limits: max 6 marks

$$= 2\pi\int_0^2 e^{\frac{y}{2}}.ydy$$ ✓

✓ ✓

$$= 2\pi\left[y.2e^{\frac{y}{2}} - \int 2e^{\frac{y}{2}}dy\right]_0^2$$

$$= 2\pi\left[2ye^{\frac{y}{2}} - 2\int e^{\frac{y}{2}}dy\right]_0^2$$

✓

$$= 2\pi\left[2ye^{\frac{y}{2}} - 2.\frac{e^{2}}{\frac{1}{2}}\right]_0^2$$

$$= 4\pi\left[ye^{\frac{y}{2}} - 2e^{\frac{y}{2}}\right]_0^2$$

✓ ✓

$$= 4\pi\left\{\left[2e^{\frac{2}{2}} - 2e^{\frac{2}{2}}\right] - \left[0 - 2e^{\frac{0}{2}}\right]\right\}$$

$$= 8\pi \quad units^3 \quad or \quad 25{,}133 \quad units^3$$ ✓

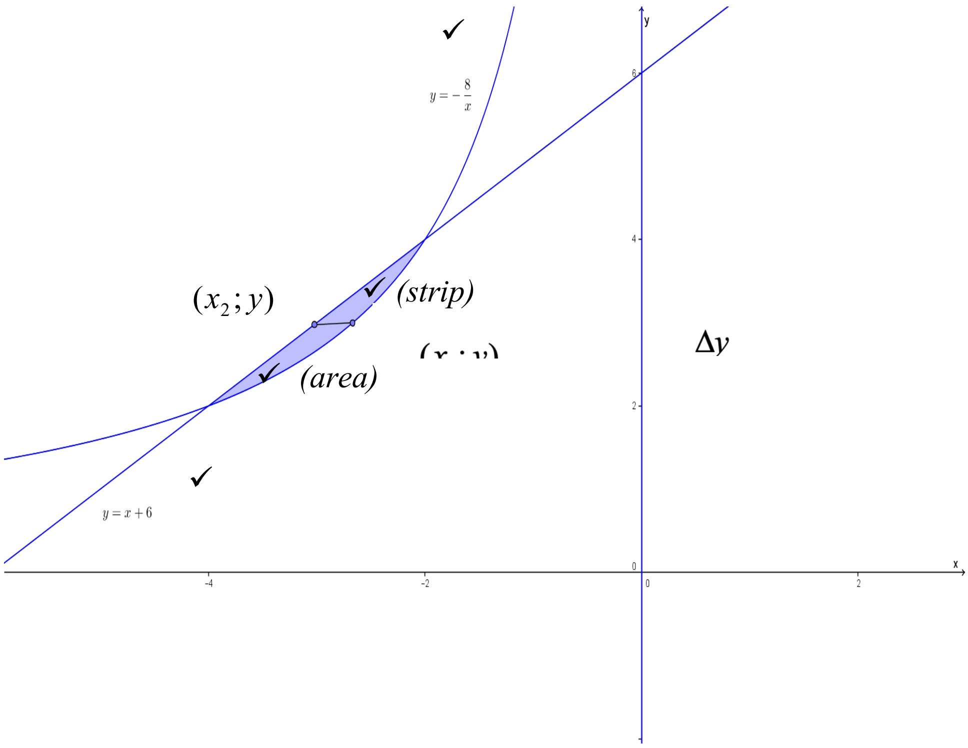
$f(y) = y$ $\quad g'(y) = e^{\frac{y}{2}}$

$f'(y) = 1$ $\quad g(y) = \dfrac{e^{\frac{y}{2}}}{\frac{1}{2}} = 2e^{\frac{y}{2}}$

(10)

5.2 5.2.1

$$x+6=-\frac{8}{x}$$
$$x^2+6x+8=0$$
$$(x+4)(x+2)=0$$
$$x=-4;\quad x=-2 \quad ✓$$
$$y=2;\quad y=4 \ ✓ \quad \therefore(-4;2) \text{ and } (-2;4)$$



(6)

5.2.2

$$\Delta A=(x_2-x_1)\Delta y \quad ✓$$

✓

$$A=\int_2^4 (x_2-x_1)dy$$

Incorrect limits: max 3 marks

$$=\int_2^4\left(-\frac{8}{y}-(y-6)\right)dy \quad ✓$$
$$=\left[-8\ln y-\frac{y^2}{2}+6y\right]_2^4 \quad ✓$$
$$=\left[-8\ln 4-\frac{4^2}{2}+6(4)\right]-\left[-8\ln 2-\frac{2^2}{2}+6(2)\right] \quad ✓$$
$$=0{,}455\ units^2 \quad ✓$$

(6)

✓ ✓

5.2.3

$$\therefore \Delta I = (x_2 - x_1)\Delta y \times y^2$$
✓

$$I = \int_2^4 \left(-\frac{8}{y} - y + 6 \right) y^2 dy \ \checkmark$$

Incorrect limits: max 5 marks

$$= \int_2^4 (-8y - y^3 + 6y^2) dy \ \checkmark$$

$$= \left[-\frac{8y^2}{2} - \frac{y^4}{4} + \frac{6y^3}{3} \right]_2^4 \ \checkmark$$

$$= \left[-4(4)^2 - \frac{(4)^4}{4} + 2(4)^3 \right] - \left[-4(2)^2 - \frac{(2)^4}{4} + 2(2)^3 \right] \ \checkmark$$

$$= \ 4\,units^4 \ \checkmark$$

(8)

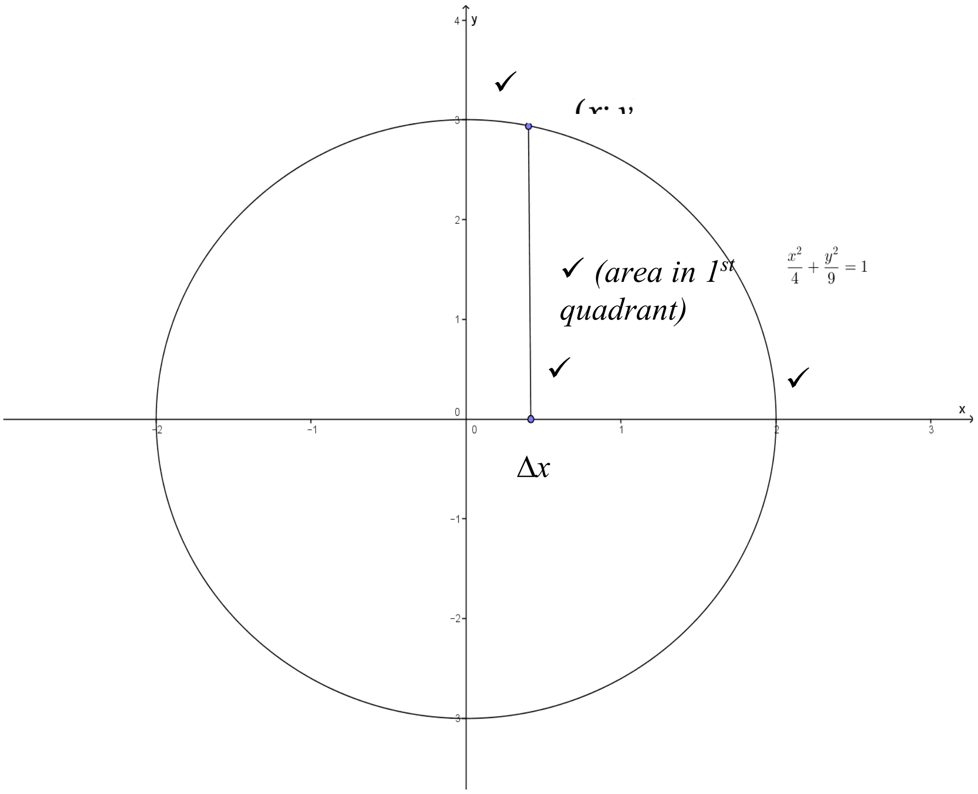
5.2.4

$$I = \frac{\overset{\checkmark}{4}}{0{,}455} \times A$$

$$= \ \overset{\checkmark}{8{,}781}\, A(units^4)$$

(2)

5.3 5.3.1



(4)

5.3.2

$$\Delta V_x = \pi y^2 \Delta x \quad \checkmark$$

$$\checkmark$$

$$V_x = \pi \int_0^2 y^2 dx$$

Incorrect limits: max 3 marks

$$= \pi \int_0^2 9(1 - \frac{x^2}{4})dx \quad \checkmark$$

$$= 9\pi \int_0^2 (1 - \frac{x^2}{4})dx \quad or \quad \frac{9}{4}\pi \int_0^2 (4 - x^2)dx$$

$$= 9\pi \left[x - \frac{x^3}{12} \right]_0^2 \quad \checkmark$$

$$= 9\pi \left[2 - \frac{2^3}{12} \right] \quad \checkmark$$

$$= \frac{36}{3}\pi \quad or\; 12\pi \quad units^3 \quad or \quad 37{,}699 \quad units^3 \quad \checkmark$$

(6)

5.3.3

$$\Delta M = \rho.\Delta V_x$$

$$= \rho.\pi y^2 \Delta x$$

$$\checkmark \qquad \checkmark$$

$$\therefore \Delta I_x = \rho.\pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}} \right)^2$$

$$\checkmark$$

$$I_x = \frac{\rho\pi}{2} \int_0^2 y^4 dx \quad \checkmark$$

Incorrect limits: max 7 marks

$$= \frac{\rho\pi}{2} \int_0^2 \left[9\left(1 - \frac{x^2}{4} \right) \right]^2 dx \quad \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^2 \left[9\left(\frac{4 - x^2}{4} \right) \right]^2 dx$$

$$= \frac{81\rho\pi}{16.2} \int_0^2 (4 - x^2)^2 dx \quad \checkmark$$

$$= \frac{81\rho\pi}{32} \int_0^2 (16 - 8x^2 + x^4)dx \quad \checkmark$$

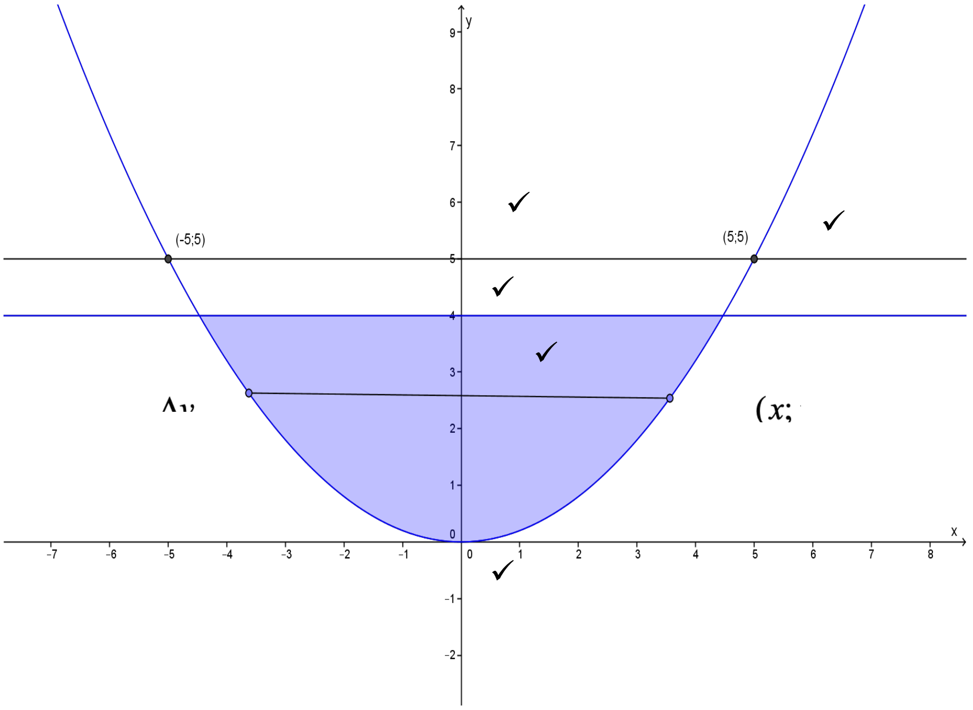
$$= \frac{81\rho\pi}{32} \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2 \quad \checkmark$$

$$= \frac{81\rho\pi}{32} \left[16(2) - \frac{8(2)^3}{3} + \frac{(2)^5}{5} \right] \quad \checkmark$$

$$= 43{,}2\pi\rho \quad units^4 \quad or \quad 135{,}717\rho \quad \checkmark$$

(10)

5.4 5.4.1



$$y = ax^2$$

$$5 = a(5)^2 \quad a = \frac{1}{5} \checkmark$$

$$y = \frac{1}{5}x^2 \checkmark \; \therefore x = \sqrt{5y} \text{ or } x = \sqrt{5}y^{\frac{1}{2}} \checkmark \; \therefore dA = 2(\sqrt{5}y^{\frac{1}{2}})dy$$

(8)

5.4.2 $\int_0^4 rdA$

$$\checkmark \quad \checkmark \quad \checkmark$$

$$= \int_0^4 (5-y)2\sqrt{5}y^{\frac{1}{2}}dy$$

Incorrect limits: max 4 marks

$$= 2\sqrt{5}\int_0^4 (5y^{\frac{1}{2}} - y^{\frac{3}{2}})dy \checkmark$$

$$= 2\sqrt{5}\left[\frac{5y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4 \checkmark$$

$$= 2\sqrt{5}\left[\frac{2}{3}.5(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}}\right]$$

$= 62{,}014$ units^3 ✓

(6)

5.4.3 $\int_0^4 r^2 dA$

Incorrect limits: max 6 marks

✓ ✓ ✓

$$= \int_0^4 (5-y)^2 2\sqrt{5} y^{\frac{1}{2}} dy$$

$$= 2\sqrt{5}\int_0^4 (25-10y+y^2) y^{\frac{1}{2}} dy \checkmark$$

$$= 2\sqrt{5}\int_0^4 (25y^{\frac{1}{2}} - 10y^{\frac{3}{2}} + y^{\frac{5}{2}}) dy \checkmark$$

$$= 2\sqrt{5}\left[\frac{25y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{10y^{\frac{5}{2}}}{\frac{5}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4 \checkmark$$

$$= 2\sqrt{5}\left[\frac{2}{3}.25(4)^{\frac{3}{2}} - \frac{2}{5}.10(4)^{\frac{5}{2}} + \frac{2}{7}(4)^{\frac{7}{2}}\right] \checkmark$$

$= 187{,}404$ units^4 ✓

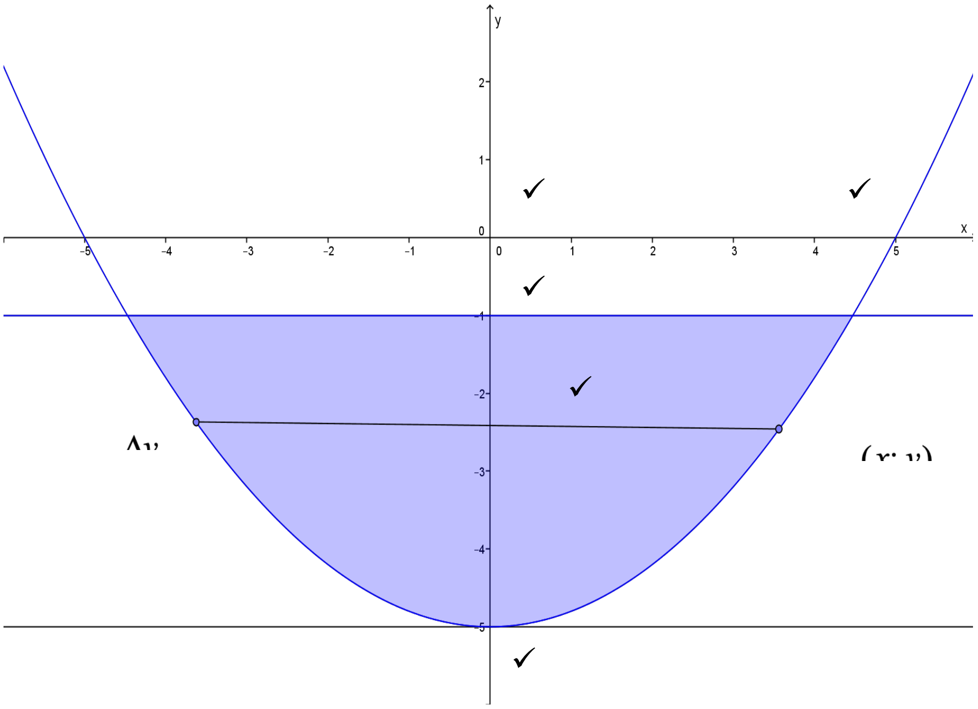
$$\overset{=}{y} = \frac{187{,}404}{62{,}014} \checkmark$$

$= 3{,}022$ *units* ✓

(10)

Alternative method

5.4 5.4.1



$y = ax^2 + bx - 5$

$0 = a(5)^2 + b(5) - 5......(5;0)$

$\therefore 1 = 5a + 6....(1)$

$0 = a(-5)^2 + b(-5) - 5......(-5;0)$

$\therefore 1 = 5a - 6....(2)$

✓

$\therefore a = \frac{1}{5} \quad and \quad b = 0$

$y = \frac{1}{5}x^2 - 5$ ✓

$\therefore x = \sqrt{5y + 25}$ or $x = \sqrt{5}(y+5)^{\frac{1}{2}}$ ✓ $\therefore dA = 2[\sqrt{5}(y+5)^{\frac{1}{2}}]dy$ (8)

5.4.2 $\int_{-5}^{-1} r dA$

✓ ✓ ✓

$$= \int_{-5}^{-1} y.2\sqrt{5}(y+5)^{\frac{1}{2}} dy$$

Incorrect limits: max 4 marks

$$= 2\sqrt{5}\int_{-5}^{-1} y(y+5)^{\frac{1}{2}} dy$$

$Let\ u = y+5$
$du = dy$
$y = u-5$

$$= 2\sqrt{5}\int_{-5}^{-1} (u-5)(u)^{\frac{1}{2}} du$$ ✓

$$= 2\sqrt{5}\int_{-5}^{-1} (u^{\frac{3}{2}} - 5u^{\frac{1}{2}}) du$$

$$= 2\sqrt{5}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 5.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{y=-5}^{y=-1}$$ ✓

$$= 2\sqrt{5}\left[\frac{2}{5}(y+5)^{\frac{5}{2}} - 5.\frac{2}{3}(y+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

$$= 2\sqrt{5}\left[\frac{2}{5}(-1+5)^{\frac{5}{2}} - 5.\frac{2}{3}(-1+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

= -62,014 units^3 ✓ (6)

5.4.3 $\int_0^4 r^2 dA$

✓ ✓ ✓

$$=\int_{-5}^{-1} y^2 . 2\sqrt{5}(y+5)^{\frac{1}{2}} dy$$

Incorrect limits: max 6 marks

$$= 2\sqrt{5}\int_{-5}^{-1} y^2 (y+5)^{\frac{1}{2}} dy \ ✓$$

Let $u = y+5$
$du = dy$
$y = (u-5)^2$

$$= 2\sqrt{5}\int_{-5}^{-1} (u-5)^2 (u)^{\frac{1}{2}} du \ ✓$$

$$= 2\sqrt{5}\int_{-5}^{-1} (u^2 - 10u + 25)u^{\frac{1}{2}} du$$

$$= 2\sqrt{5}\int_{-5}^{-1} (u^{\frac{5}{2}} - 10u^{\frac{3}{2}} + 25u^{\frac{1}{2}}) du$$

$$= 2\sqrt{5}\left[\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{10u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{25u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{y=-5}^{y=-1} \quad ✓$$

$$= 2\sqrt{5}\left[\frac{(y+5)^{\frac{7}{2}}}{\frac{7}{2}} - \frac{10(y+5)^{\frac{5}{2}}}{\frac{5}{2}} + \frac{25(y+5)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-5}^{-1} \quad ✓$$

$$= 2\sqrt{5}\left[\frac{2}{7}(-1+5)^{\frac{7}{2}} - \frac{2}{5}.10(-1+5)^{\frac{5}{2}} + \frac{2}{3}.25(-1+5)^{\frac{3}{2}}\right]$$

$= 187{,}404$ units^4 ✓

$$\bar{y} = \frac{187{,}404}{-62{,}014} \ ✓$$

$= -3{,}022$ *units* ✓

(10)

[80]

QUESTION 6

6.1

Incorrect limits: max 7 marks

$$y = 2x^2 - 4$$

$$\frac{dy}{dx} = 4x \quad ✓$$

$$\left(\frac{dy}{dx}\right)^2 = (4x)^2 \quad ✓$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + (4x)^2 \; ✓$$

$$= 1 + 16x^2$$

✓

$$S = \int_0^2 \sqrt{1 + 16x^2}\, dx \quad ✓$$

$$= \int_0^2 \sqrt{16(\frac{1}{16} + x^2)} dx \quad ✓$$

✓

$$= 4\int_0^2 \sqrt{\left(x^2 + \frac{1}{16}\right)} dx$$

$$= 4\left[\frac{x}{2}\sqrt{x^2 + \frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(x + \sqrt{x^2 + \frac{1}{16}}\right)\right]_0^2 \quad ✓$$

$$= 4\left\{\left[\frac{2}{2}\sqrt{2^2 + \frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(2 + \sqrt{2^2 + \frac{1}{16}}\right)\right] - \left[\frac{0}{2}\sqrt{0^2 + \frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(0 + \sqrt{0^2 + \frac{1}{16}}\right)\right]\right\} \; ✓$$

= 8,409 units✓ (10)

6.2 $x = a\cos^3\theta$ and $y = a\sin^3\theta$

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) \checkmark \qquad \frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta) \checkmark$$

$$\left(\frac{dx}{d\theta}\right)^2 = (-3a\cos^2\theta\sin\theta)^2 \checkmark \qquad \left(\frac{dy}{d\theta}\right)^2 = (3a\sin^2\theta\cos\theta)^2 \checkmark$$

$$\begin{aligned}\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 &= (-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2 \checkmark\\ &= 9a^2\cos^4\theta\sin^2\theta + 9a^2\sin^4\theta\cos^2\theta\\ &= 9a^2\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)\\ &= 9a^2\cos^2\theta\sin^2\theta \checkmark\end{aligned}$$

$$\text{A} = 2\pi\overset{\checkmark}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}} y\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta$$

Incorrect limits: max 11 marks

✓

$$= 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\sin^3\theta\sqrt{9a^2\cos^2\theta\sin^2\theta}\,d\theta \checkmark$$

$$= 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\sin^3\theta.3a\cos\theta\sin\theta\ d\theta \checkmark$$

$$= 6a^2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4\theta.\cos\theta\ d\theta \checkmark$$

✓

$$= 6a^2\pi\left[\frac{(\sin\theta)^5}{5}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 6a^2\pi\left[\frac{(\sin\frac{\pi}{2})^5}{5} - \frac{(\sin-\frac{\pi}{2})^5}{5}\right] \checkmark$$

$= \dfrac{12}{5}a\pi$ units² or $2{,}4a\pi$ or $7{,}54a$ units² ✓ (14)

[24]

200 ÷ 2

TOTAL: 100