higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

STRENGTH OF MATERIALS AND STRUCTURES N6

7 APRIL 2014

This marking guideline consists of 8 pages.

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QUESTION 1

1.1 $$at\ 75\ mm: a + \frac{b}{0{,}075^2} = 30 \times 10^6 \ldots\ldots\ldots (1)\ \surd$$

$$at\ 125\ mm: a + \frac{b}{0{,}125^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)\ \surd$$

$$(1) - (2): 177{,}778b - 64b = 30 \times 10^6$$

$$b = 263{,}672 \times 10^3\ \surd$$

$$a = -16{,}875 \times 10^6\ \surd$$

$$at\ 75\ mm:\ \sigma_{Hmax} = a - \frac{b}{0{,}075^2}$$

$$= -16{,}875 \times 10^6 - \frac{263{,}672 \times 10^3}{0{,}075^2}$$

$$\sigma_{Hmax} = -63{,}75MPa\ (tensile)\surd$$

$$at\ 125\ mm:\ \sigma_{Hmin} = a - \frac{b}{0{,}125^2}$$

$$= -16{,}875 \times 10^6 - \frac{263{,}675 \times 10^3}{0{,}125^2}$$

$$\sigma_{Hmin} = -33{,}75\ (tensile)\surd$$

(6)

1.2 $$\delta d_1 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$$

$$= \frac{0{,}075}{200 \times 10^9}(30 \times 10^6 - 0{,}29 \times 30 \times 10^6)\ \surd$$

$$\delta d_1 = 7{,}9875 \times 10^{-6}\ m\ \surd$$ (2)

1.3 $$\delta d_2 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$$

$$= \frac{0{,}075}{41 \times 10^9}(-63{,}75 \times 10^6 - 0{,}3 \times 30 \times 10^6)\ \surd$$

$$\delta d_2 = -133{,}0793 \times 10^{-6}\ m\ \surd$$ (2)

1.4 $\Delta d = \delta d_1 - \delta d_2$

$= 7{,}9875 \times 10^{-6} - (-133{,}0793 \times 10^{-6})$

$\Delta d = 0{,}141 \times 10^{-3}\ m\ \surd$ (1)

1.5 $F_\mu = \mu \times P \times \pi DL$

$= 0{,}2 \times 30 \times 10^6 \times \pi \times 0{,}075 \times 0{,}1\ \surd$

$F_\mu = 141{,}372\ kN\ \surd$ (2)
[13]

QUESTION 2

2.1 $M = FL + \frac{wL^2}{2}$

$= 10 \times 10^3 \times 6 + \frac{4 \times 10^3 \times 6^2}{2}\surd$

$M = 132\ kNm\surd$ (2)

2.2 $\mathrm{I} = \frac{bd^3}{12}$

$= \frac{0{,}2 \times 0{,}3^3}{12}$

$\mathrm{I} = 450 \times 10^{-6} m^4\surd$

$Y_{max} = \frac{FL^3}{3E\mathrm{I}} + \frac{wL^4}{8E\mathrm{I}}$

$= \frac{10 \times 10^3 \times 6^3}{3 \times 200 \times 10^9 \times 450 \times 10^{-6}} + \frac{4 \times 10^3 \times 6^4}{8 \times 200 \times 10^9 \times 450 \times 10^{-6}}\surd$

$= 8 \times 10^{-3} + 7{,}2 \times 10^{-3}$

$Y_{max} = 15{,}2 \times 10^{-3}\surd$ (3)

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2.3 $$\theta_{max} = \frac{FL^2}{2EI} + \frac{wL^3}{6EI}$$

$$= \frac{10 \times 10^3 \times 6^2}{2 \times 200 \times 10^9 \times 450 \times 10^{-6}} + \frac{4 \times 10^3 \times 6^3}{6 \times 200 \times 10^9 \times 450 \times 10^{-6}} \surd$$

$$= 2 \times 10^{-3} + 1{,}6 \times 10^{-3}$$

$$\theta_{max} = 3{,}6 \times 10^{-3} rad \surd$$ (2)

2.4 $$Z = \frac{M}{\sigma} = \frac{132 \times 10^3}{120 \times 10^6} = 1100 \times 10^{-6} m^3 \surd$$

$$from\ tables: I\ profile\ is\ 406 \times 178 \times 67{,}2\ kg/m \surd$$ (2)

2.5 $$\sigma = \frac{M}{Z} = \frac{132 \times 10^3 \surd}{1189 \times 10^{-6}} = 111{,}02\ MPa \surd$$ (2)

2.6 $$\frac{F_P L^3}{3EI} = \frac{FL^3}{3EI} + \frac{wL^4}{8EI}$$

$$divide\ by\ EI: \frac{F_{P\times} 6^3}{3} = \frac{10 \times 770 \times 6^3}{3} + \frac{4 \times 648 \times 6^4}{8} \surd$$

$$F_P = 19kN \surd$$ (2)
[13]

QUESTION 3

3.1

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}2^2}{4} = 31{,}416 \times 10^{-3} m^2 \surd$$

$$\mathrm{I} = \frac{\pi D^4}{64} = \frac{\pi \times 0{,}2^4}{64} = 78{,}54 \times 10^{-6} m^4 \surd$$

$$\sigma_d = \frac{F_1 + F_2}{A} = \frac{981 + F_2}{31{,}416 \times 10^{-3}} = 31{,}226 \times 10^3 + 31{,}831F_2 \surd$$

$$\sigma_b = \sigma_R - \sigma_d$$

$$= 45 \times 10^3 - (31{,}226 \times 10^3 + 31{,}831F_2) \surd$$

$$\sigma_b = 13{,}774 \times 10^3 - 31{,}831F_2 \surd$$

$$\sigma_b = \frac{F_2 \times e \times Y}{\mathrm{I}}$$

$$13{,}774 \times 10^3 - 31{,}831F_2 = \frac{F_2 \times 0{,}05 \times 0{,}1}{78{,}54 \times 10^{-6}} \surd$$

$$F_2 = 144{,}241\ N \surd$$

(7)

3.2

$$\sigma_R = \sigma_d - \sigma_b$$

$$= \frac{F_1 + F_2}{A} - \frac{F_2 \times e \times Y}{\mathrm{I}}$$

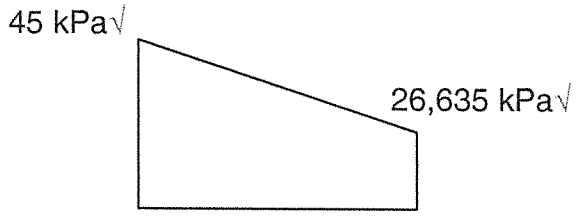
$$= \frac{981 + 144{,}241 \surd}{31{,}416 \times 10^{-3}} - \frac{144{,}241 \times 0{,}05 \times 0{,}1}{78{,}54 \times 10^{-6}} \surd$$

$$= 35{,}816 \times 10^3 - 9{,}183 \times 10^3 \surd$$

$$\sigma_R = 26{,}635 kPa \surd$$

(4)

3.3



(2)
[13]

QUESTION 4

4.1 $W_1 = \rho g A l = 2500 \times 9{,}81 \times 1 \times 3{,}2 \times 1 = 78{,}48\ kN\ \surd$

$W_2 = \rho g A l = 2500 \times 9{,}81 \times 0{,}5 \times 2 \times 3{,}2 \times 1 = 78{,}48\ kN\ \surd$

$V = W_1 + W_2 = 156{,}96\ kN\ \surd$ (3)

4.2 $C_\mu = \dfrac{1 - \sin 28}{1 + \sin 28} = 0{,}361\ \surd$

$F_g = \dfrac{\rho g h^2 C_\mu}{2} = \dfrac{1600 \times 9{,}81 \times 3{,}2^2 \times 0{,}361}{2} = 29{,}011\ kN\ \surd$ (2)

4.3 $F_p \times \dfrac{h}{2} + F_g \times \dfrac{h}{3} + W_1 x_1 + W_2 x_2 = V \times x_R$

$F_p \times 1{,}6 + 29{,}011 \times 1{,}07 + 78{,}48 \times 1{,}7 + 78{,}48 \times 0{,}5 = 156{,}96 \times 2$

$1{,}6F_p\surd + 30{,}945\surd + 130{,}8\surd + 39{,}24\surd = 313{,}92\surd$

$F_p = 68{,}88\surd$ (6)

4.4 $p = \dfrac{F_p}{C_\mu \times h} = \dfrac{70{,}594\ \surd}{0{,}361 \times 3{,}2} = 59{,}633\ kPa\ \surd$ (2)

[13]

QUESTION 5

5.1 $M_t = \sigma \times Z \times n = 100 \times 10^6 \times 1462 \times 10^{-6} \times 4 = 584{,}8\ kNm\ \surd$

$M_b = \sigma \times Z \times n = 100 \times 10^6 \times 415{,}8 \times 10^{-6} \times 16 = 665{,}28\ kNm\ \surd$ (2)

5.2 $M = \dfrac{W_c(L - l)^2}{8L}$

$584{,}8 \times 10^3 = \dfrac{W_c(3{,}6 - 1)^2}{8 \times 3{,}6}\surd$

$W_c = 2491{,}456\ kN\ \surd$

$W_T = W_c + W_F = 2491{,}456 \times 10^3 + 200 \times 10^3\surd = 2691{,}456kN\ \surd$ (4)

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5.3 $$p = \frac{W_T}{A} = \frac{2691{,}456 \times 10^3 \surd}{3{,}6 \times 3{,}6} = 207{,}674\ kPa\ \surd$$ (2)

5.4 $$F_s = \frac{W_c(L-l)}{2L} = \frac{2491{,}456 \times 10^3(3{,}6-1)}{2 \times 3{,}6} = 899{,}692\ kN\ \surd$$

$$\tau_t = \frac{F_s}{nht_1} = \frac{899{,}692 \times 10^3 \surd}{4 \times 0{,}4572 \times 0{,}0091} = 54{,}1\ MPa\ \surd$$

$$\tau_b = \frac{F_s}{nht_1} = \frac{899{,}692 \times 10^3 \surd}{16 \times 0{,}3127 \times 0{,}0066} = 27{,}246\ MPa\ \surd$$ (5)

[13]

QUESTION 6

6.1 $$\frac{bn^2}{2} = mA_s(d-n)$$

$$\frac{200n^2}{2} = 15 \times 1{,}257 \times 10^3(300-n)\surd$$

$$5{,}305 \times 10^{-3}n^2 = 300 - n$$

$$5{,}305 \times 10^{-3}n^2 + n - 300 = 0\ \surd$$

$$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(5{,}305 \times 10^{-3})(-300)}}{2 \times 5{,}305 \times 10^{-3}}\surd$$

$$n = 161{,}5\ mm\ \surd$$ (4)

6.2 $$M_s = \sigma_s A_s\left(d - \frac{n}{3}\right) = 140 \times 10^6 \times 1{,}257 \times 10^{-3} \times 0{,}246\surd = 43{,}279 kNm\ \surd$$ (2)

6.3 $$M_c = \frac{\sigma_c A_c}{2}\left(d - \frac{n}{3}\right) = \frac{5{,}2 \times 10^6 \times 0{,}2 \times 0{,}162}{2} \times 0{,}246\surd = 20{,}723 kNm\ \surd$$ (2)

6.4 $M = 20{,}723\ kNm\ (maximum\ allowed)\surd$ (1)

6.5 $$M_s = \sigma_s A_s \left(d - \frac{n}{3}\right)$$

$$20{,}723 \times 10^3 = \sigma_s \times 1{,}257 \times 10^{-3} \left(0{,}3 - \frac{0{,}162}{3}\right) \surd$$

$$\sigma_s = 67{,}0167 MPa \surd$$ (2)

6.6 $$w = \frac{8M}{L^2} = \frac{8 \times 20{,}723 \times 10^3 \surd}{6^2} = 4{,}605\ kN/m\ \surd$$ (2)
[13]

QUESTION 7

7.1 $$F_v = wx_2^2 = 6 \times 10^3 \times 77^2 = 462\ kN \surd$$

$$F_H = \sqrt{F_T^2 - F_v^2} = \sqrt{1740^2 - 462^2} \surd = 1677{,}544 kN\ \surd$$ (3)

7.2 $$d = \frac{wx_1^2}{2F_H} = \frac{6 \times 10^3 \times 63^2 \surd}{2 \times 1677{,}544 \times 10^3} = 7{,}1\ m \surd$$ (2)

7.3 $$l_1 = x_1 + \frac{2d^2}{3x_1} = 63 + \frac{2 \times 7{,}1^2}{3 \times 63} = 63{,}533\ m\ \surd$$

$$l_2 = x_2 + \frac{2(d+h)^2}{3x_2} = 71 + \frac{2 \times (7{,}1 + 3{,}5)^2}{3 \times 71} = 72{,}1\ m\ \surd$$

$$l_T = l_1 + l_2 = 63{,}533 + 77{,}973 = 135{,}6\ m\ \surd$$ (3)

7.4 $$F_{va} = R - F_v = 1062 - 462 = 600\ kN \surd$$

$$\theta = \cos^{-1}\left(\frac{F_{va}}{F_T}\right) = \cos^{-1}\left(\frac{600}{1740}\right) = 69{,}8°\ \surd$$ (2)

7.5 $$F_{Ha} = F_T \sin\theta = 1740 \sin 69{,}8 = 1633{,}279 kN\ \surd$$

$$M = (F_H - F_{Ha})H = (1677{,}544 - 1633{,}279)20\ \surd = 885{,}3 kNm\ \surd$$ (3)
[13]

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QUESTION 8

8.1 $T = 1{,}14 \times T_a = 1{,}14 \times 35 = 39{,}9\ kNm\ \surd$ (1)

8.2 $T_e = \sqrt{M^2 + T^2} = \sqrt{25^2 + 39{,}9^2}\surd = 47{,}1 kNm\ \surd$ (2)

8.3 $M_e = 0{,}5\left(M + \sqrt{M^2 + T^2}\right) = 0{,}5(25 + 47{,}1)\surd = 36{,}05 kNm\ \surd$ (2)

8.4 $\tau = \dfrac{16DT_e}{\pi(D^4 - d^4)} = \dfrac{16 \times 0{,}18 \times 47{,}1 \times 10^3\surd}{\pi(0{,}18^4 - 0{,}12^4)} = 51{,}256\ MPa\ \surd$ (2)

8.5 $\sigma = \dfrac{32DM_e}{\pi(D^4 - d^4)} = \dfrac{32 \times 0{,}18 \times 36{,}05 \times 10^3\surd}{\pi(0{,}18^4 - 0{,}12^4)} = 78{,}462\ MPa\ \surd$ (2)

[9]

TOTAL: 100

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