higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

12 AUGUST 2019

This marking guideline consists of 8 pages.

QUESTION 1

1.1 $at\ 200\ mm : a + \frac{b}{0{,}2^2} = 30 \times 10^6 \ldots\ldots\ldots (1)$ ✓

$at\ 140\ mm : a + \frac{b}{0{,}14^2} = 0 \ldots\ldots\ldots \quad (2)$ ✓

$(1) - (2) : 25b - 51{,}02b = 30 \times 10^6$

$b = -1{,}153 \times 10^6$ ✓
$a = 58{,}824 \times 10^6$ ✓

$\sigma_{Hmin} = a - \frac{b}{0{,}2^2} = 58{,}824 \times 10^6 - \frac{(-1{,}153 \times 10^6)}{0{,}2^2}$ ✓ $= 87{,}647\ MPa$ ✓ (6)

1.2 $at\ 200\ mm : a + \frac{b}{0{,}2^2} = 30 \times 10^6 \ldots\ldots\ldots (1)$ ✓

$at\ 250\ mm : a + \frac{b}{0{,}25^2} = 0 \ldots\ldots\ldots \quad (2)$ ✓

$(1) - (2) : 25b - 16b = 30 \times 10^6$

$b = 3{,}333 \times 10^6$ ✓
$a = -53{,}333 \times 10^6$ ✓

$\sigma_{Hmax} = a - \frac{b}{0{,}2^2} = -53{,}333 \times 10^6 - \frac{(3{,}333 \times 10^6)}{0{,}2^2}$ ✓ $= -136{,}667\ MPa$ ✓ (6)

1.3 $\delta d_1 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \frac{0{,}2}{200 \times 10^9}(87{,}647 \times 10^6 - 0{,}3 \times 30 \times 10^6)$

$\delta d_1 = 78{,}647 \times 10^{-6} m$ ✓ (1)

1.4 $\delta d_2 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$

$= \frac{0{,}2}{200 \times 10^9}(-136{,}667 \times 10^6 - 0{,}3 \times 30 \times 10^6)$

$\delta d_2 = -145{,}667 \times 10^{-6} m$ ✓ (1)

1.5 $\Delta d = \delta d_1 - \delta d_2$
$= 78{,}647 \times 10^{-6} — (-145{,}667 \times 10^{-6})$
$\Delta d = 0{,}224\ mm$ ✓ (1)

[15]

QUESTION 2

2.1 $M = \frac{wL^2}{8} = \frac{12 \times 10^3 \times 4^2}{8} = 24\ kNm$ ✓

$Z = \frac{M}{\sigma} = \frac{24 \times 10^3}{80 \times 10^6} = 300 \times 10^{-6} m^3$ ✓

$Lighest\ profile\ is\ 305\ \times\ 102\ \times\ 28{,}6\ kg/m$ ✓ $(\mathrm{Z} = 352{,}1 \times 10^{-6} m^3)$ (3)

2.2 $y = \frac{5wL^4}{384EI}$

$\frac{4}{360}$ ✓ $= \frac{5 \times 12 \times 10^3 \times 4^4}{384 \times 207 \times 10^9 \times I}$

$I = 17{,}391 \times 10^{-6} m^4$ ✓

$Lighest\ profile\ is\ 305\ \times\ 102\ \times 24{,}5\ kg/m$ ✓ $(\mathrm{I} = 43{,}64 \times 10^{-6} m^4)$ (3)

2.3 $Correct\ profile\ is\ 305\ \times\ 102\ \times 28{,}6 kg/m$ ✓

$This\ profile\ will\ satisfy\ both\ stress\ and\ deflection\ limits$ ✓ (2)

2.4 $\sigma = \frac{M}{Z} = \frac{24 \times 10^3}{352{,}1 \times 10^{-6}} = 68{,}162\ MPa$ ✓

$y = \frac{5wL^4}{384EI} = \frac{5 \times 12 \times 10^3 \times 4^4}{384 \times 207 \times 10^9 \times 43{,}64 \times 10^{-6}} = 4{,}428\ mm$ ✓ (2)

[10]

QUESTION 3

3.1 $A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi \times (2{,}8^2 - 2{,}6^2)}{4} = 0{,}848\ m^2$ ✓

$\mathrm{I} = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(2^4 - 1{,}9^4)}{64} = 0{,}774\ m^4$ ✓

$W = \rho Ahg = 2400 \times 0{,}848 \times h \times 9{,}81 = 19970{,}728h$ ✓

$F = p \times A_p = 1{,}2 \times 10^3 \times 2{,}8 \times h \times 0{,}6 = 2016h$ ✓

$\sigma_d = \frac{W}{A} = \frac{19970{,}728h}{0{,}848} = 23544h$ ✓

$\sigma_b = \frac{FeY}{I} = \frac{2016h \times 0{,}5h \times 1{,}4}{0{,}774} = 1823{,}233h^2$ ✓

$for\ no\ stress\ \ \sigma_b = \sigma_d$

$1823{,}233h^2 = 23544h$ ✓

$h = 14{,}559\ m$ ✓ (8)

3.2 $\sigma_d = 23544h = 23544 \times 14{,}559 = 342{,}771\ kPa = \sigma_b$ ✓

$\sigma_{max} = \sigma_d + \sigma_b = 342{,}771 + 342{,}771 = 685{,}543\ kPa$ ✓ $(compressive)$ ✓ (3)

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QUESTION 4

4.1 $W_1 = \rho gAl = 2000 \times 9{,}81 \times 1{,}2 \times 2{,}5 \times 1 = 58{,}86\ kN$ ✓

$V = W_1 + W_2 = 47{,}677 + 100 = 158{,}86\ kN$ ✓ (2)

4.2 $C_\mu = \frac{1 - sin30}{1 + sin30} = 0{,}333$ ✓

$F_g = \frac{\rho gh^2}{2} \times C_\mu = \frac{1680 \times 9{,}81 \times 2{,}5^2 \times 0{,}333}{2} = 17{,}168\ kN$ ✓ (2)

4.3 $W{\sim}M = W_1 \times x_1 + W_2 \times x_2 = 58{,}86 \times 0{,}6 + 100 \times 0{,}4 = 75{,}316\ kNm$ ✓

$F{\sim}M = F_g \times \frac{h}{3} = 17{,}168 \times 0{,}833 = 14{,}306\ kNm$ ✓

$V \times x + F{\sim}M = W{\sim}M$

$158{,}86 \times x + 14{,}306 = 75{,}316$ ✓

$x = 0{,}384\ m$ ✓

$There\ will\ be\ tension\ because\ x < \frac{B}{3}$ ✓ (falls outside middle third) (5)

4.4 $e = 0{,}5B - x = 0{,}6 - 0{,}384 = 0{,}216\ m$ ✓

$\sigma_{toe} = \frac{V}{B} + \frac{6Ve}{B^2} = \frac{158{,}86}{1{,}2} + \frac{6 \times 158{,}86 \times 0{,}216}{1{,}2^2} = 275{,}326\ kPa\ (comp)$ ✓

$\sigma_{heel} = \frac{V}{B} - \frac{6Ve}{B^2} = \frac{158{,}86}{1{,}2} - \frac{6 \times 158{,}86 \times 0{,}216}{1{,}2^2} = -10{,}559\ kPa\ (tensile)$ ✓ (3)

4.5 $\sigma_{ult} = \sigma_{max} \times FOS = 275{,}326 \times 3$ ✓ $= 825{,}978\ kPa$ ✓ (2)
[14]

QUESTION 5

5.1 $L = \sqrt{\frac{W_T}{p}} = \sqrt{\frac{2{,}4 \times 10^6 + 300 \times 10^3}{200 \times 10^3}}$ ✓ $= 3{,}674\ m$ ✓

5.2 $M = \frac{W(L-l)}{8} = \frac{2{,}4 \times 10^6 (3{,}674 - 0{,}8)}{8}$ ✓ $= 862{,}27\ kNm$ ✓

5.3 $Z = \frac{M}{\sigma \times n} = \frac{862{,}27 \times 10^3}{110 \times 10^6 \times 5} = 1567{,}764 \times 10^{-6} m^3$ ✓

$Lighest\ I-beam\ is\ 457 \times 191 \times 82\ kg/m$ ✓ $(Z = 1612 \times 10^{-6} m^3)$

5.4 $l = b \times n + 0{,}075(n-1) = 0{,}1913 \times 5 + 0{,}075 \times 4$ ✓ $= 1{,}2565\ m$ ✓

5.5 $M = \frac{W(L-l)}{8} = \frac{2{,}4 \times 10^6 (3{,}674 - 1{,}2565)}{8}$ ✓ $= 725{,}32\ kNm$ ✓

5.6 $Z = \frac{M}{\sigma \times n} = \frac{725{,}32 \times 10^3}{110 \times 10^6 \times 10} = 659{,}382 \times 10^{-6} m^3$ ✓

$Lighest\ I-beam\ is\ 356 \times 171 \times 44{,}8\ kg/m$ ✓ $(Z = 686{,}1 \times 10^{-6} m^3)$

(6 × 2) **[12]**

QUESTION 6

6.1 $$mA_s(d-n) = A_1y_1 + A_2y_2$$

$$15 \times 600 \times 10^{-6}(0{,}6-n) \checkmark = 0{,}5 \times 0{,}1(n-0{,}05) \checkmark + 0{,}2 \times 0{,}5(n-0{,}1)^2 \checkmark$$

$$100n^2 + 39n - 6{,}9 = 0$$

$$n = 0{,}132\,m \checkmark \quad (4)$$

6.2 $$\frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$$

$$\frac{120 \times 10^6}{\sigma_c} = \frac{15(0{,}6-0{,}132)}{0{,}132} \checkmark$$

$$\sigma_c = 2{,}26\,MPa \checkmark$$

$$\sigma_s = 120\,MPa \checkmark \quad (3)$$

6.3 $$\sigma_{c1} = \frac{\sigma_c(n-t)}{n} = \frac{2{,}26 \times 10^6(0{,}132-0{,}1)}{0{,}132} = 0{,}55\,MPa \checkmark \quad (1)$$

6.4 $$M_c = 0{,}5\sigma_c A_c \frac{2}{3}n - \left[0{,}5\sigma_{c1}(b-e)(n-t)\frac{2}{3}(n-t)\right]$$

$$= 0{,}5 \times 2{,}26 \times 500 \times 132 \times \frac{2}{3} \times 132 - 0{,}5 \times 0{,}55 \times 300 \times \frac{2}{3} \times 32^2 \checkmark$$

$$= 6576{,}557 - 56{,}806 \checkmark$$

$$M_c = 6519{,}752\,Nm \checkmark \quad (3)$$

6.5 $$M_s = \sigma_s A_s(d-n) = 120 \times 10^6 \times 600 \times 10^{-6}(0{,}6-0{,}132) = 33685{,}434\,Nm \checkmark$$

$$M = M_c + M_s = 6{,}5198 + 33{,}685 = 40{,}205\,kNm \checkmark \quad (2)$$

[13]

QUESTION 7

7.1 $$\frac{wx_1^2}{2d} = \frac{w(L-x_1)^2}{2(d+h)}$$

$$\frac{x_1^2}{8} = \frac{(160-x_1)^2}{14} \quad ✓$$

$$\frac{160-x_1}{x_1} = \sqrt{\frac{14}{8}} \quad ✓$$

$$x_1 = 68{,}88\ m \quad ✓ \qquad (3)$$

7.2 $$F_H = \frac{wx_1^2}{2d} = \frac{15\times10^3\times68{,}88^2}{2\times8} \ ✓ = 4447{,}944\ kN \quad ✓ \qquad (2)$$

7.3 $$F_{v2} = wx_2 = 15\times10^3\times91{,}12 = 1366{,}798\ kN \quad ✓$$

$$F_{T2} = \sqrt{F_H^2 + F_{v2}^2} = \sqrt{4447{,}944^2 + 1366{,}798^2} = 4653{,}208\ kN \quad ✓ \qquad (2)$$

7.4 $$At\ 120\ m\ from\ lower\ support: x_3 = 120 - x_1 = 51{,}12\ m \quad ✓$$

$$F_{v3} = wx_3 = 15\times10^3\times51{,}12 = 766{,}798\ kN \quad ✓$$

$$F_{T3} = \sqrt{F_H^2 + F_{v3}^2} = \sqrt{4447{,}944^2 + 766{,}798^2} = 4513{,}556\ kN \quad ✓ \qquad (3)$$

[10]

QUESTION 8

8.1 $$R_L \times 1{,}2 = 2600\times0{,}8 + 981\times0{,}4 + 600\times1{,}2\times0{,}6 \quad ✓$$
$$R_L = 2420{,}333\ N \quad ✓$$

$$R_R \times 1{,}2 = 2600\times0{,}4 + 981\times0{,}8 + 600\times1{,}2\times0{,}6 \quad ✓$$
$$R_R = 1880{,}667\ N \quad ✓ \qquad (4)$$

8.2 $$M = 2420{,}333\times0{,}4 - 600\times0{,}4\times0{,}2 \ ✓ = 920{,}133\ Nm \quad ✓ \qquad (2)$$

8.3 $$T = (T_1 - T_2)r = (2000-600)0{,}2 = 280\ Nm \quad ✓ \qquad (1)$$

8.4 $M_e = 0{,}5\left(M + \sqrt{M^2 + T^2}\right) = 0{,}5\left(920{,}13 + \sqrt{920{,}13^2 + 280^2}\right) = 940{,}963\ Nm$ ✓

$$M_e = \frac{\pi d^3 \sigma}{32}$$

$$940{,}963 = \frac{\pi \times d^3 \times 70 \times 10^6}{32}$$ ✓

$d = 51{,}542\ mm$ ✓ (3)

8.5 $$M_e = \frac{\pi(D^4 - d^4)\sigma}{32D}$$

$$940{,}963 = \frac{\pi \times [D^4 - (0{,}5)^4] \times 70 \times 10^6}{32D}$$

$$940{,}963 = \frac{\pi \times (0{,}9375D^4)70 \times 10^6}{32D}$$ ✓

$D = 52{,}662\ mm$ ✓
$d = 26{,}331\ mm$ ✓ (3)

8.6 $\%\ saving = \frac{d_s^2 - (D^2 - d^2)}{d_s^2} = \frac{51{,}542^2 - (52{,}662^2 - 26{,}331^2)}{51{,}542^2}$ ✓ $= 21{,}7\%$ ✓ (2)

[15]

TOTAL: 100