higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

6 AUGUST 2018

This marking guideline consists of 8 pages.

QUESTION 1

1.1
$$at\ 80\ mm: a - \frac{b}{0{,}08^2} = -54 \times 10^6 \ldots\ldots\ldots (1)\ \checkmark$$
$$at\ 120\ mm: a + \frac{b}{0{,}12^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)\checkmark$$
$$(1) - (2): 156{,}25b - 69{,}444b = -54 \times 10^6$$
$$b = 239{,}262 \times 10^3\ \checkmark$$
$$a = -16{,}615 \times 10^6\checkmark$$
$$at\ 80\ mm:\ \sigma_R = a + \frac{b}{0{,}08^2}$$
$$= -16{,}615 \times 10^6 - \frac{239{,}262 \times 10^3}{0{,}08^2}$$
$$\sigma_R = 20{,}769\ MPa\ \checkmark$$ (5)

1.2
$$W = p \times A = 20{,}769 \times 10^6 \times \pi \times 0{,}04^2 = 104{,}398\ kN\checkmark$$
$$m = \frac{W}{g} = \frac{104{,}398 \times 10^3}{9{,}81} = 10641{,}951\ kg\checkmark$$ (2)

1.3
$$\sigma_L = \frac{pd^2}{D^2 - d^2} = \frac{20{,}769 \times 10^6 \times 0{,}08^2}{0{,}12^2 - 0{,}08^2}\checkmark = 16{,}615\ MPa\checkmark$$ (2)

1.4
$$\epsilon = \frac{(\sigma_H - \vartheta \times \sigma_R)}{E} = \frac{-54 \times 10^6 - 0{,}29 \times 20{,}769 \times 10^6}{200 \times 10^9}\checkmark = -300{,}115 \times 10^{-6}\checkmark$$ (2)

1.5
$$\Delta d = \varepsilon \times d = -300{,}115 \times 10^{-6} \times 0{,}08\checkmark = 24{,}009 \times 10^{-6}\ m\checkmark$$ (2)

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QUESTION 2

2.1 $Consider\ the\ deflection\ limit$

$$\Delta = \frac{wl_1^4}{8EI} + \frac{wl_1^3 \times l_2}{6EI}$$

$$0{,}013 = \frac{10 \times 10^3 \times 4^4}{8 \times 200 \times 10^9 \times I} + \frac{10 \times 10^3 \times 4^3 \times 3}{6 \times 200 \times 10^9 \times I} \checkmark$$

$$I = 246{,}154 \times 10^{-6}\ m^4 \checkmark$$

$Select\ 457 \times 191 \times 67{,}1\ kg/m \checkmark$

$Consider\ the\ stress\ limit$

$$M = \frac{wl^2}{2} = \frac{10 \times 10^3 \times 4^2}{2} = 80\ kNm \checkmark$$

$$Z = \frac{M}{\sigma} = \frac{80 \times 10^3}{90 \times 10^6} = 888{,}889 \times 10^{-6}\ m^3 \checkmark$$

$Select\ 406 \times 178 \times 53{,}8\ kg/m \checkmark$

$Correct\ profile = \ 457 \times 191 \times 67{,}1\ kg/m \checkmark$

$This\ profile\ will\ satisfy\ both\ limits \checkmark$ (8)

2.2

$$\Delta = \frac{wl_1^4}{8EI} + \frac{wl_1^3 \times l_2}{6EI}$$

$$= \frac{10 \times 10^3 \times 4^4}{8 \times 200 \times 10^9 \times 294{,}1 \times 10^{-6}} + \frac{10 \times 10^3 \times 4^3 \times 3}{6 \times 200 \times 10^9 \times 294{,}1 \times 10^{-6}} \checkmark$$

$$\Delta = 10{,}88\ mm \checkmark$$

$$\sigma = \frac{M}{Z} = \frac{80 \times 10^3}{1297 \times 10^{-6}} \checkmark = 61{,}681\ MPa \checkmark$$ (4)

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QUESTION 3

3.1

$$\sigma_d = \frac{F}{A} = \frac{180 \times 10^3}{2 \times 1{,}787 \times 10^{-3}} \checkmark = 50{,}364\ MPa \checkmark$$ (2)

3.2

$$M = \frac{wL^2}{2} = \frac{2 \times 137{,}34 \times 4^2}{2} = 2{,}197\ kNm \checkmark$$

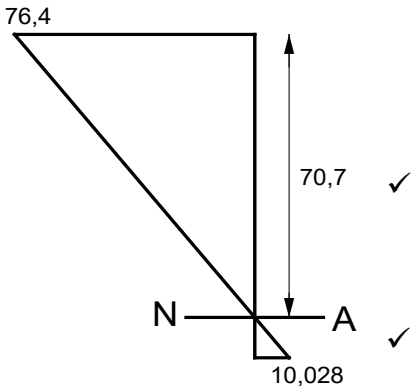
$$\sigma_{bmin} = \frac{MY}{I} = \frac{2{,}197 \times 10^3 \times 0{,}0241}{2 \times 1{,}017 \times 10^{-6}} = 26{,}0365\ MPa \checkmark\ (tensile) \checkmark$$

$$\sigma_{bmax} = \frac{MY}{I} = \frac{2{,}197 \times 10^3 \times 0{,}0559}{2 \times 1{,}017 \times 10^{-6}} = 60{,}392\ MPa \checkmark (compressive) \checkmark$$ (5)

3.3 $\sigma_{max} = \sigma_d + \sigma_b = 50{,}364 + 26{,}0365 = 76{,}4\ MPa$ ✓ $(tensile)$✓
$\sigma_{max} = \sigma_d - \sigma_b = 50{,}364 - 60{,}392 = 10{,}028\ MPa$ ✓ $(compressive)$✓ (4)

3.4 $$\frac{x}{76{,}4} = \frac{80 - x}{10{,}028}$$ ✓

$$= 70{,}7\ mm\ from\ the\ top$$✓



✓ ✓

(4)
[15]

QUESTION 4

4.1 $W_1 = \rho g A l = 2200 \times 9{,}81 \times 0{,}5 \times 4 \times y \times 1 = 43{,}164y\ kN$✓
$W_2 = \rho g A l = 2200 \times 9{,}81 \times 1{,}5 \times 4 \times 1 = 129{,}492\ kN$✓
$V = W_1 + W_2 = 43{,}164y + 129{,}492\ kN$ ✓

$$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} \quad ---(1)$$✓

$$\sigma_{min} = \frac{V}{B} - \frac{6Ve}{B^2} \quad ---(2)$$✓

$(1) + (2)$: $91{,}342 + 38{,}15 = \frac{2V}{B}$ ✓

$129{,}492 = \frac{2(43{,}164y + 129{,}492)}{1{,}5 + y}$ ✓

$129{,}492 \times (1{,}5 + y) = 2(43{,}164y + 129{,}492)$✓

$y = 1{,}5\ m$ ✓ $and\ B = 1{,}5 + 1{,}5 = 3\ m$ ✓ (10)

4.2 $V = 43{,}164y + 129{,}492 = 43{,}164 \times 1{,}5 + 129{,}492 = 194{,}238\ kN$✓

$$\sigma_d = \frac{V}{B} = \frac{194{,}238}{3} = 64{,}746\ kPa$$✓

$\sigma_b = \sigma_{max} - \sigma_d = 91{,}342 - 64{,}746$ ✓ $= 26{,}596\ kPa$✓ (4)
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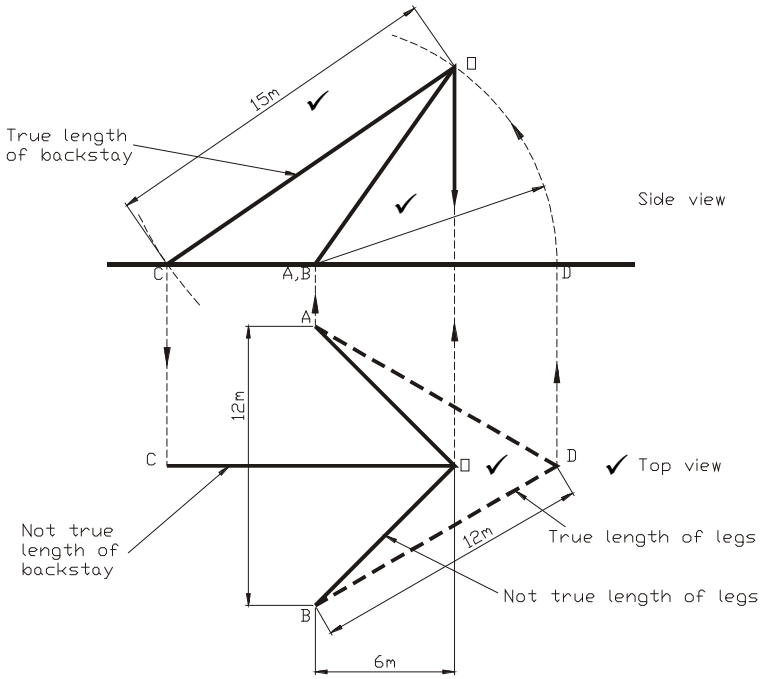
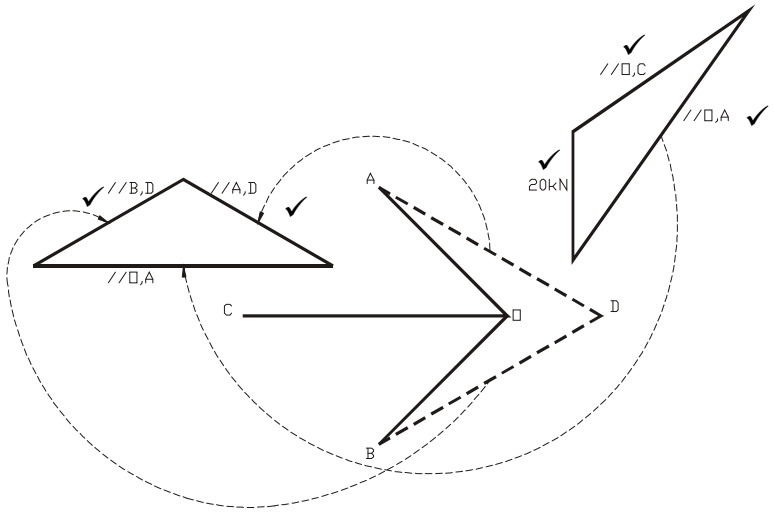
QUESTION 5

5.1



(4)

5.2



(5)

5.3

MEMBER	MAGNITUDE	NATURE
OC	33,3 kN √	TIE √
OA	27,4 kN √	STRUT √
OB	27,4 kN √	STRUT √

(3)
[12]

QUESTION 6

6.1 $\frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$

$\frac{140}{7} = \frac{15(0{,}8-n)}{n}$ ✓

$n = 0{,}343\ m$✓ (2)

6.2 $M = 0{,}5\sigma_c A_c l_a = 0{,}5 \times 7 \times 10^6 \times 0{,}35 \times 0{,}343 \times 0{,}686$✓ $= 288\ kNm$✓ (2)

6.3 $M = \sigma_s A_s l_a$

$288 \times 10^3 = 140 \times 10^6 \times A_s \times 0{,}686$✓

$A_s = 3 \times 10^{-3}\ m^2$ ✓

$Select\ 203 \times 102 \times 25.3\ kg/m$✓ (3)

6.4 $M = \sigma_s A_s l_a$

$288 \times 10^3 = \sigma_s \times 3{,}226 \times 10^{-3} \times 0{,}686$✓

$\sigma_s = 130{,}192\ MPa$✓ (2)

6.5 $M_c = 0{,}5\sigma_c A_c \frac{2}{3} n$

$= 0{,}5 \times 7 \times 10^6 \times 0{,}35 \times 0{,}343 \times \frac{2}{3} \times 0{,}343$✓

$M_c = 96\ kNm$✓

$M_s = \sigma_s A_s (d-n)$

$= 130{,}192 \times 10^6 \times 3{,}226 \times 10^{-6}(0{,}8-0{,}343)$✓

$M_s = 192\ kNm$ ✓ (4)

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QUESTION 7

7.1 $$\frac{x}{L-x} = \sqrt{\frac{d}{d+h}}$$

$$\frac{x}{130-x} = \sqrt{\frac{8}{12}} \checkmark$$

$$x = 58{,}434\ m \checkmark \quad (2)$$

7.2 $$F_H = \frac{wx_1^2}{2d} = \frac{12 \times 58{,}434^2}{2 \times 8} = 2560{,}87\ kN \checkmark$$

$$F_{V2} = wx_2 = 12 \times 71{,}566 = 858{,}796\ kN \checkmark$$

$$F_{T2} = \sqrt{F_H^2 + F_{V2}^2} = \sqrt{2560{,}87^2 + 858{,}796^2} = 2701{,}0342\ kN \checkmark \quad (3)$$

7.3 $$F_{Va} = F_H \tan\theta = 2560{,}87 \times \tan 40 = 2148{,}825\ kN \checkmark$$

$$R = F_{V2} + F_{Va} = 858{,}796 + 2148{,}825 = 3007{,}621\ kN \checkmark \quad (2)$$

7.4 $$x_3 = 80 - 58{,}434 = 21{,}566\ m \checkmark$$

$$F_{V3} = wx_3 = 12 \times 21{,}566 = 258{,}796\ kN \checkmark$$

$$F_{T3} = \sqrt{F_H^2 + F_{V3}^2} = \sqrt{2560{,}87^2 + 258{,}786^2} = 2573{,}914\ kN \checkmark \quad (3)$$

[10]

QUESTION 8

8.1 $$\tau = \frac{\tau_{ult}}{FOS} = \frac{200}{4} = 50\ MPa\checkmark$$

$$T_e = \frac{\pi(D^4 - d^4)\tau}{16D} = \frac{\pi(0{,}1^4 - 0{,}075^4) \times 50 \times 10^6}{16 \times 0{,}1}\ \checkmark = 6711{,}166\ Nm\checkmark$$

$$T_e = \sqrt{T^2 + M^2}$$

$$6711{,}166 = \sqrt{(T)^2 + (3T)^2}\checkmark$$

$$T = 2122{,}257\ Nm\checkmark$$

$$M = 6366{,}771\ Nm\checkmark$$ (6)

8.2 $$M_e = 0{,}5(M + T_e)$$

$$= 0{,}5(6366{,}771 + 6711{,}166)$$

$$M_e = 6538{,}968\ Nm\checkmark$$

$$\sigma = \frac{32 \times M_e \times D}{\pi \times (D^4 - d^4)} = \frac{32 \times 6538{,}968 \times 0{,}1}{\pi \times (0{,}1^4 - 0{,}075^4)}\checkmark = 97{,}434\ MPa\checkmark$$ (3)

8.3 $$d^3 = \frac{D^4 - d^4}{D} = \frac{0{,}1^4 - 0{,}075^4}{0{,}1} = 683{,}59375 \times 10^{-6}\checkmark$$

$$d = 88{,}1\ mm\checkmark$$ (2)

[11]

TOTAL: 100