higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

11 APRIL 2018

This marking guideline consists of 9 pages.

QUESTION 1

1.1 $\sigma_H = \varepsilon \times E = 168{,}75 \times 10^{-6} \times 200 \times 10^9$✓ $= 33{,}75\ MPa$ ✓

$at\ 125\ mm: a - \dfrac{b}{0{,}125^2} = -33{,}75 \times 10^6 \ldots\ldots\ldots (1)$ ✓

$at\ 125\ mm: a + \dfrac{b}{0{,}125^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$ ✓

$(1) + (2):\quad 2a = -33{,}75 \times 10^6$

$a = -16{,}875 \times 10^6$ ✓

$b = 263{,}672 \times 10^3$ ✓

$at\ 75\ mm:\ \sigma_{Hmax} = a - \dfrac{b}{0{,}075^2}$

$= -16{,}875 \times 10^6 - \dfrac{263{,}672 \times 10^3}{0{,}075^2}$

$\sigma_{Hmax} = -63{,}75\ MPa\ (tensile)$ ✓ (7)

1.2 $at\ 75\ mm:\ \sigma_R = a + \dfrac{b}{0{,}075^2}$

$= -16{,}875 \times 10^6 + \dfrac{263{,}672 \times 10^3}{0{,}075^2}$

$\sigma_R = 30\ MPa$ ✓

$\sigma_H = \sigma_R = 30\ MPa$ ✓ (2)

1.3 $\delta d_1 = \dfrac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \dfrac{0{,}075}{200 \times 10^9}(30 \times 10^6 - 0{,}29 \times 30 \times 10^6)$

$\delta d_1 = 7{,}9875 \times 10^{-6}\ m$ ✓ (1)

1.4 $\delta d_2 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \frac{0{,}075}{200 \times 10^9}(-63{,}75 \times 10^6 - 0{,}29 \times 30 \times 10^6)$

$\delta d_2 = -27{,}16875 \times 10^{-6}\ m$ ✓ (1)

1.5 $\Delta d = \delta d_1 - \delta d_2$

$= 7{,}9875 \times 10^{-6} - (-27{,}16875 \times 10^{-6})$

$\Delta d = 35{,}156 \times 10^{-6}\ m$ ✓ (1)
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QUESTION 2

2.1 $\Delta = \frac{5wL^4}{384EI}$

$0{,}016 = \frac{5 \times 12 \times 10^3 \times 6^4}{384 \times 200 \times 10^9 \times I}$ ✓

$I = 63{,}281 \times 10^{-6}\ m^4$✓ $= 31{,}641 \times 10^{-6}\ per\ beam$ ✓

$Choose\ 200 \times 200 \times 24\ kg/m$ ✓ (4)

2.2 $\Delta = \frac{5wL^4}{384EI}$

$= \frac{5 \times 12 \times 10^3 \times 6^4}{384 \times 200 \times 10^9 \times 66{,}62 \times 10^{-6}}$ ✓

$\Delta = 15{,}198\ mm$ ✓ (2)

2.3 $$M = \frac{wL^2}{8} = \frac{12 \times 10^3 \times 6^2}{8} = 54\ kNm\ \checkmark$$

$$\sigma_{max} = \frac{MY}{I} = \frac{54 \times 10^3 \times 0{,}1416}{66{,}62 \times 10^{-6}} = 114{,}776\ MPa\ \checkmark\ (tensile)\ \checkmark$$

$$\sigma_{min} = \frac{MY}{I} = \frac{54 \times 10^3 \times 0{,}0584}{66{,}62 \times 10^{-6}} = 47{,}337\ MPa\ \checkmark\ (compressive)\ \checkmark$$ (5)

2.4 $$\Delta = \frac{FL^3}{48EI}$$

$$(15{,}198 - 5) \times 10^{-3}\checkmark = \frac{F \times 6^3}{48 \times 200 \times 10^9 \times 66{,}62 \times 10^{-6}}$$

$$F = 30{,}196\ kN\ \checkmark$$ (2)

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QUESTION 3

3.1 $$\sigma_D = \frac{F}{A} = \frac{2 \times 10^6}{2 \times 1} = 1\ MPa\ \checkmark$$ (1)

3.2 $$I_{XX} = \frac{1 \times 2^3}{12} = 0{,}667 \times 10^{-6}\ m^4\ \checkmark$$

$$I_{YY} = \frac{2 \times 1^3}{12} = 0{,}167 \times 10^{-6}\ m^4\ \checkmark$$

$$\sigma_{XX} = \frac{FeY}{I} = \frac{2 \times 10^6 \times 0{,}5 \times 1}{0{,}667 \times 10^{-6}} = 1{,}5\ MPa\ \checkmark$$

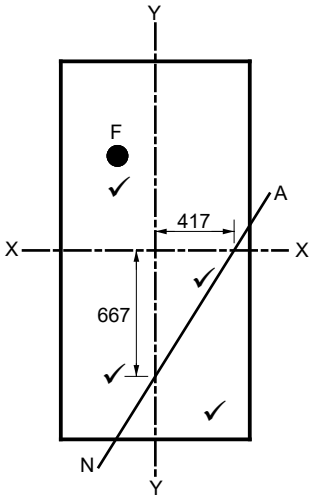
$$\sigma_{YY} = \frac{FeX}{I} = \frac{2 \times 10^6 \times 0{,}2 \times 0{,}5}{0{,}667 \times 10^{-6}} = 1{,}2\ MPa\ \checkmark$$ (4)

3.3 $$\sigma_{max} = \sigma_D + \sigma_{XX} + \sigma_{YY} = 1 + 1{,}5 + 1.2 = 3{,}7\ MPa\ \ (compressive)\ \checkmark$$

$$\sigma_{min} = \sigma_D - \sigma_{XX} - \sigma_{YY} = 1 - 1{,}5 - 1.2 = -1{,}7\ MPa\ \ (tensile)\ \checkmark$$ (2)

3.4 $Y = \frac{I_{XX}}{A \times e} = \frac{0,667}{2 \times 0,5} = 0,667\ m\ (from\ the\ XX - axis)$ ✓

$X = \frac{I_{YY}}{A \times e} = \frac{0,167}{2 \times 0,2} = 0,417\ m\ (from\ the\ YY - axis)$ ✓



(6)
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QUESTION 4

4.1 $W_1 = \rho g A l = 2200 \times 9,81 \times 2 \times 6 \times 1 = 258,984\ kN$ ✓

$W_2 = \rho g A l = 2200 \times 9,81 \times 0.5 \times 1 \times 6 \times 1 = 64,746\ kN$ ✓

$V = W_1 + W_2 = 323,73\ kN$ ✓ (3)

4.2 $W - M = W_1 x_1 + W_2 x_2$

$= 258,984 \times 2 + 64,746 \times 0,667$ ✓

$W - M = 561,132\ kNm$ ✓ (2)

4.3 $\sigma_{max} = \frac{\sigma_{ult}}{FOS} = \frac{621}{3} = 207\ kPa$ ✓

$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2}$

$207 = \frac{323,73}{3} + \frac{6 \times 323,73 \times e}{3^2}$ ✓

$e = 0,459\ m$ ✓

$x = 0,5B - e = 1,5 - 0,459 = 1,041\ m$ ✓ (4)

4.4 $V \times x_R + F_M = W_M \ (taking\ moments\ about\ the\ toe)$

$323{,}73 \times 1{,}041 + F_M = 561{,}132$ ✓

$F_M = 224{,}172\ kNm$ ✓

$$F_M = \frac{\rho g h^2}{2} \times \frac{h}{3}$$

$$224{,}172 = \frac{10^3 \times 9{,}81 \times h^3}{6}$$ ✓

$h = 5{,}156\ m$ ✓ (4)

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QUESTION 5

5.1 $M = \sigma \times Z \times n = 180 \times 1959 \times 3$ ✓ $= 1057{,}86\ kNm$ ✓ (2)

5.2 $$M = \frac{W_c(L-l)^2}{8L}$$

$$1057{,}86 = \frac{W_c(3-0{,}8)^2}{8 \times 3}$$ ✓

$W_c = 5245{,}587\ kN$ ✓

$$A_c = \frac{W_c}{\sigma_c} = \frac{5245{,}587 \times 10^3}{140 \times 10^6} = 37{,}468 \times 10^{-3}\ m^2$$ ✓

$Select\ 356 \times 406 \times 340\ kg/m$ ✓

$$\sigma = \frac{W}{A} = \frac{5245{,}587 \times 10^3}{43{,}27 \times 10^{-3}} = 121{,}229\ MPa$$ ✓ (5)

5.3 $l = 3 \times 192{,}5 + 2 \times 75 = 728{,}4\ mm$ ✓

$$n = \frac{M}{\sigma \times Z} = \frac{1057{,}86 \times 10^3}{180 \times 10^6 \times 562{,}9 \times 10^{-6}} = 10{,}44$$ ✓ $say\ 11$ ✓ (3)

5.4 $$\sigma = \frac{M}{Z \times n} = \frac{1057{,}86 \times 10^3}{562{,}9 \times 10^{-6} \times 11} = 170{,}846\ MPa$$ ✓ (1)

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QUESTION 6

6.1 $$\frac{en^2}{2} = mA_s(d-n)$$

$$\frac{0{,}2 \times n^2}{2} = 15 \times 800 \times 10^{-6}(0{,}8-n)\checkmark$$

$$100 \times 10^{-3}\, n^2 = 12 \times 10^{-3} - 9{,}6 \times 10^{-3}\, n \ \checkmark$$

$$n = 301{,}72\ mm \ \checkmark \quad (3)$$

6.2 $$l_a = d - \frac{n}{3} = 0{,}18 - \frac{0{,}0968}{3} = 0{,}148\ m$$

$$M = 0{,}5\sigma_c A_c l_a = 0{,}5 \times 7 \times 10^6 \times 0{,}2 \times 0{,}302 \times 0{,}699 \ = 147{,}722\ kNm \checkmark$$

$$M = \sigma_s A_s l_a = 140 \times 10^6 \times 800 \times 10^{-6} \times 0{,}699 \ = 78{,}336\ kNm \ \checkmark$$

$$Therefore\ maximum\ M = 78{,}336\ kNm \ \checkmark \quad (3)$$

6.3 $$M_c = 0{,}5\sigma_c A_c l_a$$

$$78{,}336 \times 10^3 = 0{,}5 \times \sigma_c \times 450 \times 0{,}2 \times 0{,}302 \times 0{,}699 \ \checkmark$$

$$\sigma_c = 3{,}712\ MPa \ \checkmark \quad (2)$$

6.4 $$M_c = 0{,}5\sigma_c A_c \times \frac{2}{3}n$$

$$= 0{,}5 \times 3{,}712 \times 10^6 \times 0{,}2 \times 0{,}302 \times \frac{2}{3} \times 0{,}302 \ \checkmark$$

$$M_c = 22{,}528\ kNm \ \checkmark$$

$$M_s = \sigma_s A_s (d-n)$$

$$= 140 \times 10^6 \times 800 \times 10^{-6}(0{,}8-0{,}302) \ \checkmark$$

$$M_s = 55{,}807\ kNm \ \checkmark \quad (4)$$

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QUESTION 7

7.1 $y_0 = \frac{F_H}{w} = \frac{30 \times 10^3}{20} = 1500\ m$ ✓

$F_{T2} = \frac{F_H}{\cos\theta} = \frac{30 \times 10^3}{15} = 31{,}0583\ kN$ ✓

$l_2 = y_0 \tan\theta = 1500 \times \tan 15 = 401{,}924\ m$ ✓

$l_1 = l_T - l_2 = 650 - 401{,}924 = 248{,}0762\ m$ ✓

$y_1 = \sqrt{l_1^2 + y_0^2} = \sqrt{248{,}0762^2 + 1500^2} = 1520{,}376\ m$ ✓

$F_{T1} = wy_1 = 20 \times 1520{,}376 = 30{,}408\ kN$ ✓ (6)

7.2 $F_{V2} = F_H \tan\theta = 30 \times 10^3 \times \tan 15 = 8{,}0385\ kN$ ✓

$F_{Va} = F_{T2} \sin\alpha = 31{,}0583 \times 10^3 \times \sin 60 = 26{,}897\ kN$ ✓

$R_V = F_{V2} + F_{Va} = 8{,}0385 + 26{,}897 = 34{,}936\ kN$ ✓

$F_{Ha} = F_{T2} \cos\alpha = 31{,}0583 \times 10^3 \cos 60 = 15{,}529\ kN$ ✓

$R_H = F_{Hc} - F_{Ha} = 30 \times 10^3 - 15{,}529 \times 10^3 = 14{,}471\ kN$ ✓

$R = \sqrt{R_H^2 + R_V^2} = \sqrt{14{,}471^2 + 34{,}936^2} = 37{,}814\ kN$ ✓ (6)

7.3 $M = R_H \times H = 14{,}471 \times 20$ ✓ $= 289{,}417\ kNm$ ✓ (2)

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QUESTION 8

8.1 $d = \sqrt[3]{\frac{16T_e}{\pi \times \tau}} = \sqrt[3]{\frac{16 \times 2 \times 10^3}{\pi \times 60 \times 10^6}}$ ✓ $= 55{,}371\ mm$ ✓

$d = \sqrt[3]{\frac{32M_e}{\pi \times \sigma}} = \sqrt[3]{\frac{32 \times 3 \times 10^3}{\pi \times 90 \times 10^6}}$ ✓ $= 69{,}763\ mm$ ✓

$Minimum\ diameter\ required = 69{,}763\ mm$ ✓
$This\ diameter\ will\ satisfy\ both\ stress\ limits$ ✓ (6)

8.2 $M_e = 0{,}5(M + T_e)$

$3 = 0{,}5(M + 2)$ ✓

$M = 4\ kNm$ ✓

$w = \frac{8M}{L^2} = \frac{8 \times 4}{4^2} = 2\ kN/m$ ✓ (3)

8.3 $\sigma = 90\ MPa$ ✓

$\tau = \frac{16T_e}{\pi d^3} = \frac{16 \times 2 \times 10^3}{\pi \times 0{,}0698^3}$ ✓ $= 30\ MPa$ ✓ (3)

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TOTAL: 100