



**higher education  
& training**

Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

# **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**STRENGTH OF MATERIALS AND STRUCTURES N6**

**29 NOVEMBER 2019**

**This marking guideline consists of 8 pages.**

**QUESTION 1**

$$1.1 \quad \text{at } 100 \text{ mm} : a - \frac{b}{0,1^2} = 46 \times 10^6 \dots \dots \dots (1) \checkmark$$

$$\text{at } 100 \text{ mm} : a + \frac{b}{0,1^2} = 30 \times 10^6 \dots (2) \checkmark$$

$$(1) + (2) : 2a = 76 \times 10^6$$

$$a = 38 \times 10^6 \checkmark$$

$$b = -0,08 \times 10^6 \checkmark$$

$$\begin{aligned} \text{at } 200 \text{ mm} : \sigma_H &= a - \frac{b}{0,2^2} \\ &= 38 \times 10^6 - \frac{-0,08 \times 10^6}{0,2^2} \end{aligned}$$

$$\sigma_H = 40 \text{ MPa (compressive)} \checkmark$$

$$\begin{aligned} \text{at } 200 \text{ mm} : \sigma_R &= a + \frac{b}{0,2^2} \\ &= 38 \times 10^6 + \frac{-0,08 \times 10^6}{0,2^2} \end{aligned}$$

$$\sigma_R = 36 \text{ MPa (compressive)} \checkmark$$

(6)

$$1.2 \quad \text{at } 200 \text{ mm} : a + \frac{b}{0,2^2} = 36 \times 10^6 \dots \dots \dots (3) \checkmark$$

$$\text{at } 400 \text{ mm} : a + \frac{b}{0,4^2} = 0 \dots \dots \dots (4) \checkmark$$

$$(3) - (4) : 25b - 6,25b = 36 \times 10^6$$

$$b = 1,92 \times 10^6 \checkmark$$

$$a = -12 \times 10^6 \checkmark$$

$$\begin{aligned} \text{at } 200 \text{ mm} : \sigma_H &= a - \frac{b}{0,2^2} \\ &= -12 \times 10^6 - \frac{1,92 \times 10^6}{0,2^2} \end{aligned}$$

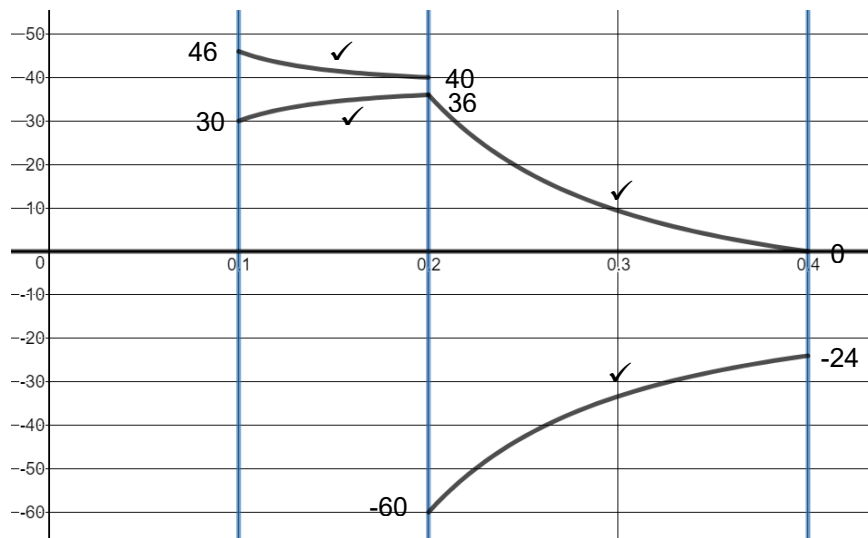
$$\sigma_H = -60 \text{ MPa (tensile)} \checkmark$$

$$\begin{aligned} \text{at } 400 \text{ mm} : \sigma_H &= a - \frac{b}{0,4^2} \\ &= -12 \times 10^6 - \frac{1,92 \times 10^6}{0,4^2} \end{aligned}$$

$$\sigma_H = -24 \text{ MPa (tensile)} \checkmark$$

(6)

1.3



(4)  
[16]

**QUESTION 2**

2.1

$$y_1 = \frac{wl_1^4}{8EI} + \frac{wl_1^3 \times l_2}{6EI}$$

$$= \frac{20 \times 10^3 \times 1,25^4}{8 \times 200 \times 10^9 \times I} + \frac{20 \times 10^3 \times 1,25^3 \times 1,25}{6 \times 200 \times 10^9 \times I} \checkmark$$

$$y_1 = \frac{71,208 \times 10^{-9}}{I} \checkmark$$

$$y_2 = \frac{FL^3}{3EI} = \frac{2 \times 10^3 \times 2,5^3}{3 \times 200 \times 10^9 \times I} = \frac{52,083 \times 10^{-9}}{I} \checkmark$$

$$y_T = y_1 + y_2$$

$$7 \times 10^{-3} = \frac{71,208 \times 10^{-9}}{I} + \frac{52,083 \times 10^{-9}}{I} \checkmark$$

$$I = 17,613 \times 10^{-6} m^4 \checkmark$$

$$I = \frac{\pi(D^4 - d^4)}{64}$$

$$17,613 \times 10^{-6} = \frac{\pi((2d)^4 - d^4)}{64} \checkmark$$

$$d = 70 \text{ mm} \checkmark$$

$$D = 140 \text{ mm} \checkmark$$

(8)

2.2 For  $I = 17,613 \times 10^{-6} m^4$  select  $203 \times 102 \times 25,3 \text{ kg/m}$

(1)

2.3

$$M = FL + \frac{wl^2}{2}$$

$$= 2 \times 10^3 \times 2,5 + \frac{20 \times 10^3 \times 1,25^2}{2} \checkmark$$

$$M = 20,625 \text{ kNm} \checkmark$$

$$\sigma = \frac{M}{Z} = \frac{20,625 \times 10^3}{226,1 \times 10^{-6}} = 91,221 \text{ MPa} \checkmark$$

(3)  
[12]

**QUESTION 3**

3.1

$$A = \pi r^2 = \pi \times 0,015^2 = 706,858 \times 10^{-6} \text{ m}^2 \checkmark$$

$$\sigma_d = \frac{F}{A} = \frac{F}{706,858 \times 10^{-6}} = 1414,711F \checkmark$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 0,03^4}{64} = 39,761 \times 10^{-9} \text{ m}^4 \checkmark$$

$$\sigma_b = \frac{F \times e \times Y}{I} = \frac{F \times 0,09 \times 0,015}{39,761 \times 10^{-9}} = 33953,055F \checkmark$$

$$\sigma_{Rmax} = \sigma_d + \sigma_b$$

$$90 \times 10^6 = 1414,711F + 33953,055F \checkmark$$

$$F = 2544,69 \text{ N} \checkmark \leftrightarrow m = 259,4 \text{ kg} \checkmark$$

(7)

3.2

$$\sigma_d = 1414,711F = 3,6 \text{ MPa (tensile)} \checkmark$$

$$\sigma_b = 33953,055F = 86,4 \text{ MPa} \checkmark$$

$$\sigma_{min} = \sigma_b - \sigma_d = 82,8 \text{ MPa} \checkmark \text{ (compressive)} \checkmark$$

(4)  
[11]

**QUESTION 4**

$$4.1 \quad W = \rho g A l = 2500 \times 9,81 \times 2,1 \times h \times 1 = 51502,5h \quad \checkmark$$

$$F_w = \frac{\rho g h^2}{2} = \frac{1000 \times 9,81 \times h^2}{2} = 4905h^2 \quad \checkmark$$

$$\text{Moments about the toe: } V \times x_R + F_g \times \frac{h}{3} = W \times x_1$$

$$51502,5h \times 0,6 \quad \checkmark + 4905h^2 \times 0,333h \quad \checkmark = 51502,5h \times 1,05 \quad \checkmark$$

$$h = 3,765 \text{ m} \quad \checkmark \quad (6)$$

$$4.2 \quad W \sim M = W \times x_1 = (51502,5 \times 3,765) \times 1,05 = 203,601 \text{ kNm} \quad \checkmark$$

$$F \sim M = F_w \times \frac{h}{3} = 4905h^2 \times \frac{h}{3} = 4905 \times 3,765^2 \times \frac{3,765}{3} = 87,257 \text{ kNm} \quad \checkmark$$

$$FOS = \frac{W \sim M}{F \sim M} = \frac{203,601}{87,257} = 2,3 \text{ (safe)} > 2 \quad \checkmark \quad (3)$$

$$4.3 \quad e = 0,5B - x = 1,05 - 0,6 = 0,45 \text{ m} \quad \checkmark$$

$$V = W = 51502,5h = 51502,5 \times 3,765 = 193,905 \text{ kN} \quad \checkmark$$

$$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} = \frac{193,905}{2,1} + \frac{6 \times 193,905 \times 0,45}{2,1^2} = 211,053 \text{ kPa (comp)} \quad \checkmark$$

$$\sigma_{min} = \frac{V}{B} - \frac{6Ve}{B^2} = \frac{193,905}{2,1} - \frac{6 \times 193,905 \times 0,45}{2,1^2} = -26,382 \text{ kPa (tensile)} \quad \checkmark \quad (4)$$

**[13]****QUESTION 5**

$$5.1 \quad W_c = 137 \times 9,81 \times 4 = 5,376 \text{ kN} \quad \checkmark$$

$$F_c = \sigma A = 9 \times 10^6 \times 17,42 \times 10^{-3} = 156,78 \text{ kN} \quad \checkmark$$

$$W_T = F_c + W_c + W_f = 156,78 + 5,376 + 25 = 187,156 \text{ kN} \quad \checkmark$$

$$p = \frac{W_T}{A} = \frac{187,156 \times 10^3}{1,2^2} = 129,969 \text{ kPa} \quad \checkmark$$

$$\sigma_{ult} = p \times FOS = 129,969 \times 3 \quad \checkmark = 389,908 \text{ kPa} \quad \checkmark \quad (6)$$

$$5.2 \quad t = \frac{L - l}{\sqrt{2}} = \frac{1,2 - 0,5}{\sqrt{2}} \quad \checkmark = 0,495 \text{ m} \quad \checkmark \quad (2)$$

$$5.3 \quad d = \frac{p}{\rho g} \left( \frac{1 - \sin \theta}{1 + \sin \theta} \right)^2 = \frac{138,889 \times 10^3}{1600 \times 9,81} \left( \frac{1 - \sin 30}{1 + \sin 30} \right)^2 \quad \checkmark = 0,983 \text{ m} \quad \checkmark \quad (2)$$

**[10]**

**QUESTION 6**

$$6.1 \quad \frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$$

$$\frac{140}{8} = \frac{15(0,38-n)}{n} \quad \checkmark$$

$$n = 0,175 \text{ m} \quad \checkmark \quad (2)$$

$$6.2 \quad M_s = \sigma_s A_s (d-n) = 140 \times 10^6 \times 6400 \times 10^{-6} (0,38 - 0,175) \quad \checkmark$$

$$M_s = 183,335 \text{ kNm} \quad \checkmark$$

$$M_c = M - M_s = 300 - 183,335 = 116,665 \text{ kNm} \quad \checkmark \quad (3)$$

$$6.3 \quad \sigma_{c1} = \frac{\sigma_c(n-t)}{n} = \frac{8(0,175-0,1)}{0,175} \quad \checkmark = 3,439 \text{ MPa} \quad \checkmark \quad (2)$$

$$6.4 \quad M_c = \frac{1}{2} \sigma_c b n \frac{2}{3} n - \frac{1}{2} \sigma_{c1} (b-e)(n-t) \frac{2}{3} (n-t)$$

$$116665 = \frac{8 \times 10^6 \times b \times 0,175^2}{3} - \frac{3,439 \times 10^6 \times (b-0,27)(0,175-0,1)^2}{3} \quad \checkmark$$

$$116665 = 82026,0355b \quad \checkmark - 6513,665b \quad \checkmark + 1758,69 \quad \checkmark$$

$$b = 1,522 \text{ m} \quad \checkmark \quad (5)$$

**[12]**

**QUESTION 7**

$$7.1 \quad w = \frac{3024}{2 \times 36} = 42 \text{ kN/m} \checkmark \quad (1)$$

$$7.2 \quad F_H = \frac{wx^2}{2d} = \frac{42 \times 10^3 \times 18^2}{2 \times 3} = 2268 \text{ kN} \checkmark$$

$$F_v = wx = 21 \times 10^3 \times 18 = 378 \text{ kN} \checkmark$$

$$F_T = \sqrt{F_H^2 + F_v^2} = \sqrt{2268^2 + 378^2} = 2299,284 \text{ kN} \checkmark \quad (3)$$

$$7.3 \quad \sigma = \frac{\sigma_{ult}}{FOS} = \frac{320}{8} = 40 \text{ MPa} \checkmark$$

$$A = \frac{F}{\sigma} = \frac{2299,284 \times 10^3}{40 \times 10^6} = 57,482 \times 10^{-3} \text{ m}^2 \checkmark$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{57,482 \times 10^{-3}}{\pi}} = 27,1 \text{ mm} \checkmark \quad (3)$$

$$7.4 \quad \text{At 10 m from lower support: } x_2 = 18 - 10 = 8 \text{ m} \checkmark$$

$$F_{v2} = wx_2 = 21 \times 10^3 \times 8 = 168 \text{ kN} \checkmark$$

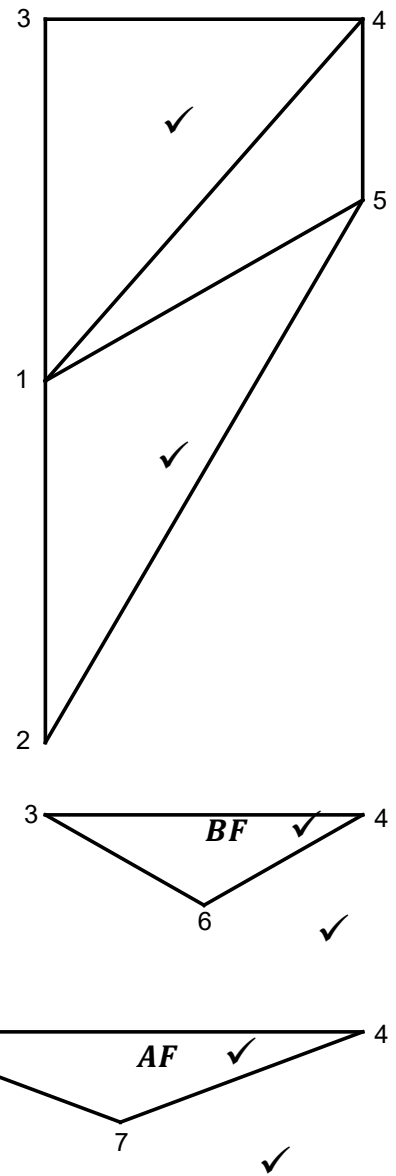
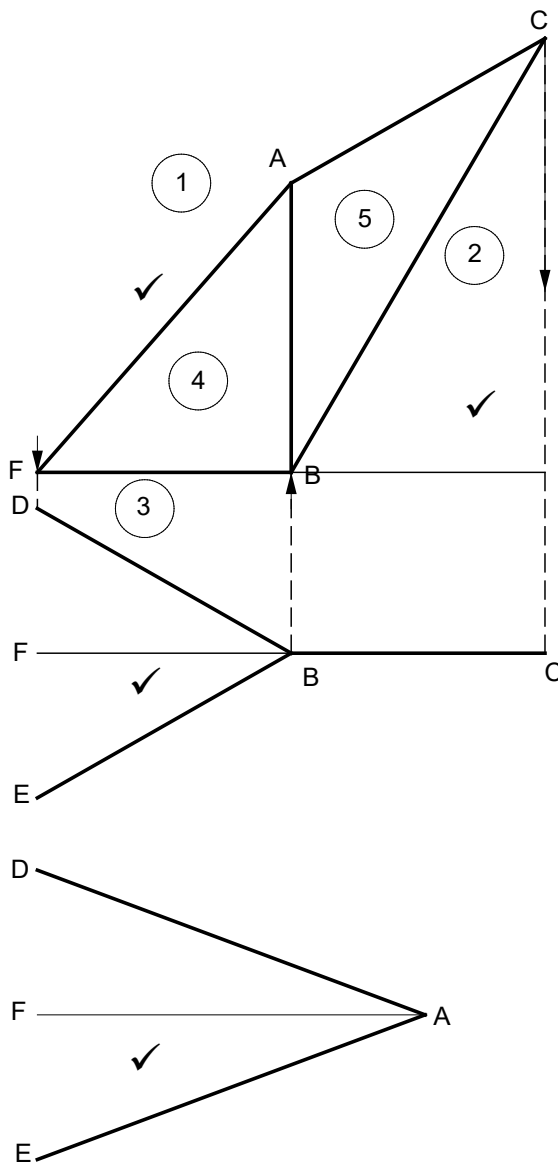
$$F_{T2} = \sqrt{F_H^2 + F_{v2}^2} = \sqrt{2268^2 + 168^2} = 2274,214 \text{ kN} \checkmark \quad (3)$$

**[10]**

**QUESTION 8**

8.1 **SPACE DIAGRAMS** 8.1 and 8.2

**VECTOR DIAGRAMS**



(10)

8.3

MEMBER	MAGNITUDE	NATURE
AB (4-5)	25 kN ✓	Strut ✓
AC (1-5)	50 kN ✓	Tie ✓
BC (2-5)	86,6 kN ✓	Strut ✓
DB (3-6) and EB (6-4)	25 kN ✓	Strut ✓
DA (1-7) and EA (7-4)	35,4 kN ✓	Tie ✓
R <sub>B</sub> (2-3)	100 kN ✓	Upwards ✓

(NOTE: ✓ = Half mark)

(6)

[16]

**TOTAL: 100**