higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N6

28 MAART 2018

Hierdie nasienriglyn bestaan uit 15 bladsye.

VRAAG 1

1.1 $\dfrac{\partial z}{\partial x} = $ ✓ ✓

           ✓ ✓
       $\dfrac{\partial z}{\partial y} = xy^{x-1}$

1.2 $z = \tan x \sec y$

       $\dfrac{\partial z}{\partial x} = \sec^2 x \sec$ ✓

       $\dfrac{\partial^2 z}{\partial y \partial x} = \sec^2 x \sec y \tan y$ ✓

       $\dfrac{\partial z}{\partial y} = \tan x \sec y \tan y$ ✓

       $\dfrac{\partial^2 z}{\partial x \partial y} = \sec^2 x \sec y \tan y$ ✓

       $\therefore \dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y}$

1.3 $y = 2 \sin \theta$ $x = 2 \cos \theta$

       $\dfrac{dy}{d\theta} = 2 \cos \theta$ ✓ $\dfrac{dx}{d\theta} = -2 \sin \theta$ ✓

       $\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \dfrac{2 \cos \theta}{-2 \sin \theta} = -\cot \theta$ ✓

       Wanneer $x = \dfrac{\pi}{4}$ $\dfrac{dy}{dx} = -\dfrac{1}{\tan \dfrac{\pi}{4}} = $ ✓

                                                                   (3 × 2)   **[6]**

VRAAG 2

2.1 $\int \sin^4 4x \, dx$

$= \int (\sin^2 4x)^2 \, dx$ ✔

$= \int (\frac{1}{2} - \frac{1}{2}\cos 8x)^2 \, dx$ ✔ ✔

$= \int (\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{4}\cos^2 8x) \, dx$ ✔

$= \int \left(\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{4}(\frac{1}{2} + \frac{1}{2}\cos 16x) \right) dx$ ✔

$= \int (\frac{1}{4} - \frac{1}{2}\cos 8x + \frac{1}{8} + \frac{1}{8}\cos 16x) \, dx$ ✔

$= \int (\frac{3}{8} - \frac{1}{2}\cos 8x + \frac{1}{8}\cos 16x) \, dx$ ✔

$= \frac{3}{8}x - \frac{1}{2}.\frac{1}{8}\sin 8x + \frac{1}{8}.\frac{1}{16}\sin 16x + C$ ✔ ✔ ✔

$= \frac{3}{8}x - \frac{1}{16}\sin 8x + \frac{1}{128}\sin 16x + C$

> Of gebruik
> $\frac{1}{4}\int \cos^2 8x \, dx = \frac{1}{4}\left(\frac{x}{2} - \frac{\sin 16x}{32} \right)$
> $= \frac{x}{8} - \frac{1}{128}\sin 16x$

(5)

2.2 $\int \frac{\cos 3x}{e^{2x}} \, dx$

$\int e^{-2x}\cos 3x \, dx \qquad f(x) = e^{-2x} \quad g'(x) = \cos 3x$

$= e^{-2x}\frac{\sin 3x}{3} - \int -2e^{-2x}\frac{\sin 3x}{3} \, dx$ ✔ ✔

$= e^{-2x}\frac{\sin 3x}{3} + \frac{2}{3}\int e^{-2x}\sin 3x \, dx \qquad\qquad f(x) = e^{-2x} \quad g'(x) = \sin 3x$ ✔

$= e^{-2x}\frac{\sin 3x}{3} + \frac{2}{3}\left[e^{-2x}.-\frac{\cos 3x}{3} - \int -2e^{-2x} -\frac{\cos 3x}{3} \, dx \right]$ ✔ ✔

$= e^{-2x}\frac{\sin 3x}{3} - \frac{2}{9}$ ✔ $\cos 3x - \frac{4}{9}\int e^{-2x}\cos 3x \, dx$ ✔

$\therefore \int e^{-2x}\cos 3x \, dx + \frac{4}{9}\int e^{-2x}\cos 3x \, dx = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x$ ✔ ✔

$\frac{13}{9}\int e^{-2x}\cos 3x \, dx = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x$

$\int e^{-2x}\cos 3x \, dx = \frac{9}{13}\left[\frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x \right] + C$ ✔

$or = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x) + C$

Alternatief vir 2.2 : Begin met $f(x) = \cos 3x$ en $g'(x) = e^{-2x}$

(5)

2.3 $\int \tan^3 5x(\sec^2 5x - 1)dx$

$\int (\tan^3 5x \sec^2 5x - \tan^3 5x)dx$ ✔

                                      ✔

$\int (\tan^3 5x \sec^2 5x - \tan 5x \tan^2 5x)dx$

$\int (\tan^3 5x \sec^2 5x - \tan 5x(\sec^2 5x - 1))dx$ ✔

                                      ✔

$\int (\tan^3 5x \sec^2 5x - \tan 5x \sec^2 5x + \tan 5x)dx$

✔   ✔   ✔

$\dfrac{1}{5}\dfrac{\tan^4 5x}{4} - \dfrac{1}{5}\dfrac{\tan^2 5x}{2} + \dfrac{1}{5}\ln(\sec 5x) + C$ (4)

2.4 $\int \dfrac{1}{\sqrt{1 + 3x - x^2}}dx$

$-x^2 + 3x + 1$

$= -\left[x^2 - 3x - 1\right]$ ✔

                          ✔   ✔

$= -\left[\left\{x^2 - 3x + \left(\dfrac{3}{2}\right)^2\right\} - \left(\dfrac{3}{2}\right)^2 - 1\right]$

$= -\left[\left(x - \dfrac{3}{2}\right)^2 - \dfrac{13}{4}\right]$ ✔

$= \dfrac{13}{4} - \left(x - \dfrac{3}{2}\right)^2$ ✔

$$\int \frac{1}{\sqrt{\frac{13}{4}-\left(x-\frac{3}{2}\right)^2}}dx \quad \checkmark$$

$$\frac{1}{b^2}\sin^{-1}\frac{bx}{a}+C$$

$$=\frac{1}{1}\sin^{-1}\frac{1\left(x-\frac{3}{2}\right)}{\sqrt{\frac{13}{4}}}+C \quad \checkmark$$

$$=\sin^{-1}\frac{2\left(x-\frac{3}{2}\right)}{\sqrt{13}}+C$$

$$=\sin^{-1}\frac{(2x-3)}{\sqrt{13}}+C$$

Of gebruik die formule $\quad ax^2+bx+c=\dfrac{4ac-b^2}{4a}+a\left(x+\dfrac{b}{2a}\right)^2$

$$-x^2+3x+1$$
$$\qquad\checkmark \qquad\qquad \checkmark$$

$$=\frac{4(-1)1-(3)^2}{4(-1)}+(-1)\left(x+\frac{3}{2.-1}\right)^2$$

$$\qquad\qquad \checkmark$$
$$=\frac{\overset{\checkmark}{-4-9}}{-4}-\left(x-\frac{3}{2}\right)^2$$

$$=\frac{13}{4}-\left(x-\frac{3}{2}\right)^2$$

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VRAAG 3

3.1 $\int \dfrac{(x+3)(x+4)}{(x+1)(x-2)}dx$

gebruik lang deling

$$\frac{(x+3)(x+4)}{(x+1)(x-2)} = \frac{x^2+7x+12}{x^2-x-2} \checkmark = 1 + \frac{8x+14}{(x+1)(x-2)} \checkmark$$

$$\frac{8x+14}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} \quad \checkmark$$

$$8x+14 = A(x-2) + B(x+1) \quad \checkmark$$

$$x=2 \qquad 8(2)+14 = 0 + B(2+1) \qquad\qquad B=10 \quad \checkmark$$

$$x=-1 \qquad 8(-1)+14 = \quad A(-1-2) \qquad\qquad A=-2 \quad \checkmark$$

$$\therefore \int \frac{(x+3)(x+4)}{(x+1)(x-2)}dx = \int 1 + \frac{8x+14}{(x+1)(x-2)}dx$$

$$= \int 1 + \frac{-2}{x+1} + \frac{10}{x-2} \, dx \quad \checkmark$$

$$\checkmark \qquad \checkmark \qquad \checkmark\ \checkmark$$

$$= x - 2\ln(x+1) + 10\ln(x-2) + C$$

Alternatief 3.1 (Nota : Aangesien die noemer en teller van tweede graad met koëffisiënt 1 is, sal die koëffisiënt 1 wees en die res sal lineêr wees. Die rasionale breuke verkry kan sonder langdeling in parsiële breuke opgelos word. Daarvolgens kan die volgende aanvaar word)

$$\frac{(x+3)(x+4)}{(x+1)(x-2)} = 1 + \frac{A}{x+1} + \frac{B}{x-2} \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$(x+3)(x+4) = 1(x+1)(x-2) + A(x-2) + B(x+1) \quad \checkmark \quad \checkmark$$

$$x=-1 \qquad (-1+3)(-1+4) = +A(-1-2) \qquad A=-2 \quad \checkmark$$

$$x=2 \qquad (2+3)(2+4) = 1B(2+1) \qquad B=10 \quad \checkmark$$

$$\therefore \int \frac{(x+3)(x+4)}{(x+1)(x-2)}dx = \int 1 + \frac{-2}{x+1} + \frac{10}{x-2} \, dx \quad \checkmark$$

$$\checkmark \qquad \checkmark \qquad \checkmark$$

$$= x - 2\ln(x+1) + 10\ln(x-2) + C$$

(6)

3.2 $\dfrac{3x^2 - 2x - 4}{\left(2x^21\right)\left(x-1\right)} = \dfrac{Ax+B}{2x^2+1} + \dfrac{C}{x-1}$

$3x^2 - 2x - 4 = \left(Ax + B\right)\left(x - 1\right) + C\left(2x^2 + 1\right)$ ✓

$x = 1 \qquad 3 - 2 - 4 = C\left(2 + 1\right) \qquad C = -1$ ✓

$3x^2 - 2x - 4 = Ax^2 + Bx - Ax - B + 2Cx^2 + C$

$x^2: \quad A + 2C = 3 \qquad \therefore A - 2 = 3 \qquad A = 5$ ✓

$x: \qquad B - A = -2 \quad \therefore B = A - 2 = 5 - 2 = 3 \quad B = 3$ ✓

$\displaystyle\int \dfrac{3x^2 - 2x - 4}{\left(2x^21\right)\left(x-1\right)}\,dx = \int \dfrac{5x+3}{2x^2+1}\,dx + \int \dfrac{-1}{x-1}\,dx$ ✓

$\qquad\qquad = \displaystyle\int \dfrac{5x}{2x^2+1}\,dx + \int \dfrac{3}{2x^2+1}\,dx + \int \dfrac{-1}{x-1}\,dx$ ✓

✓ ✓ ✓ ✓

$\qquad\qquad = \dfrac{5}{4}\ln\left(2x^2+1\right) + \dfrac{3}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right) - \ln\left(x-1\right) + C$

(6)

[12]

VRAAG 4

4.1 $x\dfrac{dy}{dx} + 2y = xe^x$

$\dfrac{dy}{dx} + 2\dfrac{y}{x} = e^x$ ✓

✓ ✓ ✓ ✓

$e^{\int p\,dx} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

$\displaystyle\int Qe^{\int p\,dx}\,dx = \int e^x x^2\,dx$

✓ ✓

$\qquad = x^2 e^x - \displaystyle\int 2xe^x\,dx$

✓

$\qquad = x^2 e^x - \left[2xe^x - \displaystyle\int 2e^x\,dx\right]$

$\qquad = x^2 e^x - 2xe^x + 2e^x$ ✓

$yx^2 = x^2 e^x - 2xe^x + 2e^x + C$ ✓

(5)

4.2 $\dfrac{d^2 y}{dx^2} + 4\dfrac{dy}{dx} - 5y = 2e^x$

$r^2 + 4r - 5 = 0$ ✔

$(r+5)(r-1) = 0$

    ✔ ✔

$r = -5 \quad r = 1$

$y_c = Ae^{-5x} + Be^x$ ✔

$y = Cxe^x$ ✔

$\dfrac{dy}{dx} = Cxe^x + Ce^x$ ✔

$\dfrac{d^2 y}{dx^2} = Cxe^x + Ce^x + Ce^x$

   $= Cxe^x + 2Ce^x$ ✔

$\therefore Cxe^x + 2Ce^x + 4\left(Cxe^x + Ce^x\right) - 5Cxe^x = 2e^x$ ✔

$Cxe^x + 2Ce^x + 4Cxe^x + 4Ce^x - 5Cxe^x = 2e^x$

$6Ce^x = 2e^x \quad \therefore C = \dfrac{1}{3}$ ✔

$y_p = \dfrac{1}{3} xe^x$

$y = y_c + y_p$

$y = Ae^{-5x} + Be^x + \dfrac{1}{3} xe^x$ ✔

$x = 0 \ and \ y = 1 \quad 1 = A + B \dots\dots\dots\dots(1)$

$\dfrac{dy}{dx} = -5Ae^{-5x} + Be^x + \dfrac{1}{3} xe^x + \dfrac{1}{3} e^x$ ✔

$x = 0 \ and \ \dfrac{dy}{dx} = 0 \qquad 0 = -5A + B + \dfrac{1}{3} \qquad 5A - B = \dfrac{1}{3} \dots\dots(2)$

$(2)+(1) \qquad 6A = \dfrac{4}{3} \qquad\qquad A = \dfrac{2}{9}$ ✔ $\quad \therefore B = 1 - A = \dfrac{7}{9}$ ✔

$y = \dfrac{2}{9} e^{-5x} + \dfrac{7}{9} e^x + \dfrac{1}{3} xe^x$ ✔

                          (7)

                          [12]

> *or* $y = Ae^x + Be^{-5x}$
>
> *Also* $y = 2Cxe^x$ wat sal
>
> gee $C = \dfrac{1}{6}$ Daarvolgens
>
> $y_p = 2Cxe^x = 2\dfrac{1}{6} xe^x = \dfrac{1}{3} xe^x$

5.1   5.1.1

✓ elke grafiek
✓ area
✓ strook

(2)

5.1.2
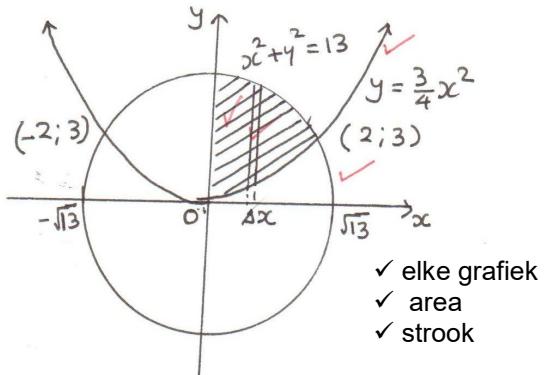
$$A = \int_a^b y_1 - y_2 \, dx$$

$$= \int_0^2 \sqrt{13 - x^2} - \frac{3}{4}x^2$$

$$= \left[\frac{13}{2} \sin^{-1} \frac{x}{\sqrt{13}} + \frac{x}{2}\sqrt{13 - x^2} - \frac{3}{4}\frac{x^3}{3} \right]_0^2$$

$$= \frac{13}{2} \sin^{-1} \frac{2}{\sqrt{13}} + \frac{2}{2}\sqrt{13 - 2^2} - \frac{3}{4}\frac{x^3}{3} - 0 \quad ✓$$

$$= 4{,}822 \, units^2 \quad ✓ \quad ✓$$

$$y = 4x - x^2 \qquad\qquad y = x$$
$$\therefore x = 4x - x^2$$
$$x^2 - 3x = 0$$
$$x(x - 3) = 0$$
$$x = 0 \quad x = 3$$
$$y = 0 \quad y = 3 \qquad (0;0) \quad (3;3)$$

(4)

5.2   5.2.1

✓ elke grafiek
✓ area
✓ strook
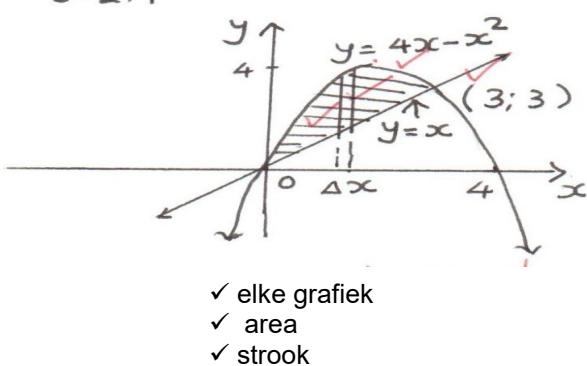
$$V_x = \pi \int_a^b y_1^2 - y_2^2 \, dx$$

$$= \pi \int_0^3 \left(4x - x^2\right)^2 - x \, dx$$

$$= \pi \int_0^3 16x^2 - 8x^3 + x^4 - x^2 \, dx$$

$$= \pi \int_0^3 15x^2 - 8x^3 + x^4 \, dx \qquad (3)$$

5.2.2
$$= \pi \left[\frac{15x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5} \right]_0^3$$

$$= \pi \left[5(3)^3 - 2(3)^4 + \frac{(3)^5}{5} - 0 \right]$$

$$= 21,6\pi \qquad = 21\frac{3}{5}\pi \qquad = \frac{08}{5}\pi$$

$$= 67,858 \, units^3$$

$$V_{m-y} = \int_a^b r \, dv$$

$$= \int_a^b x 2\pi \left(\frac{y_1 + y_2}{2} \right) (y_1 - y_2) \, dv$$

$$= \pi \int_a^b x(y_1^2 - y_2^2) \, dx \qquad \checkmark \ \checkmark \ \checkmark$$

> 1 regmerk vir elke stap of 3 regmerke indien slegs hierdie laaste stap

$$(4)$$

5.2.3
$$= \pi \int_0^3 x\left(15x^2 - 8x^3 + x^4\right) dx \qquad note: y_1^2 - y_2^2 \ from \ 5.2.2$$

$$= \pi \int_0^3 \left(15x^3 - 8x^4 + x^5\right) dx \qquad \checkmark$$

$$= \pi \left[\frac{15x^4}{4} - \frac{8x^5}{5} + \frac{x^6}{6} \right]_0^3 \qquad \checkmark$$

$$= \pi \left[\frac{15(3)^4}{4} - \frac{8(3)^5}{5} + \frac{(3)^6}{6} - 0 \right] \qquad \checkmark$$

$$= \pi 36,45 \, units^4 \qquad = \pi \frac{729}{20} \, units^4 \qquad = \pi 39\frac{9}{20} \, units^4$$

$$= 114,51 \, lunits^4$$

$$\overline{x} = \frac{\pi 36,45 \, units}{21,6\pi \, units^3} \qquad \checkmark \ 1,688 \, units \qquad \checkmark$$

$$(5)$$

5.3   5.3.1
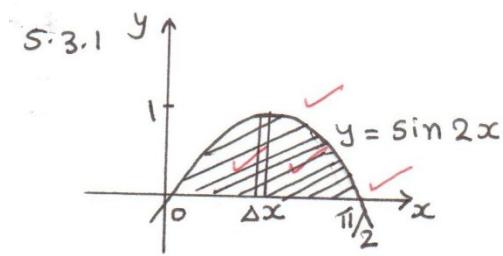
✓ vorm
✓ *x* snypunt   (2)
✓ stoip

$$A = \int_0^{\frac{\pi}{2}} \sin 2x\, dx$$

$$= \left[-\frac{\cos 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= -\frac{\cos 2\frac{\pi}{2}}{2} - \left(-\frac{\cos 2(0)}{2} \right)$$

$$= 1\ units^2$$

5.3.2 $$I_x = \int_a^b r^2 dA$$

$$I_x = \int_0^{\frac{\pi}{2}} x^2 \sin 2x\, dx$$

$$= \left[x^2(-\frac{1}{2}\cos 2x) - \int 2x(-\frac{1}{2}\cos 2x)dx \right]_0^{\frac{\pi}{2}}$$

$$= \left[-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x\, dx \right]_0^{\frac{\pi}{2}}$$ (3)

5.3.3 $$= \left[-\frac{1}{2}x^2 \cos 2x + \left\{ x\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \right\} \right]_0^{\frac{\pi}{2}}$$

$$= \left[-\frac{1}{2}x^2 \cos 2x + \left\{ x\frac{1}{2}\sin 2x + \frac{1}{4}\cos 2x \right\} \right]_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\left(\frac{\pi}{2}\right)^2 \cos 2\left(\frac{\pi}{2}\right) + \left\{ \left(\frac{\pi}{2}\right)\frac{1}{2}\sin 2\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos 2\left(\frac{\pi}{2}\right) \right\} - \left\{ 0 + 0 + \frac{1}{4} \right\}$$

$$= 0{,}7337\, units^4$$ (5)

5.4.1
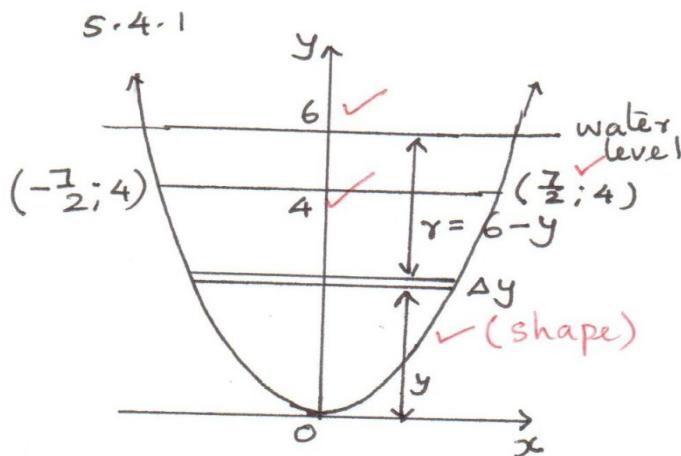
$y = ax^2$ ✓ $4 = a\left(\dfrac{7}{2}\right)^2$ ✓ $4 = a\left(\dfrac{49}{4}\right)$ $a = \dfrac{16}{49}$ ✓

$\therefore y = \dfrac{16}{49}x^2$ $x^2 = \dfrac{49}{16}y$ $x = \dfrac{7}{4}y^{\frac{1}{2}}$ ✓

Eerste moment van area $= \displaystyle\int_a^b r\,dA = \int_0^4 (6-y)\,2x\,dy$

✓

$= \displaystyle\int_0^4 (6-y)\,2\dfrac{7}{4}y^{\frac{1}{2}}\,dy$ ✓

$= \dfrac{7}{2}\displaystyle\int_0^4 (6-y)\,y^{\frac{1}{2}}\,dy$

$= \dfrac{7}{2}\displaystyle\int_0^4 \left(6y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy$ ✓

$= \dfrac{7}{2}\left[\dfrac{6y^{\frac{3}{2}}}{\dfrac{3}{2}} - \dfrac{y^{\frac{5}{2}}}{\dfrac{5}{2}}\right]_0^4$

$= \dfrac{7}{2}\left[4y^{\frac{3}{2}} - \dfrac{2}{5}y^{\frac{5}{2}}\right]_0^4$ ✓

$= \dfrac{7}{2}\left[4(4)^{\frac{3}{2}} - \dfrac{2}{5}(4)^{\frac{5}{2}} - 0\right]$ ✓ $= 67,2m^3$ ✓

Tweede moment=

$\displaystyle\int_a^b r^2\,dA = \int_0^4 (6-y)^2\,2.\dfrac{7}{4}y^{\frac{1}{2}}\,dy$ ✓ ✓ ✓

$= \dfrac{7}{2}\displaystyle\int_0^4 (36 - 12y + y^2).y^{\frac{1}{2}}\,dy$ ✓

$= \dfrac{7}{2}\displaystyle\int_0^4 (36y^{\frac{1}{2}} - 12y^{\frac{3}{2}} + y^{\frac{5}{2}})\,dy$ ✓

$$= \frac{7}{2}\left[36y^{\frac{3}{2}}\frac{2}{3} - 12y^{\frac{5}{2}}\frac{2}{5} + y^{\frac{7}{2}}\frac{2}{7}\right]_0^4 \quad \checkmark$$

$$= \frac{7}{2}\left[36(4)^{\frac{3}{2}}\frac{2}{3} - 12(4)^{\frac{5}{2}}\frac{2}{5} + (4)^{\frac{7}{2}}\frac{2}{7} - 0\right] \quad \checkmark$$

$$= \frac{7}{2}\left[24(4)^{\frac{3}{2}} - \frac{24}{5}(4)^{\frac{5}{2}} + (4)^{\frac{7}{2}}\frac{2}{7} - 0\right]$$

$$= 262,4m^4 \quad \checkmark$$

$$\overline{y} = \frac{262,4}{67,2}\,m \quad \checkmark$$

$$= 3,905 \text{ m}$$

5.4.1 (alternative)
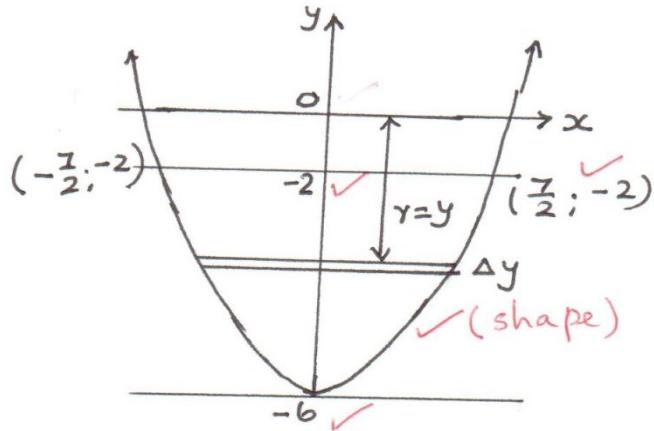
$$y = ax^2 - 6 \quad \checkmark$$

$$-2 = a\left(\frac{7}{2}\right)^2 - 6 \quad \checkmark \qquad a = \frac{16}{49} \quad \checkmark$$

$$y = \frac{16}{49}x^2 - 6 \qquad x^2 = \frac{49}{16}(y+6) \qquad x = \frac{7}{4}(y+6)^{\frac{1}{2}} \quad \checkmark$$

(4)

5.4.2   **(Alternatief)**
First moment

$$= \int_{-6}^{-2} y2\left\{\frac{7}{4}(y+6)^{\frac{1}{2}}\right\}dy \qquad \checkmark \qquad \checkmark$$

$$= \frac{7}{2}\int_{-6}^{-2} y(y+6)^{\frac{1}{2}}dy \qquad\qquad u=y+6 \quad y=u-6 \quad dy=du$$

$$y=-2 \quad then \; u=4 \; and \; y=-6 \; then \; u=0$$

$$= \frac{7}{2}\int_{0}^{4}(u-6)u^{\frac{1}{2}}du \qquad \checkmark$$

$$= \frac{7}{2}\int_{0}^{4}\left(u^{\frac{3}{2}}-6u^{\frac{1}{2}}\right)du \qquad\qquad \checkmark$$

$$= \frac{7}{2}\left[\frac{2}{5}u^{\frac{5}{2}}-6\frac{2}{3}u^{\overset{\checkmark}{\frac{3}{2}}}\right]_{4}^{0} = \frac{7}{2}\left[0-\left\{\frac{2}{5}(4)^{\frac{5}{2}}-4(4)^{\frac{3}{2}}\right\}\right]_{0}^{4} \qquad \checkmark$$

$$= --67,2 \; m^3 \qquad \checkmark$$

(3)

5.4.3   **Tweede moment (Alternatief)**

$$= \int_{a}^{b} r^2 dA$$

$$\qquad\qquad\qquad\qquad \overset{\checkmark\quad\checkmark}{}$$

$$= \int_{-6}^{-2} y^2 2\left\{\frac{7}{4}(y+6)^{\frac{1}{2}}\right\}dy \qquad u=y+6 \qquad y=u-6 \qquad dy=du$$

$$y=-2 \; then \; u=4 \; and \; y=-6 \; then \; \checkmark$$

$$= \frac{7}{2}\int_{0}^{4}(u-6)^2 u^{\frac{1}{2}}du \qquad \checkmark$$

$$= \frac{7}{2}\int_{0}^{0}(u^2-12u+36)u^{\frac{1}{2}}du \qquad \checkmark$$

$$= \frac{7}{2}\int_{4}^{0}(u^{\frac{5}{2}}-12u^{\frac{3}{2}}+36u^{\frac{1}{2}})du \qquad \checkmark$$

$$= \frac{7}{2}\left[(\frac{2}{7}u^{\frac{7}{2}}-12u^{\frac{5}{2}}\frac{2}{5}+36u^{\frac{3}{2}}\frac{2}{3})\right]_{4}^{0} \qquad \checkmark$$

$$= \frac{7}{2}\left[0-\left\{\frac{2}{7}(4)^{\frac{7}{2}}-12(4)^{\frac{5}{2}}\frac{2}{5}+36(4)^{\frac{3}{2}}\frac{2}{3}\right\}\right] \qquad \checkmark$$

$$= 262,4 m^4 \qquad \checkmark$$

$$\overset{=}{y} = \frac{262,4}{-67,2}\overset{\checkmark}{=}-3,905m \qquad \checkmark$$

(5)
[40]

VRAAG 6

6.1 $y = \ln(\sec x)$

$$\frac{dy}{dx} = \frac{1}{\sec x} \sec x \tan x \quad = \tan x \quad \checkmark$$

$$\left(\frac{dy}{dx}\right)^2 = \left(\tan x\right)^2 \quad \checkmark$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad \checkmark$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x}\, dx \quad \checkmark$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x}\, dx \quad \checkmark$$

$$= \int_0^{\frac{\pi}{3}} \sec x \, dx \quad \checkmark$$

$$= \left[\ln\left(\sec x + \tan x\right)\right]_0^{\frac{\pi}{3}} \quad \checkmark$$

$$= \ln\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) - \ln\left(\sec 0 + \tan 0\right) \quad \checkmark$$

$$= 1,317 \, units \quad \checkmark \ \checkmark$$

(6)

6.2 $x = 2\cos\theta$ $\qquad\qquad$ $y = 2\sin\theta$

$$\frac{dx}{d\theta} = -2\sin\theta \qquad\qquad \frac{dy}{d\theta} = 2\cos\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 = 4\sin^2\theta \qquad \left(\frac{dy}{d\theta}\right)^2 = 4\cos^2\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 4\sin^2\theta + 4\cos^2\theta$$

$$= 4(1) = 4$$

$$A_y = 2\pi\int_{\theta_1}^{\theta_2} x\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$= 2\pi\int_0^{\frac{\pi}{2}} 2\cos\theta\sqrt{4}\ d\theta \quad \checkmark$$

$$= 8\pi\left[\sin\theta\right]_0^{\frac{\pi}{2}} \quad \checkmark$$

$$= 8\pi\left[\sin\frac{\pi}{2} - \sin 0\right] \quad \checkmark$$

$$= 8\pi = 25,133\, units^2 \quad \checkmark$$

(6)

[12]

TOTAAL: **100**