higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

21 April 2021

This marking guideline consists of 8 pages.

QUESTION 1: THICK CYLINDERS

1.1 $\text{at } 100 \text{ mm: } a + \frac{b}{0{,}1^2} = 100 \times 10^6 \ldots\ldots\ldots (1)$✓

$\text{at } 200 \text{ mm: } a + \frac{b}{0{,}2^2} = 40 \times 10^6 \ldots\ldots\ldots\ldots (2)$✓

$(1) - (2): 100b - 25b = 60 \times 10^6$

$b = 800 \times 10^3$✓

$a = 20 \times 10^6$✓

$\text{at } 100 \text{ mm: } \sigma_H = a - \frac{b}{0{,}1^2}$

$= 20 \times 10^6 - \frac{800 \times 10^3}{0{,}1^2}$

$\sigma_H = -60 \text{ MPa (tensile)}$✓

$\text{at } 200 \text{ mm: } \sigma_H = a - \frac{b}{0{,}2^2}$

$= 20 \times 10^6 - \frac{800 \times 10^3}{0{,}2^2}$

$\sigma_H = 0$✓ (6)

1.2 $\delta d_1 = \frac{d}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \frac{0{,}1}{200 \times 10^9}(-60 \times 10^6 - 0{,}3 \times 100 \times 10^6)$

$\delta d_1 = -45 \times 10^{-6} \text{ m (increase)}$✓

$d = 100 + 0{,}045 = 100{,}045 \text{ mm}$✓ (2)

1.3 $\delta d_2 = \frac{D}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \frac{0{,}2}{200 \times 10^9}(0 - 0{,}3 \times 40 \times 10^6)$

$\delta d_2 = 12 \times 10^{-6} \text{ m (increase)}$✓

$D = 200 + 0{,}012 = 200{,}012 \text{ mm}$✓ (2)

1.4 $t = \frac{D-d}{2} = \frac{200{,}012 - 100{,}045}{2} = 49{,}9835 \text{ mm}$✓ (1)

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QUESTION 2: TENSION IN CABLES

2.1 $F_T = \sigma_s A_s + \sigma_a A_a$

$= 42 \times 50 + 14 \times 45$✓

$F_T = 8\,400 \text{ N}$✓ (2)

2.2 $y_2 = \frac{F_T}{w} = \frac{8\,400}{60} = 140 \text{ m}$✓

$y_0 = \sqrt{y_2^2 - \ell_2^2} = \sqrt{140^2 - 88^2} = 108{,}885 \text{ m}$✓

$y_1 = \sqrt{y_0^2 + \ell_1^2} = \sqrt{108{,}885^2 + 62^2} = 125{,}3 \text{ m}$✓

$h = y_2 - y_1 = 14{,}7 \text{ m}$✓ (4)

2.3 $x_2 = y_0 ln\left(\frac{y_2 + \ell_2}{y_0}\right) = 108{,}885 \times ln\left(\frac{140 + 88}{108{,}885}\right) = 80{,}472 \text{ m}$✓ (1)

2.4 $y_3 = \frac{F_3}{w} = \frac{7\,800}{60} = 130 \text{ m}$✓

$\ell_3 = \sqrt{y_3^2 - y_0^2} = \sqrt{130^2 - 108{,}885^2} = 71{,}0211 \text{ m}$✓

$x_3 = y_0 ln\left(\frac{y_3 + \ell_3}{y_0}\right) = 108{,}885 \times ln\left(\frac{130 + 71{,}0211}{108{,}885}\right) = 66{,}759 \text{ m (from TP)}$✓

$x = x_2 - x_3 = 80{,}472 - 66{,}759 = 13{,}713 \text{ m (from highest support)}$✓ (4)

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QUESTION 3: BENDING AND DEFLECTION OF BEAMSs

3.1 $I_{XX} = 2 \times I_{xx} = 2 \times 19{,}11 \times 10^{-6} = 38{,}22 \times 10^{-6}\ \text{m}^4$✓

$I_{YY} = 2 \times \left(I_{yy} + Ar^2\right)$

$= 2 \times (1{,}478 \times 10^{-6} + 3{,}218 \times 10^{-3} \times 0{,}0499^2)$✓

$I_{YY} = 18{,}982 \times 10^{-6}\ \text{m}^4$✓ (3)

3.2 Consider the stress limit:

$M = \sigma z = 80 \times 10^6 \times 2 \times 191{,}1 \times 10^6 = 30{,}576\ \text{kNm}$✓

$$M = \frac{WL}{4} + \frac{wL^2}{8}$$

$$30\,576 = \frac{W \times 4}{4} + \frac{496{,}386 \times 4^2}{8}$$

$W = 29{,}583\ \text{kN}$✓

Consider the deflection limit:

$$\Delta = \frac{WL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$10 \times 10^{-3} = \frac{W \times 4^3}{48 \times 200 \times 38{,}22 \times 10^3} + \frac{5 \times 496{,}386 \times 4^4}{384 \times 200 \times 38{,}22 \times 10^3}$$ ✓

$W = 56{,}089\ \text{kN}$✓

⇒ Maximum allowed load is 29,583 kN✓ (5)

3.3 $\sigma = 80$ MPa as this was the limiting factor✓

$$\Delta = \frac{29\,583 \times 4^3}{48 \times 200 \times 38{,}22 \times 10^3} + \frac{5 \times 496{,}386 \times 4^4}{384 \times 200 \times 38{,}22 \times 10^3} = 5{,}377\ \text{mm}$$✓ (2)

3.4

$$\Delta = \frac{FL^3}{48EI}$$

$$(5{,}377 - 2) \times 10^{-3}\text{✓} = \frac{F \times 4^3}{48 \times 200 \times 38{,}22 \times 10^3}$$

$F = 19{,}359\ \text{kN}$✓ (2)

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QUESTION 4: DIRECT AND BENDING STRESSES

4.1 $$Y_t = \frac{a_1y_1 + a_2y_2}{a_T} = \frac{8 \times 10^{-3} \times 0{,}02 + 8 \times 10^{-3} \times 0{,}12}{16 \times 10^{-3}}$$ ✓✓

$Y_t = 0{,}07$ m✓ and $Y_c = 0{,}13$ m (3)

4.2 $$I_1 = \frac{bd^3}{12} + Ah^2 = \frac{0{,}2 \times 0{,}04^3}{12} + 8 \times 10^{-3} \times 0{,}05^2 = 21{,}1 \times 10^{-6}\ \text{m}^4$$ ✓

$$I_2 = \frac{bd^3}{12} + Ah^2 = \frac{0{,}05 \times 0{,}16^3}{12} + 8 \times 10^{-3} \times 0{,}05^2 = 37{,}1 \times 10^{-6}\ \text{m}^4$$ ✓

$I_{XX} = I_1 + I_2 = 21{,}1 \times 10^{-6} + 37{,}1 \times 10^{-6} = 58{,}2 \times 10^{-6}$ m^4✓ (3)

4.3 $$\sigma_d = \frac{F}{A} = \frac{40 \times 10^3}{16 \times 10^{-3}} = 2{,}5\ \text{MPa}$$ ✓ (tensile)✓ (2)

4.4 $$\sigma_{bt} = \frac{FeY_t}{I} = \frac{40 \times 10^3 \times 0{,}37 \times 0{,}07}{58{,}2 \times 10^{-6}} = 22{,}251\ \text{MPa (tensile)}$$ ✓

$$\sigma_{bc} = \frac{FeY_c}{I} = \frac{40 \times 10^3 \times 0{,}37 \times 0{,}13}{58{,}2 \times 10^{-6}} = 41{,}323\ \text{MPa (compressive)}$$ ✓

$\sigma_{Rt} = \sigma_d + \sigma_{bt} = 3{,}125 + 22{,}251 = 25{,}376$ MPa✓ (tensile)✓

$\sigma_{Rc} = \sigma_d - \sigma_{bc} = 3{,}125 - 41{,}323 = 38{,}198$ MPa✓ (compressive)✓ (6)

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QUESTION 5: RETAINING WALLS

5.1 $$F_w = \frac{\rho g h^2}{2} = \frac{1\,000 \times 9{,}81 \times 6^2}{2} = 176{,}58\ \text{kN}$$ ✓

$W_1 = \rho g A l = 2\,100 \times 9{,}81 \times 0{,}5 \times 1 \times 6 = 61{,}803$ kN✓

$W_2 = \rho g A l = 2\,100 \times 9{,}81 \times 2 \times 6 = 247{,}212$ kN✓

$V = W_1 + W_2 = 61{,}803 + 247{,}212 = 309{,}015$ kN✓ (4)

5.2 $$F{\sim}M = F_w \times \frac{h}{3} = 176{,}58 \times 2 = 353{,}16\ \text{kNm}$$ ✓

$W{\sim}M = W_1x_1 + W_2x_2 = 61{,}803 \times 0{,}667 + 247{,}212 \times 2$✓ $= 535{,}626$ kNm✓ (3)

5.3 $FOS = \frac{W{\sim}M}{F{\sim}M} = \frac{535{,}626}{353{,}16} = 1{,}517$ ✓ (not safe , should be ≥ 2)✓ (2)

5.4 $Vx + F{\sim}M = W{\sim}M$

$309{,}015x + 353{,}16 = 535{,}626$✓

$x = 0{,}59$ m from toe✓

There will be tension because $x < \frac{B}{3}$✓ (falls outside middle third) (3)

5.5 $e = 0{,}5B - x = 1{,}5 - 0{,}59 = 0{,}91$ m✓

$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} = \frac{309{,}015}{3} + \frac{6 \times 309{,}015 \times 0{,}91}{3^2} = 290{,}474$ kPa✓

$\sigma_{ult} = 3 \times \sigma_{max} = 871{,}422$ kPa✓ (3)

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QUESTION 6: FOUNDATIONS

6.1 $W_T = 340 \times 9{,}81 \times 4 + 50 \times 10^3 = 63{,}342$ kN✓

$L = \sqrt{\frac{W_T}{p_g}} = \sqrt{\frac{63{,}342}{30}} = 1{,}453$ m✓ (2)

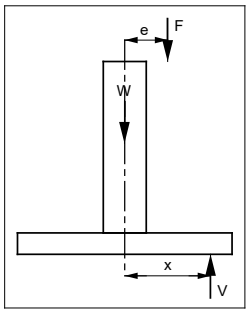
6.2 $\sigma_d = \frac{W_T}{A_c} = \frac{63{,}342 \times 10^3}{43{,}27 \times 10^{-3}}$✓ $= 1{,}464$ MPa✓ (2)

6.3 $e = 100 + \frac{h}{2} = 100 + \frac{406{,}4}{2} = 303{,}2$ mm✓

Take moments about centre: $V \times x = F \times e$✓

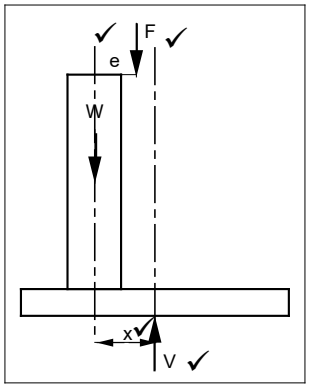
$63{,}342 \times x = 50 \times 0{,}3032$✓

$x = 239{,}337$ mm✓



(4)

6.4



(4)

[12]

QUESTION 7: REINFORCED CONCRETE

7.1

$$\frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$$

$$\frac{140}{7} = \frac{15(d-n)}{n}$$

$d = 2{,}333n \ldots\ldots\ldots (1)$✓

$$M = 0{,}5\sigma_c A_c \left(d - \frac{n}{3}\right)$$

$174 \times 10^3 = 0{,}5 \times 7 \times 10^6 \times 0{,}4 \times n(d - 0{,}333n) \ldots\ldots\ldots (2)$✓

(1) into (2): $174 \times 10^3 = 0{,}5 \times 7 \times 10^6 \times 0{,}4 \times n(2{,}333n - 0{,}333n)$✓

$n = 0{,}249$ m✓

$d = 2{,}333n = 0{,}582$ m✓

$D = d + r + c = 582 + 10 + 30 = 622$ mm✓ (6)

7.2

$$\ell_a = d - \frac{n}{3} = 0{,}582 - \frac{0{,}249}{3} = 0{,}499 \text{ m}$$✓

$$A_s = \frac{M}{\sigma_s \times l_a} = \frac{174 \times 10^3}{140 \times 10^6 \times 0{,}499} = 2{,}493 \times 10^{-3} \text{ m}^2$$✓

$A_r = \pi r^2 = \pi \times 0{,}01^2 = 314{,}159 \times 10^{-6}$ m²✓

$$\text{number of rods} = \frac{A_s}{A_r} = \frac{2{,}493 \times 10^{-3}}{314{,}159 \times 10^{-6}} = 7{,}9 \text{ say } 8 \text{ rods}$$✓ (4)

7.3

$$M_c = 0{,}5\sigma_c A_c \times \frac{2}{3}n$$

$= 0{,}5 \times 7 \times 10^6 \times 0{,}4 \times 0{,}249 \times \frac{2}{3} \times 0{,}249$✓

$M_c = 58$ kNm✓

$$M_s = \sigma_s A_s (d-n)$$

$= 140 \times 10^6 \times 2{,}493 \times 10^{-3}(0{,}582 - 0{,}249)$✓

$M_s = 1\,167$ kNm✓ (4)

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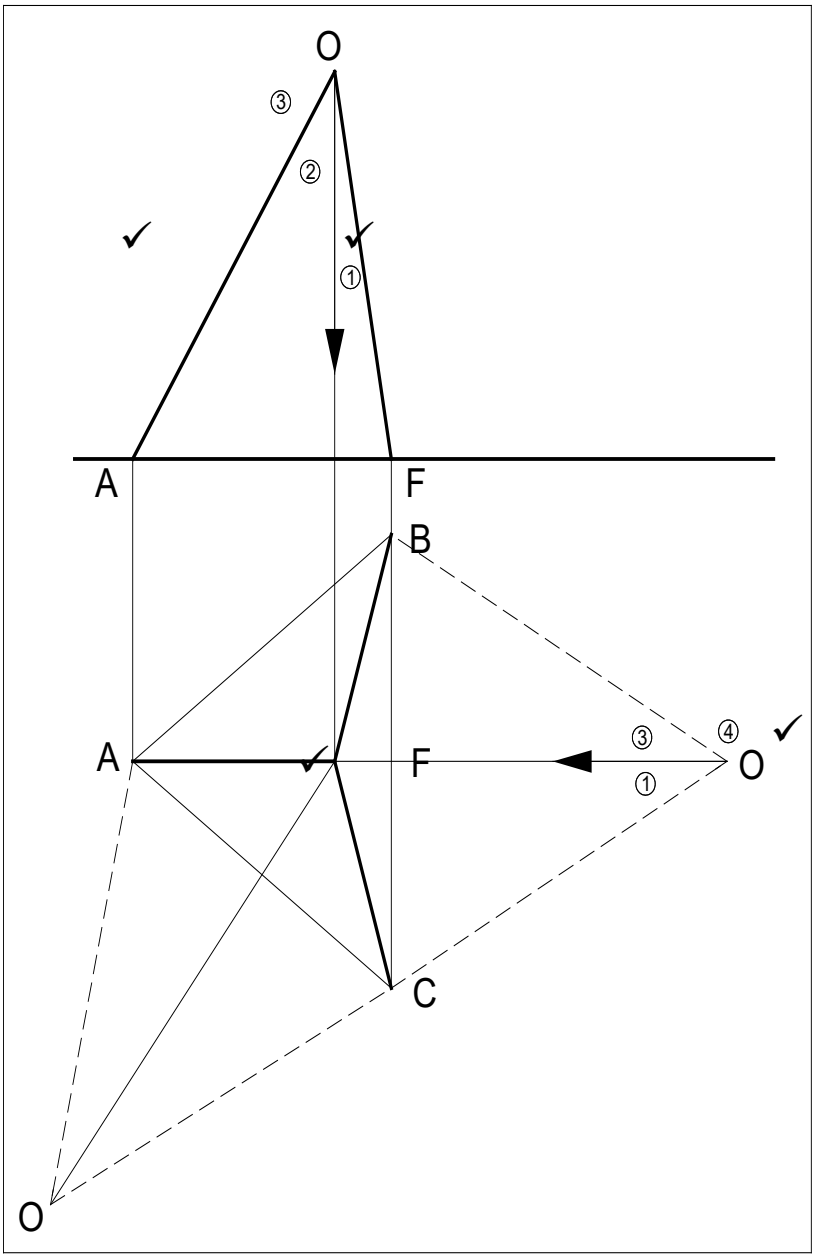
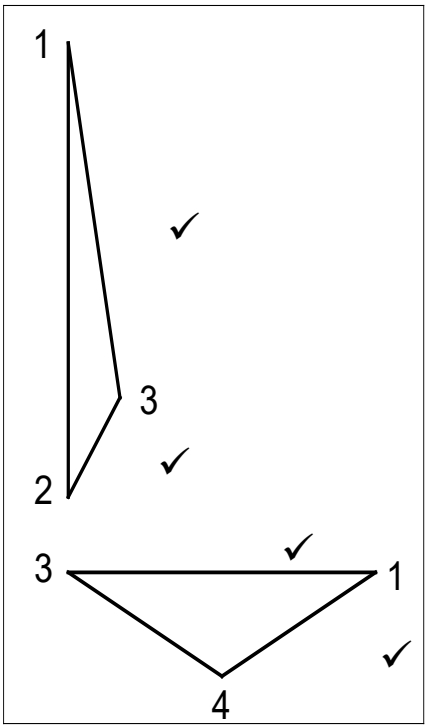
QUESTION 8: STRUCTURAL FRAMEWORKS

8.1



8.2



8.1 See sketch (4)

8.2 See sketch (4)

8.3

MEMBER	MAGNITUDE	NATURE
OA (2-3)	7,7 kN√	Strut√
OB (3-4)	13,7 kN√	Strut√
OC (1-4)	13,7 kN√	Strut√

(3)
[11]

TOTAL: 100