higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

AUGUST EXAMINATION

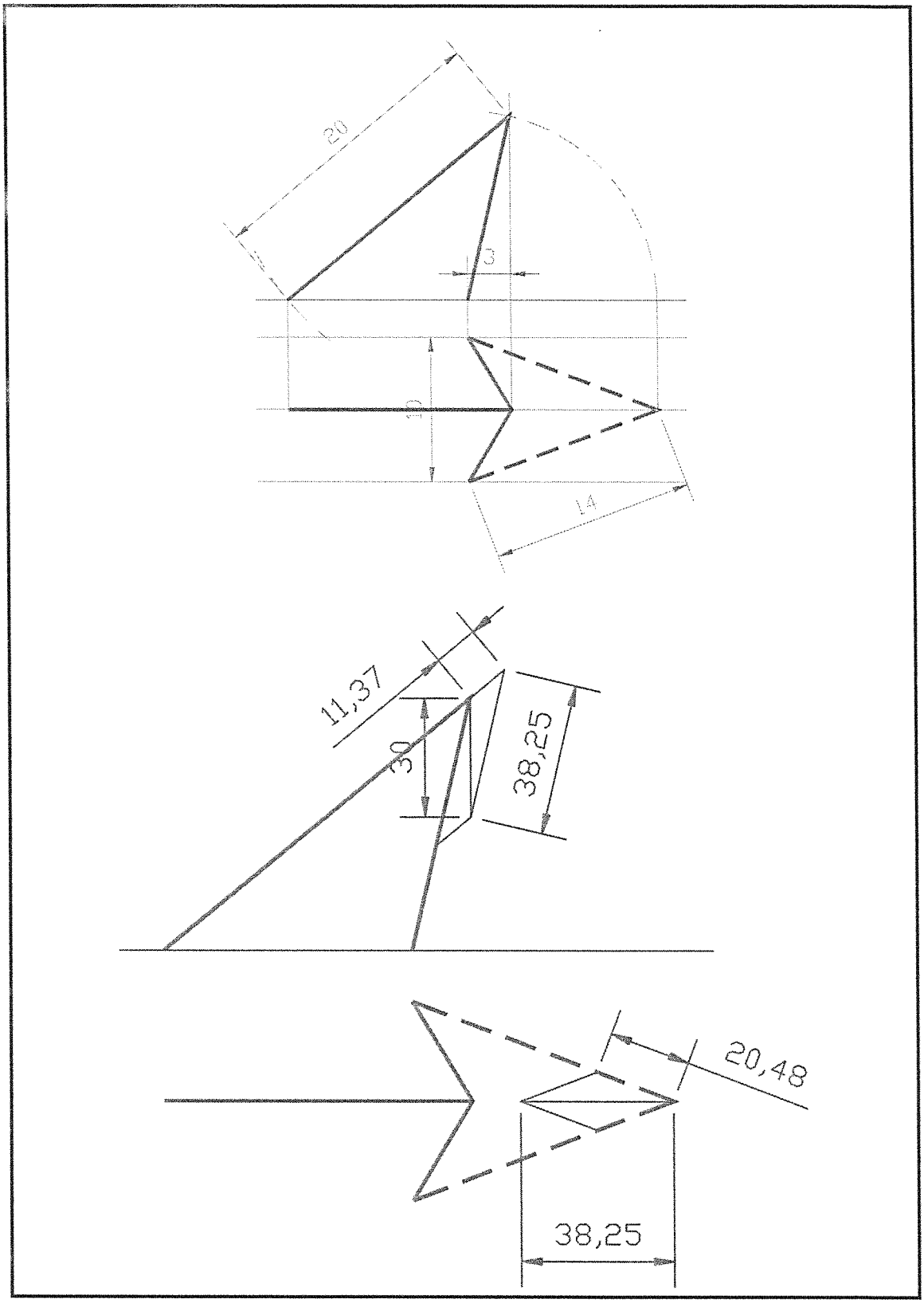
STRENGTH OF MATERIALS AND STRUCTURES N6

4 AUGUST 2014

This marking guideline consists of 7 pages.

QUESTION 1

1.1



(8)

1.2 *Force in backstay* = 11,37*kN* √
Force in legs = 20,48 *kN each* √

(2)
[10]

QUESTION 2

2.1 $$at\ 150\ mm: a + \frac{b}{0{,}15^2} = 48 \times 10^6 \ldots\ldots\ldots (1)\surd$$

$$at\ 200\ mm: a + \frac{b}{0{,}2^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)\surd$$

$$(1) - (2): 44{,}444b - 25b = 48 \times 10^6$$

$$b = 2{,}469 \times 10^{6}\surd$$

$$a = -61{,}714 \times 10^{6}\surd$$

$$at\ 150\ mm: \sigma_{Hmax} = a - \frac{b}{0{,}15^2}$$

$$= -61{,}714 \times 10^6 - \frac{2{,}469 \times 10^6}{0{,}15^2}\surd$$

$$\sigma_{ult} = -171{,}429MPa\ (tensile)\surd$$ (6)

2.2 $$\sigma_{Hmax} = \frac{\sigma_{ult}}{SF} = \frac{171{,}429 \times 10^{6}\surd}{3} = 57{,}143MPa\ \surd$$

$$at\ 150\ mm: a - \frac{b}{0{,}15^2} = -57{,}143 \times 10^6 \ldots\ldots\ldots (1)\surd$$

$$at\ 280\ mm: a + \frac{b}{0{,}28^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)\surd$$

$$(1) - (2): -44{,}444b - 12{,}755b = -57{,}143 \times 10^6$$

$$b = 999{,}0114 \times 10^{3}\surd$$

$$a = -12{,}746 \times 10^{6}\surd$$

$$at\ 150\ mm: \sigma_R = a + \frac{b}{0{,}15^2}$$

$$= -12{,}746 \times 10^6 + \frac{999{,}0114 \times 10^3}{0{,}15^2}\surd$$

$$\sigma_R = 31{,}658MPa\ \surd$$ (8)

[14]

QUESTION 3

3.1 $$I = \frac{\pi D^4}{64} = \frac{\pi \times 0{,}23^4}{64} = 137{,}366 \times 10^{-6} m^4 \surd$$

$$Y_{max} = \frac{FL^3}{3EI} = \frac{45 \times 10^3 \times 3^3}{3 \times 200 \times 10^9 \times 137{,}366 \times 10^{-6}} = 14{,}74 \times 10^{-3} m \surd$$ (4)

3.2 $$Y_{max} = \frac{FL^3}{3EI} + \frac{Fl^3}{3EI} + \frac{Fl^2}{2EI} \times l_f$$

$$14{,}74 \times 10^{-3} = \frac{F \times 3^3}{3 \times 27{,}47 \times 10^6} + \frac{F \times 2^3}{3 \times 27{,}47 \times 10^6} + \frac{F \times 2^2}{2 \times 27{,}47 \times 10^6} \times 1$$

$$14{,}74 \times 10^{-3} \surd = 3{,}276 \times 10^{-7} F \surd + 9{,}708 \times 10^{-8} F \surd + 7{,}281 \times 10^{-8} F \surd$$

$$F = 29{,}629 kN \surd$$ (5)

3.3 $$M = F \times L + F \times l = 29{,}633 \times 10^3 \times 3 + 29{,}633 \times 10^3 \times 2 \surd = 148{,}165 kNm \surd$$

$$\sigma = \frac{M \times Y}{I} = \frac{148{,}165 \times 10^3 \times 0{,}115 \surd}{137{,}677 \times 10^{-6}} = 123{,}761 MPa \surd$$ (4)

[13]

QUESTION 4

4.1 $$Y_c = \frac{a_1 y_1 + a_2 y_2}{a_T} = \frac{8 \times 10^{-3} \times 0{,}02 + 8 \times 10^{-3} \times 0{,}12}{16 \times 10^{-3} \surd \surd}$$

$$Y_c = 0{,}07m \surd \ and \ Y_c = 0{,}13m$$ (3)

4.2 $$I_1 = \frac{bd^3}{12} + ah^2 = \frac{0{,}2 \times 0{,}04^3}{12} + 8 \times 10^{-3} \times 0{,}05^2 \surd = 21{,}1 \times 10^{-6} m^4 \surd$$

$$I_2 = \frac{bd^3}{12} + ah^2 = \frac{0{,}05 \times 0{,}16^3}{12} + 8 \times 10^{-3} \times 0{,}05^2 \surd = 37{,}1 \times 10^{-6} m^4 \surd$$

$$I_{XX} = I_1 + I_2 = 21{,}1 \times 10^{-6} + 37{,}1 \times 10^{-6} = 58{,}2 \times 10^{-6} m^4 \surd$$ (5)

4.3 $$\sigma_D = \frac{F}{A} = \frac{50 \times 10^3 \surd}{16 \times 10^{-3}} = 3{,}125 MPa \surd (tensile) \surd$$ (3)

4.4 $\sigma_{bt} = \frac{FeY_t}{I} = \frac{50 \times 10^3 \times 0,37 \times 0,07}{58,2 \times 10^{-6}} = 22,251 Mpa(tensile)$ √

$\sigma_{bc} = \frac{FeY_c}{I} = \frac{50 \times 10^3 \times 0,37 \times 0,13}{58,2 \times 10^{-6}} = 41,323 Mpa(compressive)$ √

$\sigma_{Rt} = \sigma_D + \sigma_{bt} = 3,125 + 22,251 = 25,376 MPa(tensile)$ √

$\sigma_{Rc} = \sigma_D - \sigma_{bc} = 3,125 - 41,323 = 38,198 MPa(compressive)$ √ (4)
[15]

QUESTION 5

5.1 $W_1 = \rho g A l = 2000 \times 9,81 \times 2 \times 6 \times 1 = 235,44 kN$ √

$W_2 = \rho g A l = 2000 \times 9,81 \times 0.5 \times 3 \times 6 \times 1 = 176,58 kN$ √

$V_R = W_1 + W_2 = 412,02 kN$ √ (3)

5.2 $W_1 x_1 = 235,44 \times 4 = 941,76 kNm$ √

$W_2 x_2 = 176,58 \times 2 = 353,16 kNm$ √

$\sum W - M = W_1 x_1 + W_2 x_2 = 1294,92 kNm$ √ (3)

5.3 $F_w = \frac{\rho g h^2}{2} = \frac{1000 \times 9,81 \times 6^2}{2} = 176,58 kN$ √

$\sum F - M = F_w \times \frac{h}{3} = 176,58 \times 2$ √ $= 353,16 kNm$ √ (3)

5.4 $V_R x_R + \sum F - M = \sum W - M$

$412,02 \times x_R + 353,16 = 1294,92$ √

$x_R = 2,286 m$ √ (2)

5.5 $e = \frac{B}{2} - x_R = 2,5 - 2,286 = 0,214 m$ √

$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} = \frac{412,02}{5} + \frac{6 \times 412,02 \times 0,214}{5^2}$ √ $= 103,565 kPa$ √ (3)
[14]

QUESTION 6

6.1 $F_c = \sigma A = 500 \times 10^3 \times 0{,}6^2 = 180kN$ √

$W_T = F_c + W_c + W_f = 180 + 20 + 29 = 229kN$ √

$L = \sqrt{\frac{W_T}{p}} = \sqrt{\left(\frac{229}{180}\right)\sqrt{}} = 1{,}128m$ √ (4)

6.2 $t = \frac{L - l}{\sqrt{2}} = \frac{1{,}128 - 0{,}6}{\sqrt{2\sqrt{}}} = 0{,}373m$ √ (2)

6.3 $d = \frac{p}{\rho g}\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)^2 = \frac{180}{1600 \times 9{,}81}\left(\frac{1 - \sin 30}{1 + \sin 30}\right)^{2\sqrt{}} = 1{,}274m$ √ (2)
[8]

QUESTION 7

7.1 $\frac{\sigma_s}{\sigma_c} = \frac{m(d - n)}{n}$

$\frac{115}{5} = \frac{15(0{,}6 - n)}{n\sqrt{}}$

$23n = 9 - 15n$ √

$n = 0{,}237m$ √ (3)

7.2 $M_c = 0{,}5\sigma_c A_c\left(d - \frac{n}{3}\right) = 0{,}5 \times 5 \times 10^6 \times 0{,}4 \times 0{,}237\left(0{,}6 - \frac{0{,}237}{3}\right)$ √

$M_c = 123{,}477kNm$ √ (2)

7.3 $M = \frac{wL^2}{8} + \frac{FL}{4}$

$123{,}477 \times 10^3 = \frac{30 \times 10^3 \times 4^2}{8} + \frac{F \times 4}{4\sqrt{}}$

$F = 63{,}477kNm$ √ (2)

7.4 $\sigma_s A_s = 0{,}5\sigma_c A_c$

$115 \times A_s = 0{,}5 \times 5 \times 0{,}4 \times 0{,}237$ √

$A_s = 2{,}0609 \times 10^{-3}m^{2\sqrt{}}$ (2)

7.5 $$A_1 = \frac{\pi d^2}{4} = \frac{\pi 0{,}025^2}{4\sqrt{}} = 4{,}909 \times 10^{-4} m^2 \sqrt{}$$

$$x = \frac{A_s}{A_1} = \frac{2{,}0609 \times 10^{-3}}{4{,}909 \times 10^{-4}\sqrt{}} = 4{,}19 \quad say \quad 5\ bars\ \sqrt{}$$

(4)
[13]

QUESTION 8

8.1 $$T = \frac{P \times 1{,}12}{2\pi N} = \frac{150 \times 10^3 \times 1{,}12}{2\pi \times 10} = 2673{,}803 Nm \sqrt{}$$

(1)

8.2 $$M = \frac{Fab}{L} = \frac{7800 \times 0{,}4 \times 0{,}6}{1} = 1872 Nm \sqrt{}$$

(1)

8.3 $$T_e = \sqrt{T^2 + M^2} = \sqrt{(2673{,}803^2 + 1872^2)\sqrt{}} = 3263{,}986 Nm\ \sqrt{}$$

(2)

8.4 $$M_e = 0{,}5\left(M + \sqrt{T^2 + M^2}\right) = 0{,}5(1872 + 3263{,}986)\sqrt{} = 2567{,}993 Nm\ \sqrt{}$$ (2)

8.5 $$consider\ shear\ stress: \quad \frac{\pi d^3 \tau}{16} = T_e$$

$$\frac{\pi \times d^3 \times 45 \times 10^6}{16} = 3263{,}986 \sqrt{}$$

$$d = 71{,}752 \times 10^{-3} m \sqrt{}$$

$$consider\ bending\ stress: \frac{\pi d^3 \sigma}{32} = M_e$$

$$\frac{\pi \times d^3 \times 60 \times 10^6}{32} = 2567{,}993 \sqrt{}$$

$$d = 75{,}825 \times 10^{-3} m \sqrt{}$$

$$minimum\ diameter: \quad d = 75{,}825 \times 10^{-3} m \sqrt{}$$ (5)

8.6 $$\tau = \frac{16 T_e}{\pi d^3} = \frac{16 \times 3263{,}986\ \sqrt{}}{\pi (75{,}825 \times 10^{-3})^3} = 38{,}131 MPa\ \sqrt{}$$

(2)
[13]

TOTAL: 100