higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

STRENGTH OF MATERIALS AND STRUCTURES N6

12 August 2021

This marking guideline consists of 10 pages.

QUESTION 1

1.1 $\text{at } 200 \text{ mm}: a + \frac{b}{0{,}2^2} = 25 \times 10^6 \ldots\ldots\ldots (1)$ ✓

$\text{at } 200 \text{ mm}: a - \frac{b}{0{,}2^2} = -5 \times 10^6 \ldots\ldots \quad (2)$ ✓

$(1) + (2): \ 2a = 20 \times 10^6$

$a = 10 \times 10^6$ ✓

$b = 600 \times 10^3$ ✓

$\text{at } 300 \text{ mm}: \ \sigma_R = a + \frac{b}{0{,}3^2}$

$= 10 \times 10^6 + \frac{600 \times 10^3}{0{,}3^2}$

$\sigma_R = 16{,}667 \text{ MPa}$ ✓ (5)

1.2 $a\text{t } 300 \text{ mm}: \sigma_H = a - \frac{b}{0{,}3^2}$

$= 10 \times 10^6 - \frac{600 \times 10^3}{0{,}3^2}$

$\sigma_H = 3{,}333 \text{ MPa}$ ✓ (compressive) ✓ (2)

1.3 $\sigma_H = 0 \text{ where: } a - \frac{b}{D_x^2} = 0$

$10 \times 10^6 - \frac{600 \times 10^3}{D_x^2} = 0$

$D_x = 244{,}949 \text{ mm}$ ✓ (1)

1.4 $\text{at } 300 \text{ mm}: a + \frac{b}{0{,}3^2} = 16{,}667 \times 10^6 \ldots\ldots\ldots (1)$ ✓

$\text{at } 300 \text{ mm}: a - \frac{b}{0{,}3^2} = -65 \times 10^6 \ldots\ldots\ldots\ldots\ldots (2)$ ✓

$(1) + (2): \ 2a = -48{,}333 \times 10^6$

$a = -24{,}167 \times 10^6$ ✓

$b = 3{,}675 \times 10^6$ ✓

$\text{at } D: \quad a + \frac{b}{D^2} = 0$

$-24{,}167 \times 10^6 + \frac{3{,}675 \times 10^6}{D^2} = 0$

$D = 389{,}9 \text{ mm}$ ✓ (5)

[13]

QUESTION 2

2.1 $y_B = \frac{F_{TB}}{w} = \frac{6000}{40} = 150 \text{ m}$ ✓

$y_A = y_B - h = 150 - 6 = 144 \text{ m}$ ✓

$F_{TA} = wy_A = 40 \times 144 = 5\,760 \text{ N}$ ✓ (3)

2.2 $y_0 = y_A - d = 144 - 4 = 140 \text{ m}$ ✓

$\ell_A = \sqrt{y_A^2 - y_0^2} = \sqrt{144^2 - 140^2} = 33{,}705 \text{ m}$ ✓

$\ell_B = \sqrt{y_B^2 - y_0^2} = \sqrt{150^2 - 140^2} = 53{,}852 \text{ m}$ ✓

$\ell_T = \ell_A + \ell_B = 87{,}556 \text{ m}$ ✓ (4)

2.3 $F_{vc} = w\ell_A = 40 \times 33{,}705 = 1\,348{,}184$ N ✓

$F_{va} = F_{TA}\cos\alpha = 5\,760 \times \cos 30 = 4988{,}306$ N ✓

$F_{va} = F_{va} + F_{vc} = 6336{,}49$ N ✓ (3)

2.4 $F_{Hc} = wy_0 = 40 \times 140 = 5\,600$ N $= F_{Ha}$ ✓

$F_{Ta} = \dfrac{F_{Ha}}{\sin\theta} = \dfrac{5\,600}{\sin 30} = 11\,200$ N ✓ (2)

[12]

QUESTION 3

3.1 $\Delta_1 = \dfrac{w\ell_1^4}{8EI} + \dfrac{w\ell_1^3 \times \ell_2}{6EI}$

$= \dfrac{10 \times 10^3 \times 2{,}5^4}{8 \times 200 \times 10^9 \times I} + \dfrac{10 \times 10^3 \times 2{,}5^3 \times 1{,}5}{6 \times 200 \times 10^9 \times I}$ ✓

$\Delta_1 = \dfrac{439{,}453 \times 10^{-9}}{I}$ ✓

$\Delta_2 = \dfrac{FL^3}{3EI} = \dfrac{20 \times 10^3 \times 4^3}{3 \times 200 \times 10^9 \times I} = \dfrac{2{,}133 \times 10^{-6}}{I}$ ✓

$\Delta_T = \Delta_1 + \Delta_2$

$11 \times 10^{-3} = \dfrac{439{,}453 \times 10^{-9}}{I} + \dfrac{2{,}133 \times 10^{-6}}{I}$ ✓

$I = 233{,}89 \times 10^{-6}\ m^4$ ✓

$I = \dfrac{\pi(D^4 - d^4)}{64}$

$233{,}89 \times 10^{-6} = \dfrac{\pi((2d)^4 - d^4)}{64}$ ✓

$d = 133{,}502$ mm ✓
$D = 267{,}004$ mm ✓

(8)

3.2 For $I = 233{,}89 \times 10^{-6}\ \text{m}^4$ select $305 \times 305 \times 118\ \text{kg/m}$ ✓ (1)

3.3
$$M = FL + \frac{w\ell^2}{2}$$
$$= 20 \times 10^3 \times 4 + \frac{10 \times 10^3 \times 2{,}5^2}{2}$$ ✓
$$\text{M} = 111{,}25\ \text{kNm}$$ ✓

$$\sigma = \frac{M}{Z} = \frac{111{,}25 \times 10^3}{1755 \times 10^{-6}} = 63{,}39\ \text{MPa}$$ ✓

(3)
[12]

QUESTION 4

4.1
$$A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi(3^2 - 2{,}5^2)}{4} = 2{,}16\ \text{m}^2$$ ✓

$$W = \rho g A h = 2\,500 \times 9{,}81 \times 2{,}16 \times 15 = 794{,}553\ \text{kN}$$ ✓

$$\sigma_D = \frac{W}{A} = \frac{794{,}553 \times 10^3}{2{,}16} = 367{,}875\ \text{kPa (compressive)}$$ ✓

(3)

4.2
$$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(3^4 - 2{,}5^4)}{64} = 2{,}059\ \text{m}^4$$ ✓

$$M = F \times e = 60 \times 10^3 \times 7{,}5 = 450\ \text{kNm}$$ ✓

$$\sigma_b = \frac{MY}{I} = \frac{450 \times 10^3 \times 1{,}5}{2{,}059} = 327{,}892\ \text{kPa}$$ ✓

(3)

4.3 $\sigma_{max} = \sigma_d + \sigma_b = 695{,}767\ \text{kPa}$ ✓ (compressive) ✓

$\sigma_{min} = \sigma_d - \sigma_b = 39{,}983\ \text{kPa}$ ✓ (compressive) ✓

(4)

4.4 $Y_0 = \frac{\sigma_d \times I}{M} = \frac{367{,}875 \times 10^3 \times 2{,}059}{450 \times 10^3} = 1{,}683$ m (from the centroid) ✓

$Y = 1{,}683 + 1{,}5 = 3{,}183$ mm (outside the profile) ✓

OR

$\frac{\sigma_{max}}{x} = \frac{\sigma_{min}}{x - D}$

$\frac{695{,}767}{x} = \frac{39{,}983}{x - 3}$ ✓

$x =$ 3,183 mm (outside the profile) ✓

(2)
[12]

QUESTION 5

5.1 $W_1 = \rho g A \ell = 2200 \times 9{,}81 \times 0{,}5 \times 5 \times b \times 1 = 53{,}955b$ kN ✓

$W_2 = \rho g A \ell = 2200 \times 9{,}81 \times 2 \times 5 \times 1 = 215{,}82$ kN ✓

$V = W_1 + W_2 = 53{,}955b + 215{,}82$ kN ✓

$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} - - - (1)$ ✓

$\sigma_{min} = \frac{V}{B} - \frac{6Ve}{B^2} - - - (2)$ ✓

$(1) + (2)$: $107{,}91 + 35{,}97 = \frac{2V}{B}$ ✓

$143{,}88 = \frac{2(53{,}955b + 215{,}82)}{2 + b}$ ✓

$143{,}88 \times (2 + b) = 2(53{,}955b + 215{,}82)$ ✓

$b = 4$ m ✓ and $B = 4 + 2 = 6$ m ✓

(10)

5.2 $V = 53{,}955b + 215{,}82 = 53{,}955 \times 4 + 215{,}82 = 431{,}64 \text{ kN}$ ✓

$\sigma_d = \frac{V}{B} = \frac{431{,}64}{6} = 71{,}94 \text{ kPa}$ ✓

$\sigma_b = \sigma_{max} - \sigma_d = 107{,}91 - 71{,}94$ ✓ $= 35{,}97 \text{ kPa}$ ✓

(4)
[14]

QUESTION 6

6.1 $A = \frac{W_T}{p} = \frac{3 \times 10^6}{187{,}5 \times 10^3} = 16 \text{ m}^2$ ✓

$L = \sqrt{A} = \sqrt{16} = 4 \text{ m}$ ✓

(2)

6.2 $M = \frac{W(L - \ell)}{8} = \frac{2{,}5 \times 10^6 (4 - 0{,}8)}{8} = 1 \text{ MNm}$ ✓

$Z = \frac{M}{\sigma \times n} = \frac{1 \times 10^6}{100 \times 10^6 \times 5} = 2\,000 \times 10^{-6} \text{ m}^3$ ✓

lighest I − beam is $533 \times 210 \times 92{,}5 \text{ kg/m}$ ✓

(3)

6.3 $\ell = b \times n + 0{,}075(n - 1) = 0{,}2093 \times 5 + 0{,}075 \times 4 = 1{,}3465 \text{ m}$ ✓

The given base plate dimension of 1,2 m is not sufficient and must be changed to 1,3465 m ✓

(2)

6.4 $M = \frac{W(L - l)}{8} = \frac{2{,}5 \times 10^6 (4 - 1{,}3465)}{8} = 829{,}219 \text{ kNm}$ ✓

$Z = \frac{M}{\sigma \times n} = \frac{829{,}219 \times 10^3}{100 \times 10^6 \times 10} = 829{,}219 \times 10^{-6} \text{ m}^3$ ✓

lighest I − beam is $406 \times 178 \times 53{,}8 \text{ kg/m}$ ✓ (3)

6.5 $\sigma_T = \frac{M}{Z \times n} = \frac{1 \times 10^6}{2076 \times 10^{-6} \times 5} = 96{,}339 \text{ MPa}$ ✓

$\sigma_B = \frac{M}{Z \times n} = \frac{829{,}219 \times 10^3}{927{,}4 \times 10^{-6} \times 10} = 89{,}413 \text{ MPa}$ ✓ (2)

[12]

QUESTION 7

7.1 $\frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$

$\frac{138}{6} = \frac{15(0{,}6-n)}{n}$ ✓

$23n = 9 - 15n$ ✓

$n = 0{,}237 \text{ m}$ ✓ (3)

7.2 $l_a = d - \frac{n}{3} = 0{,}6 - \frac{0{,}237}{3} = 0{,}521 \text{ m}$ ✓

$M = \sigma_s A_s l_a = 138 \times 10^6 \times 2 \times 10^{-3} \times 0{,}521 = 143{,}811 \text{ kNm}$ ✓ (2)

7.3 $M = \frac{FL}{4} + \frac{wL^2}{8}$

$143{,}811 = \frac{60 \times 4}{4}$ ✓ $+ \frac{w \times 4^2}{8}$ ✓

$w = 41{,}905 \text{ kN/m}$ ✓ (3)

7.4 $M = 0{,}5\sigma_c A_c l_a$

$143{,}811 \times 10^3 = 0{,}5 \times 6 \times 10^6 \times b \times 0{,}237 \times 0{,}521$ ✓

$b = 0{,}388 \text{ m}$ ✓ (2)

7.5 $M_c = 0{,}5\sigma_c A_c \frac{2}{3} n$

$= 0{,}5 \times 6 \times 10^6 \times 0{,}388 \times 0{,}237 \times \frac{2}{3} \times 0{,}237$ ✓

$M_c = 43{,}579 \text{ kNm}$ ✓ (2)

7.6 $M_s = \sigma_s A_s (d-n)$

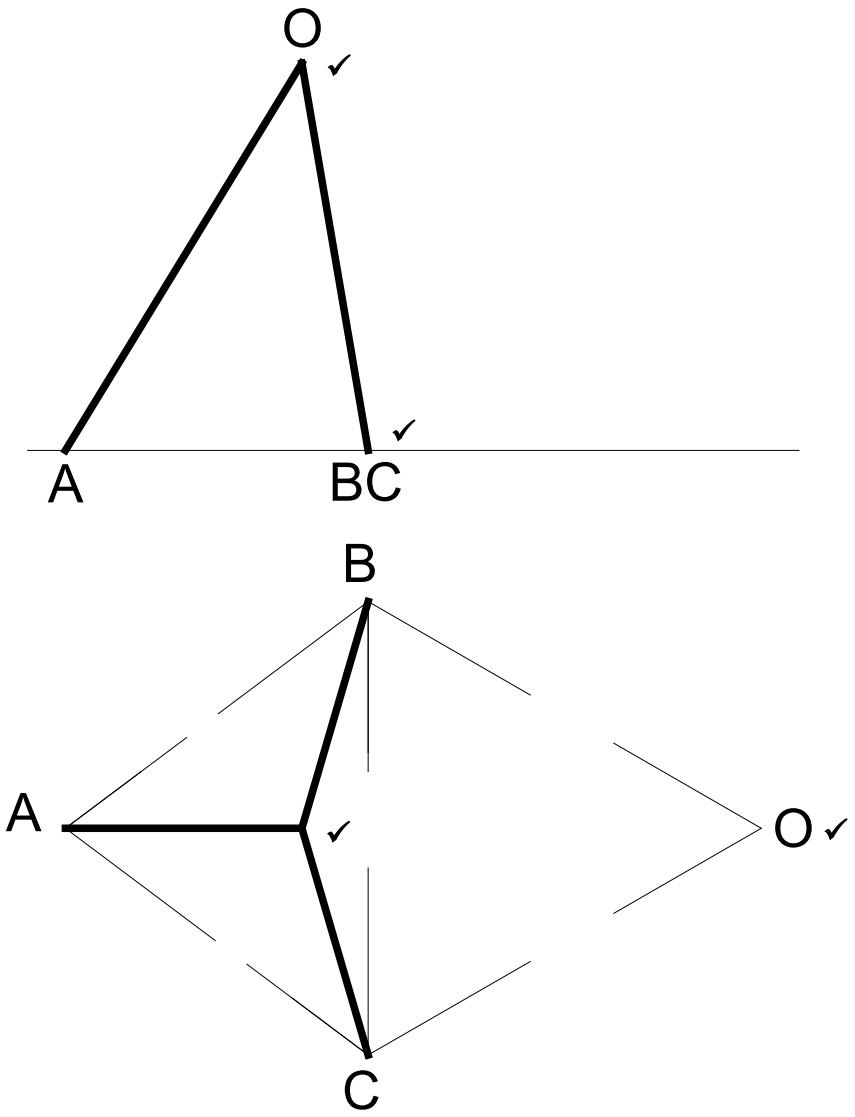
$= 138 \times 10^6 \times 2 \times 10^{-3}(0{,}6 - 0{,}237)$ ✓

$M_s = 100{,}232\ kNm$ ✓ (2)

[14]

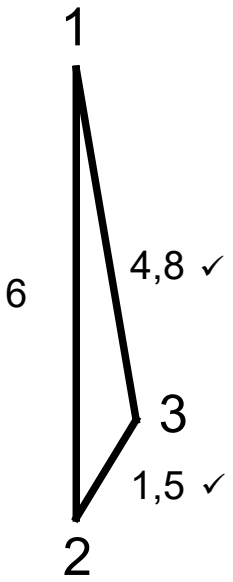
QUESTION 8

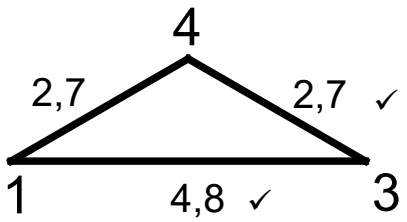
8.1



(4)

8.2





MEMBER	MAGNITUDE	NATURE
OA (2-3)	9 kN ✓	Strut
OB (3-4)	16,2 kN ✓	Strut
OC (1-4)	16,2 kN ✓	Strut

(7)
[11]

TOTAL: 100