higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

STRENGTH OF MATERIALS AND STRUCTURES N6

25 NOVEMBER 2014

This marking guideline consists of 7 pages.

QUESTION 1

1.1 $$at\ 200\ mm: a + \frac{b}{0{,}2^2} = 25 \times 10^6 \ldots\ldots\ldots (1)\surd$$

$$at\ 120\ mm: a + \frac{b}{0{,}12^2} = 0 \ldots\ldots\ldots \quad (2)\surd$$

$$(1) - (2): 25b - 69{,}444b = 25 \times 10^6$$

$$b = -562{,}5 \times 10^3 \surd$$
$$a = 39{,}0625 \times 10^6 \surd$$

$$\sigma_{Hmin} = a - \frac{b}{0{,}2^2} = 39{,}0625 \times 10^6 - \frac{(-562{,}5 \times 10^3)}{0{,}2^2} \surd = 53{,}125 MPa\ \surd$$ (6)

1.2 $$at\ 200\ mm: a + \frac{b}{0{,}2^2} = 25 \times 10^6 \ldots\ldots\ldots (1)\surd$$

$$at\ 260\ mm: a + \frac{b}{0{,}26^2} = 0 \ldots\ldots\ldots \quad (2)\surd$$

$$(1) - (2): 25b - 14{,}793b = 25 \times 10^6$$

$$b = 2{,}449 \times 10^3 \surd$$
$$a = -36{,}232 \times 10^6 \surd$$

$$\sigma_{Hmax} = a - \frac{b}{0{,}2^2} = -36{,}232 \times 10^6 - \frac{(2{,}449 \times 10^3)}{0{,}2^2} \surd = -97{,}464 MPa\ \surd$$ (6)

1.3 $$\delta d_1 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$$

$$= \frac{0{,}2}{200 \times 10^9}(53{,}125 \times 10^6 - 0{,}3 \times 25 \times 10^6)$$

$$\delta d_1 = 4{,}5625 \times 10^{-5} m\ \surd$$ (1)

1.4 $$\delta d_2 = \frac{D_c}{E}(\sigma_H - \vartheta \times \sigma_R)$$

$$= \frac{0{,}2}{200 \times 10^9}(-97{,}464 \times 10^6 - 0{,}3 \times 25 \times 10^6)$$

$$\delta d_2 = -1{,}04964 \times 10^{-4} m\ \surd$$ (1)

1.5 $\Delta d = \delta d_1 - \delta d_2$
$= 4{,}5625 \times 10^{-5} - (-1{,}04964 \times 10^{-4})$
$\Delta d = 0{,}151 \times 10^{-3} m$ √ (1)
[15]

QUESTION 2

2.1 $M = Fl + \frac{wL^2}{2} = 10 \times 10^3 + \frac{521{,}892 \times 5^2}{2}$ √ $= 36{,}524 kNm$ √ (2)

2.2 $\sigma_t = \frac{MY_t}{I} = \frac{36{,}524 \times 10^3 \times 52 \times 10^{-3}}{15{,}222 \times 10^{-6}}$ √ $= 124{,}77 MPa (tensile)$ √

$\sigma_c = \frac{MY_c}{I} = \frac{36{,}524 \times 10^3 \times 98 \times 10^{-3}}{15{,}222 \times 10^{-6}}$ √ $= 235{,}143 MPa (compressive)$ √ (4)

2.3 $Y_1 = \frac{Fl^2}{2EI} \times l_f = \frac{10 \times 10^3 \times 3^2}{2 \times 200 \times 10^9 \times 15{,}222 \times 10^{-6}} \times 2$ √ $= 29{,}562 \times 10^{-3} m$ √

$Y_2 = \frac{wL^3}{6EI} = \frac{521{,}892 \times 5^3}{6 \times 200 \times 10^9 \times 15{,}222 \times 10^{-6}}$ √ $= 3{,}571 \times 10^{-3} m$ √

$Y_T = Y_1 + Y_2 = 33{,}133 \times 10^{-3} m$ √ (5)
[11]

QUESTION 3

3.1 $A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi(2^2 - 1{,}9^2)}{4} = 0{,}306 m^2$ √

$W = \rho g A h = 7500 \times 9{,}81 \times 0{,}306 \times 30 = 675{,}419 kN$ √

$\sigma_D = \frac{W}{A} = \frac{675{,}419}{0{,}306} = 2{,}207 MPa (compressive)$ √ (3)

3.2 $I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi(2^4 - 1{,}9^4)}{64} = 145{,}686 \times 10^{-3} m^4$ √

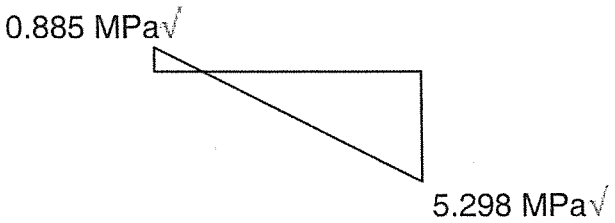
$\sigma_b = \frac{FeY}{I} = \frac{30 \times 10^3 \times 15 \times 1}{145{,}686 \times 10^{-3}}$ √ $= 3{,}0888 MPa$ √ (3)

3.3 $\sigma_{max} = \sigma_d + \sigma_b = 2{,}204 + 3{,}0888$ √ $= 5{,}298 MPa$ $(compressive)$ √

$\sigma_{min} = \sigma_d - \sigma_b = 2{,}204 - 3{,}0888$ √ $= 0{,}885 MPa$ $(tensile)$ √ (4)

3.4 $Y_D = \frac{\sigma_d \times I}{F \times e} = \frac{2{,}204 \times 10^6 \times 145{,}686 \times 10^{-3}}{30 \times 10^3 \times 15}$ √ $= 0{,}714 m$ √ (2)

3.5



(2)
[14]

QUESTION 4

4.1 $W_1 = \rho g A l = 2100 \times 9{,}81 \times 4 \times 2 \times 1 = 164{,}808kN$ √

$W_2 = \rho g A l = 2100 \times 9{,}81 \times 4 \times 3 \times 1 = 247{,}212kN$ √

$W_3 = \rho g A l = 1000 \times 9{,}81 \times 4 \times 1 \times 1 = 39{,}24kN$ √

$V = W_1 + W_2 + W_3 = 451{,}26kN$ √ (4)

4.2 $F_w = \frac{\rho g h^2}{2} = \frac{1000 \times 9{,}81 \times 4^2}{2} = 78{,}48kN$ √

$\sum F - M = F_w \times \frac{h}{3} = 78{,}48 \times \frac{4}{3}$√ $= 104{,}64kNm$ √ (3)

4.3 $W_1 x_1 = 164{,}808 \times 1 = 164{,}808kNm$ √

$W_2 x_2 = 247{,}212 \times 4 = 988{,}848kNm$ √

$W_3 x_3 = 39{,}24 \times 4 = 156{,}96kNm$ √

$\sum W - M = W_1 x_1 + W_2 x_2 + W_3 x_3 = 1310{,}616kNm$ √ (4)

4.4 $FOS = \frac{\sum W - M}{\sum F - M} = \frac{1310{,}616}{104{,}64}$√ $= 12{,}525$ √ (2)

4.5 $FOS = \frac{\mu V}{F} = \frac{0{,}4 \times 451{,}26}{78{,}48}$√ $= 2{,}3$ √

(2)
[15]

QUESTION 5

5.1 $$L = \sqrt{\frac{W_T}{p}} = \sqrt{\frac{2{,}5 \times 10^6 + 150 \times 10^3}{200 \times 10^3}} \surd = 3{,}64m \surd$$ (2)

5.2 $$M = \frac{W(L-l)}{8} = \frac{2{,}5 \times 10^6(3{,}64 - 0{,}8)}{8} \surd = 887{,}5kNm \surd$$ (2)

5.3 $$Z = \frac{M}{\sigma \times n} = \frac{887{,}5 \times 10^3}{100 \times 10^6 \times 4} \surd = 2218{,}75 \times 10^{-6} m^3 \surd$$

$lighest\ I-beam\ is\ 533X210X101\ kg/m$ √ (3)

5.4 $$l = b \times n + 0{,}075(n-1) = 0{,}2101 \times 4 + 0{,}075 \times 3 \surd = 1{,}065m \surd$$ (2)

5.5 $$M = \frac{W(L-l)}{8} = \frac{2{,}5 \times 10^6(3{,}64 - 1{,}065)}{8} \surd = 804{,}563kNm \surd$$ (2)

5.6 $$Z = \frac{M}{\sigma \times n} = \frac{804{,}563 \times 10^3}{100 \times 10^6 \times 16} \surd = 502{,}85 \times 10^{-6} m^3 \surd$$

$lighest\ I-beam\ is\ 305X165X40{,}5\ kg/m$ √ (3)
[14]

QUESTION 6

6.1 $$y_B = \frac{F_{TB}}{w} = \frac{4000}{25} = 160m \surd$$

$$y_A = y_B - h = 160 - 3 = 157m \surd$$

$$F_{TA} = wy_A = 25 \times 157 = 3925N \surd$$ (3)

6.2 $$y_0 = y_A - d = 157 - 4 = 153m \surd$$

$$l_A = \sqrt{y_A^2 - y_0^2} = \sqrt{157^2 - 153^2} = 35{,}214m \surd$$

$$l_B = \sqrt{y_B^2 - y_0^2} = \sqrt{160^2 - 153^2} = 46{,}81m \surd$$

$$l_T = l_A + l_B = 82{,}024m \surd$$ (4)

6.3 $$x_A = y_0 ln\left(\frac{y_A + l_A}{y_0}\right) = 153 \times ln\left(\frac{157 + 35{,}214}{153}\right) = 34{,}91m \ \surd$$

$$x_B = y_0 ln\left(\frac{y_B + l_B}{y_0}\right) = 153 \times ln\left(\frac{160 + 46{,}81}{153}\right) = 46{,}108m \ \surd$$

$$x_T = x_A + x_B = 81{,}018m \ \surd$$

(3)

6.4 $$F_{vc} = wl_A = 25 \times 35{,}214 = 880{,}35N \ \surd$$

$$F_{va} = F_{TA} \cos\alpha = 3925 \times \cos 60 = 1962{,}5N \ \surd$$

$$F_{va} = F_{va} + F_{vc} = 2842{,}85N \ \surd$$

(3)

6.5 $$F_{Hc} = wy_0 = 25 \times 153 = 3825N = F_{Ha}\surd$$

$$F_{Hc} = \frac{F_{Hc}}{\sin\theta} = \frac{3825}{\sin 70} = 4070{,}5N \ \surd$$

(2)
[15]

QUESTION 7

7.1 *Take moments about R bearing:*

$$R_L \times 1 = 2600 \times 0{,}7 + 1962 \times 0{,}2 + 600 \times 1 \times 0{,}5 \ \surd$$

$$R_L = 2512{,}4N \ \surd$$

Take moments about L bearing:

$$R_R \times 1 = 2600 \times 0{,}3 + 1962 \times 0{,}8 + 600 \times 1 \times 0{,}5 \ \surd$$

$$R_R = 2649{,}6N \ \surd$$

(4)

7.2 $$M = 2512{,}4 \times 0{,}3 - 600 \times 0{,}3 \times 0{,}15\surd = 726{,}72Nm \ \surd$$

(2)

7.3 $$T = (T_1 - T_2)r = (2000 - 600)0{,}2\surd = 280Nm \ \surd$$

(2)

7.4 $M_e = 0,5\left(M + \sqrt{M^2 + T^2}\right) = 0,5\left(726,72 + \sqrt{726,72^2 + 280^2}\right) = 752,758Nm$ √

$$M_e = \frac{\pi d^3 \sigma}{32}$$

$$752,758 = \frac{\pi \times d^3 \times 65 \times 10^6}{32} \surd$$

$$d = 49 \times 10^{-3} m \surd$$ (3)

7.5 $$M_e = \frac{\pi(D^4 - d^4)}{32D}$$

$$752,758 = \frac{\pi \times [D^4 - (0,6)^4]}{32D}$$

$$752,758 = \frac{\pi \times (0,8704D^4)}{32D} \surd$$

$$D = 51,366 \times 10^{-3} m \surd$$

$$d = 30,82 \times 10^{-3} m \surd$$ (3)

7.6 $$\%\ saving = \frac{d_s^2 - (D^2 - d^2)}{d_s^2} = \frac{49^2 - (51,366^2 - 30,82^2)\surd}{49^2} = 29,5\% \surd$$ (2)

[16]

TOTAL: 100