higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 November 2016 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE

MATHEMATICS N6

TIME: 3 HOURS

MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Use only BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = \tan(x^3y^2) + \text{cosec}(xy^2)$ calculate the following:

1.1.1 $\frac{\partial z}{\partial x}$ (2)

1.1.2 $\frac{\partial z}{\partial y}$ (1)

1.2 The parametric equations of a function are given as $x = t^2$ and $y = 2t^5$.

Calculate the value of $\frac{d^2y}{dx^2}$ (3)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \arcsin 3x$ (2)

2.2 $y = \frac{2}{\sec^4 2x}$ (4)

2.3 $y = \frac{1}{4x^2 + 12x + 24}$ (4)

2.4 $y = \text{cosec}^5 4x.\cos^3 4x$ (4)

2.5 $y = x^3.\sin 2x$ (4)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{3x^2 - 3x + 1}{(2x+1)^2(x-1)}\,dx$ (6)

3.2 $\int \frac{x^4 + x^2 - 2}{x^3 + x}\,dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of

$t\frac{dy}{dt} - 2y = t^2 - t + 1$, if $t = 1$ when $y = -\frac{1}{2}$. (5)

4.2 Calculate the particular solution of

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 6e^{\frac{x}{2}}$, if $y = 1$ when $x = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$. (7)

[12]

QUESTION 5

5.1 5.1.1 Make a neat sketch of the graph $y = 2\ln x$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, the line $y = 2$, the x-axis and the y-axis rotates about the x-axis. (2)

5.1.2 Calculate the volume generated if the area described in QUESTION 5.1.1 rotates about the x-axis. (5)

5.2 5.2.1 Calculate the points of intersection of the two curves $y = x + 6$ and $xy = -8$. Make a neat sketch of the two curves and show the area bounded by the curves in the second quadrant. Show the representative strip/element PARALLEL to the x-axis that you will use to calculate the area bounded by the curves. (3)

5.2.2 Calculate the area bounded by the two curves in the second quadrant described in QUESTION 5.2.1. (3)

5.2.3 Calculate the second moment of area when the area bounded by the two curves in the second quadrant described in QUESTION 5.2.1 is rotated about the x-axis. (4)

5.2.4 Express the answer in QUESTION 5.2.3 in terms of the area. (1)

5.3 5.3.1 Make a neat sketch of the graph $9x^2 + 4y^2 = 36$ and show the representative strip/element PERPENDICULAR to the x-axis that you will use to calculate the volume of the solid generated when the area bounded by the curve for $0 \le x \le 2$ rotates about the x-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x-axis. (5)

5.4 5.4.1 A water canal in the shape of a parabola is 5 m deep, 10 m wide at the top and full of water. A vertical retaining wall is placed in the canal with its top 1 m below the water surface.

Sketch the retaining wall and show the representative strip/element that you will use to calculate the area moment of the wall about the water level.

Calculate the relation between the two variables x and y. (4)

5.4.2 Calculate, by using integration, the area moment of the retaining wall about the water level in QUESTION 5.4.1. (3)

5.4.3 Calculate, by using integration, the second moment of area of the retaining wall described in QUESTION 5.4.1, about the water level as well as the depth of the centre of pressure on the retaining wall. (5)

[40]

QUESTION 6

6.1 Calculate the length of the curve $y = 2x^2 - 4$ between $x = 0$ and $x = 2$. (5)

6.2 Calculate the surface area of revolution generated by rotating the two curves, $x = a\cos^3\theta$ and $y = a\sin^3\theta$ about the x-axis, between $\theta = \frac{\pi}{2}$ and $\theta = -\frac{\pi}{2}$. (7)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n\, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a\, \mathrm{cosec}^2\, ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\mathrm{cosec}\, ax$	$-a\, \mathrm{cosec}\, ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2 dx \; ; V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi\int_a^b xydy$$

$$V_y = \pi\int_a^b x^2 dy \; ; V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\; dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\; dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$