



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NATIONALE SERTIFIKAAT

WISKUNDE N6

6 APRIL 2021

Hierdie nasienriglyn bestaan uit 23 bladsye.

VRAAG 1

1.1 1.1.1 $z = a^{3x-2y}$
 $\frac{\partial z}{\partial x} = a^{3x-2y} \cdot 3 \ln a \checkmark$ (1)

1.1.2 $\frac{\partial^2 z}{\partial y \partial x} = 3 \ln a \cdot a^{3x-2y} \cdot -2 \ln a \checkmark \checkmark$
 $= -6(\ln a)^2 a^{3x-2y}$
 OF $-6 \ln^2 a \cdot a^{3x-2y}$ (2)

1.2 1.2.1 $x = \frac{1}{1+2t} = (1+2t)^{-1}$
 $\frac{dx}{dt} = -(1+2t)^{-2} \cdot 2 = -\frac{2}{(1+2t)^2} \checkmark$ OF $x = \frac{1}{1+2t} = \frac{1}{y} \therefore y = \frac{1}{x} \checkmark$
 $y = 1+2t$ $\frac{dy}{dx} = -\frac{1}{x^2}$
 $\frac{dy}{dt} = 2$ $= -\left(\frac{1}{x}\right)^2$
 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{dy}{dx} \cdot \frac{dt}{dx}$ $= -y^2$ because $y = \frac{1}{x}$
 $= 2 \cdot \frac{-(1+2t)^2}{2} = -(1+2t)^2 \checkmark$ $= -(1+2t)^2 \checkmark$ (2)

1.2.2 $\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx}$ OF $\frac{dy}{dx} = -\frac{1}{x^2} = -x^{-2} \checkmark$
 $= -2(1+2t) \cdot 2 \cdot \frac{-(1+2t)^2}{2} \checkmark$ $\frac{d^2 y}{dx^2} = 2x^{-3} \checkmark = 2\left(\frac{1}{x}\right)^3 = 2y^3 = 2(1+2t)^3$
 $= 2(1+2t)^3 \checkmark$ (1)
[6]

VRAAG 2

$$\begin{aligned}
2.1 \quad \int y \, dx &= \int \sin^3 2x (1 - \sin^2 2x) \, dx \\
&= \int \sin^3 2x \cos^2 2x \, dx \quad \checkmark \\
&= \int \sin^2 2x \sin 2x \cos^2 2x \, dx \quad \checkmark \\
&= \int (1 - \cos^2 2x) \sin 2x \cos^2 2x \, dx \quad \checkmark \\
&= \int \sin 2x \cos^2 2x - \sin 2x \cos^4 2x \, dx \quad \checkmark \\
&= -\frac{1}{2} \frac{\cos^3 2x}{3} + \frac{1}{2} \frac{\cos^5 2x}{5} + C \quad \checkmark
\end{aligned}$$

Alternatief

$$\begin{aligned}
\int y \, dx &= \int \sin^3 2x (1 - \sin^2 2x) \, dx \\
&= \int \sin^3 2x - \sin^5 2x \, dx \\
&= \int \sin^2 2x \sin 2x - (\sin^2 2x)^2 \sin 2x \, dx \\
&= \int (1 - \cos^2 2x) \sin 2x - (1 - \cos^2 2x)^2 \sin 2x \, dx \quad \checkmark \\
&= \int (1 - \cos^2 2x) \sin 2x - (1 - 2\cos^2 2x + \cos^4 2x) \sin 2x \, dx \quad \checkmark \\
&= \int \sin 2x - \cos^2 2x \sin 2x - \sin 2x + 2\cos^2 2x \sin 2x - \cos^4 2x \sin 2x \, dx \quad \checkmark \\
&= \int \cos^2 2x \sin 2x - \cos^4 2x \sin 2x \, dx \quad \checkmark \\
&= -\frac{1}{2} \frac{\cos^3 2x}{3} + \frac{1}{2} \frac{\cos^5 2x}{5} + C \quad \checkmark
\end{aligned}$$

of $u = \cos 2x$ $dx = \frac{du}{-2 \sin 2x}$

(5)

$$\begin{aligned}
2.2 \quad \int y \, dx &= \int \tan^3 3x \sec^4 3x \, dx \\
&= \int \tan^3 3x \sec^2 3x \sec^2 3x \, dx \\
&= \int \tan^3 3x (1 + \tan^2 3x) \sec^2 3x \, dx \quad u = \tan 3x \quad dx = \frac{du}{3 \sec^2 3x} \quad \checkmark \\
&= \int u^3 (1 + u^2) \sec^2 3x \frac{du}{3 \sec^2 3x} \quad \checkmark \\
&= \frac{1}{3} \int u^3 + u^5 \, du \\
&= \frac{1}{3} \left(\frac{u^4}{4} + \frac{u^6}{6} \right) \quad \checkmark \\
&= \frac{1}{3} \left(\frac{\tan^4 3x}{4} + \frac{\tan^6 3x}{6} \right) + C \quad \checkmark \quad \text{or} \quad \frac{\tan^4 3x}{12} + \frac{\tan^6 3x}{18} + C
\end{aligned}$$

Alternatief 1

$$\begin{aligned} \int y \, dx &= \int \tan^3 3x \sec^4 3x \, dx \\ &= \int \tan^3 3x \sec^2 3x \sec^2 3x \, dx \quad \checkmark \quad n^3 3x(1 + \tan^2 3x) \sec^2 3x \, dx \quad \checkmark \\ &= \int \tan^3 3x \sec^2 3x + \tan^5 3x \sec^2 3x \, dx \quad \checkmark \quad \frac{\tan^4 3x}{12} + \frac{\tan^6 3x}{18} + C \quad \checkmark \end{aligned}$$

by inspection

Alternative 2

$$\begin{aligned} \int y \, dx &= \int \tan^3 3x \sec^4 3x \, dx \\ &= \int \frac{\sin^3 3x}{\cos^3 3x} \frac{1}{\cos^4 3x} \, dx \quad \checkmark \\ &= \int \frac{\sin^2 3x \sin 3x}{\cos^7 3x} \, dx \\ &= \int \frac{(1 - \cos^2 3x) \sin 3x}{\cos^7 3x} \, dx \quad u = \cos 3x \quad \frac{du}{dx} = -3 \sin 3x \quad dx = \frac{du}{-3 \sin 3x} \quad \checkmark \\ &= \int \frac{(1 - u^2) \sin 3x}{u^7} \frac{du}{-3 \sin 3x} \\ &= -\frac{1}{3} \int u^{-7} - u^{-5} \, du \quad \checkmark \\ &= -\frac{1}{3} \left[\frac{u^{-6}}{-6} - \frac{u^{-4}}{-4} \right] + C \\ &= -\frac{1}{3} \left[-\frac{1}{6} \sec^6 3x + \frac{1}{4} \sec^4 3x \right] + C \quad \checkmark \end{aligned}$$

Alternative 3

$$\begin{aligned} \int y \, dx &= \int \tan^3 3x \sec^4 3x \, dx \\ &= \int \tan 3x \tan^2 3x \sec^4 3x \, dx \quad \checkmark \\ &= \int \tan 3x (\sec^2 3x - 1) \sec^4 3x \, dx \\ &= \int \sec^6 3x \tan 3x - \sec^4 3x \tan 3x \, dx \quad \checkmark \\ &= \int \sec^5 3x \sec 3x \tan 3x - \sec^3 3x \sec 3x \tan 3x \, dx \quad \checkmark \\ &= \int \frac{1}{3} \sec^5 3x (3 \sec 3x \tan 3x) - \frac{1}{3} \sec^3 3x (3 \sec 3x \tan 3x) \, dx \\ &= \frac{1}{3} \frac{\sec^6 3x}{6} - \frac{1}{3} \frac{\sec^4 3x}{4} + C \quad \checkmark \end{aligned}$$

$$\begin{aligned}
\frac{1}{18}\sec^6 3x - \frac{1}{12}\sec^4 3x + C &= \frac{1}{18}(\sec^2 3x)^3 - \frac{1}{12}(\sec^2 3x)^2 + C \\
&= \frac{1}{18}(1 + \tan^2 3x)^3 - \frac{1}{12}(1 + \tan^2 3x)^2 + C \\
&= \frac{1}{18}(1 + 3\tan^2 3x + 3\tan^4 3x + \tan^6 3x) - \frac{1}{12}(1 + 2\tan^2 3x + \tan^4 3x) + C \\
&= \frac{1}{18} + \frac{1}{6}\tan^2 3x + \frac{1}{6}\tan^4 3x + \frac{1}{18}\tan^6 3x - \frac{1}{12} - \frac{1}{6}\tan^2 3x - \frac{1}{12}\tan^4 3x + C \\
&= \frac{1}{12}\tan^4 3x + \frac{1}{18}\tan^6 3x + \frac{1}{18} - \frac{1}{12} + C = \frac{1}{12}\tan^4 3x + \frac{1}{18}\tan^6 3x + C
\end{aligned}$$

(4)

2.3

$$\begin{aligned}
&\int y \, dx \\
&= \int (x+1)^2 \ln(x+1)^2 \, dx \quad f(x) = \ln(x+1)^2 \quad g'(x) = (x+1)^2 \\
&\quad \quad \quad \quad \quad \quad \quad \quad f'(x) = \frac{2}{x+1} \quad \quad \quad g(x) = \frac{(x+1)^3}{3} \quad \checkmark \\
&= \ln(x+1)^2 \frac{(x+1)^3}{3} - \int \frac{2}{x+1} \frac{(x+1)^3}{3} \, dx \quad \checkmark \\
&= \ln(x+1)^2 \frac{(x+1)^3}{3} - \frac{2}{3} \int (x+1)^2 \, dx \quad \checkmark \\
&= \ln(x+1)^2 \frac{(x+1)^3}{3} - \frac{2}{3} \frac{(x+1)^3}{3} + C \quad \checkmark \\
&\text{or } = \frac{2}{3}(x+1)^3 \ln(x+1) - \frac{2}{9}(x+1)^3 + C \quad \text{or } = \frac{(x+1)^3}{3} \left[2\ln(x+1) - \frac{2}{3} \right] + C \\
&\text{OR } \int y \, dx = \int (x+1)^2 \ln(x+1)^2 \, dx = 2 \int (x+1)^2 \ln(x+1) \, dx \quad \text{follow up}
\end{aligned}$$

Alternatief

$$\int y dx = \int (x+1)^2 \ln(x+1)^2 dx$$

$$= \int (x+1)^2 \cdot 2 \ln(x+1) dx \checkmark$$

$$f(x) = \ln(x+1) \Rightarrow f'(x) = \frac{1}{x+1}$$

$$g'(x) = (x+1)^2 \Rightarrow g(x) = \frac{(x+1)^3}{3}$$

$$= 2 \left[\frac{(x+1)^3}{3} \cdot \ln(x+1) - \int \frac{1}{x+1} \cdot \frac{(x+1)^3}{3} dx \right] \checkmark$$

$$= 2 \left[\frac{(x+1)^3}{3} \cdot \ln(x+1) - \int \frac{(x+1)^2}{3} dx \right] \checkmark$$

$$= 2 \left[\frac{(x+1)^3}{3} \cdot \ln(x+1) - \frac{(x+1)^3}{9} \right] + C \checkmark$$

Let ook op

$$-\int \frac{(x+1)^2}{3} dx = -\frac{1}{3} \int (x^2 + 2x + 1) dx$$

$$= -\frac{1}{3} \left[\frac{x^3}{3} + x^2 + x \right] + C$$

(4)

2.4

$$\frac{9}{4} - 3x - x^2 = -\left[x^2 + 3x - \frac{9}{4} \right]$$

$$= -\left[x^2 + 3x + \left(\frac{3}{2}\right)^2 - \frac{9}{4} - \left(\frac{3}{2}\right)^2 \right]$$

$$= -\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} \right] \checkmark$$

$$= \frac{9}{4} - \left(x + \frac{3}{2}\right)^2 \checkmark$$

$$\begin{aligned}
& \int y \, dx \\
&= \int \frac{1}{\sqrt{\frac{9}{4} - 3x - x^2}} \, dx = \int \frac{1}{\sqrt{\frac{9}{2} - \left(x + \frac{3}{2}\right)^2}} \, dx \\
&= \frac{1}{1} \sin^{-1} \frac{x + \frac{3}{2}}{\frac{3}{\sqrt{2}}} + C \quad \checkmark \\
&= \sin^{-1} \frac{2x+3}{2\frac{3}{\sqrt{2}}} + C \quad \text{or} \quad \sin^{-1} \frac{2x+3}{3\sqrt{2}} + C \quad \checkmark
\end{aligned}$$

Alternatief

$$\begin{aligned}
ax^2 + bx + c &= a \left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \\
\Rightarrow -x^2 - 3x + \frac{9}{4} &= -1 \left(x + \frac{3}{2}\right)^2 + \frac{4(-1)\left(\frac{9}{4}\right) - (-3)^2}{4(-1)} \quad \checkmark \\
&= \frac{9}{4} - 1 \left(x + \frac{3}{2}\right)^2 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
\int \frac{1}{\sqrt{-x^2 - 3x + \frac{9}{4}}} \, dx &= \int \frac{1}{\sqrt{\frac{9}{4} - 1 \left(x + \frac{3}{2}\right)^2}} \, dx \quad \checkmark \\
&= \frac{1}{1} \sin^{-1} \left[\frac{x + \frac{3}{2}}{\frac{3}{\sqrt{2}}} \right] + C \quad \checkmark
\end{aligned}$$

(4)

$$\begin{aligned}
2.5 \quad \int y \, dx &= \int \sin^2 2x - \cos^2 2x \, dx \\
&= -\int \cos^2 2x - \sin^2 2x \, dx \\
&= -\int \cos 4x \, dx \\
&= -\frac{\sin 4x}{4} + C \quad \checkmark
\end{aligned}$$

Alternatief 1

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$\frac{x}{2} - \frac{\sin 4x}{8} - \left(\frac{x}{2} + \frac{\sin 4x}{8} \right) + C$$

$$\frac{x}{2} - \frac{\sin 4x}{8} - \frac{x}{2} - \frac{\sin 4x}{8} + C$$

$$-\frac{\sin 4x}{4} + C \quad \checkmark$$

Gebruik die formuleblad.

Alternatief 2

$$\int y \, dx = \int \sin^2 2x - \cos^2 2x \, dx$$

$$= \int \frac{1}{2} - \frac{1}{2} \cos 4x - \left(\frac{1}{2} + \frac{1}{2} \cos 4x \right)$$

$$= -\int \cos 4x \, dx = -\frac{\sin 4x}{4} + C \quad \checkmark$$

Alternatief 3

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$\int \sin^2 2x - (1 - \sin^2 2x) \, dx$$

$$\int 2\sin^2 2x - 1 \, dx$$

$$2 \left(\frac{x}{2} - \frac{\sin 4x}{8} \right) - x + C$$

$$x - \frac{\sin 4x}{4} - x + C$$

$$-\frac{\sin 4x}{4} + C \quad \checkmark$$

Alternatief 4

$$\int \sin^2 2x - \cos^2 2x \, dx$$

$$= \int 1 - \cos^2 2x - \cos^2 2x \, dx$$

$$= \int 1 - 2\cos^2 2x \, dx$$

$$= x - 2 \left(\frac{x}{2} + \frac{\sin 4x}{8} \right) + C$$

$$= x - x - \frac{\sin 4x}{4} + C$$

$$= -\frac{\sin 4x}{4} + C \quad \checkmark$$

(1)
[18]

VRAAG 3

3.1

$$\int \frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} dx$$

$$\frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1} + \frac{D}{x - 1} \quad \checkmark$$

$$3x^2 - 6x - 1 = (Ax + B)(x + 1)(x - 1) + C(x^2 + 1)(x - 1) + D(x^2 + 1)(x + 1) \dots \dots \dots (1) \quad \checkmark$$

$$3x^2 - 6x - 1 = (Ax + B)(x^2 - 1) + C(x^3 + x - x^2 - 1) + D(x^3 + x + x^2 + 1)$$

$$3x^2 - 6x - 1 = Ax^3 + Bx^2 - Ax - B + Cx^3 + Cx - Cx^2 - C + Dx^3 + Dx + Dx^2 + D$$

Equating x^3 $A + C + D = 0 \dots \dots \dots (2)$

Equating x^2 $B - C + D = 3 \dots \dots \dots (3)$

Equating x $-A + C + D = -6 \dots \dots \dots (4)$

Equating constants: $-B - C + D = -1 \dots \dots \dots (5)$

$(3) - (5) \Rightarrow 2B = 4 \quad \therefore B = 2 \quad \checkmark$

$(2) - (4) \Rightarrow 2A = 6 \quad \therefore A = 3$

$(2) + (4) \Rightarrow 2C + 2D = -6 \quad \therefore D = -3 - C$

Substituting B and D in (5) $-2 - C - 3 - C = -1 \quad \therefore C = -2 \quad \checkmark$

$\therefore D = -3 + 2$

$D = -1 \quad \checkmark$

$$\int \frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} dx = \int \frac{3x + 2}{x^2 + 1} dx + \int \frac{-2}{x + 1} dx + \int \frac{-1}{x - 1} dx$$

$$= \int \frac{3x}{x^2 + 1} dx + \int \frac{2}{x^2 + 1} dx - \int \frac{2}{x + 1} dx - \int \frac{1}{x - 1} dx$$

$$= \frac{3}{2} \ln(x^2 + 1) + 2 \tan^{-1} x - 2 \ln(x + 1) - \ln(x - 1) + C \quad \checkmark$$

Alternatief 1

$$\frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1} + \frac{D}{x - 1} \quad \checkmark$$

$$3x^2 - 6x - 1 = (Ax + B)(x + 1)(x - 1) + C(x^2 + 1)(x - 1) + D(x^2 + 1)(x + 1) \dots \dots \dots (1) \quad \checkmark$$

When $x = 1$ $-4 = 4D$ $\therefore D = -1$

When $x = -1$ $8 = -4C$ $\therefore C = -2$

When $x = 0$ $-1 = -B - C + D \Rightarrow -1 = -B + 2 - 1 \quad \therefore B = 2$

Equating x^3 $A + C + D = 0 \Rightarrow A - 2 - 1 = 0 \quad \therefore A = 3 \quad \checkmark$

$A = 3$	$B = 2$	$C = -2$	$D = -1$
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$$\int \frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} dx = \int \frac{3x + 2}{x^2 + 1} dx + \int \frac{-2}{x + 1} dx + \int \frac{-1}{x - 1} dx \quad \checkmark$$

$$= \int \frac{3x}{x^2 + 1} dx + \int \frac{2}{x^2 + 1} dx - \int \frac{2}{x + 1} dx - \int \frac{1}{x - 1} dx \quad \checkmark$$

$$= \frac{3}{2} \ln(x^2 + 1) + 2 \tan^{-1} x - 2 \ln(x + 1) - \ln(x - 1) + C \quad \checkmark$$

Alternatief 2

$$\frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 - 1} \quad \checkmark$$

$$3x^2 - 6x - 1 = (Ax + B)(x^2 - 1) + (Cx + D)(x^2 + 1) \dots \dots \dots (1) \quad \checkmark$$

When $x = 1$ $-4 = (C + D)2 \quad \therefore C + D = -2 \dots \dots \dots (2)$

When $x = -1$ $8 = (-C + D)2 \quad \therefore -C + D = 4 \dots \dots \dots (3)$

(2) + (3) $2D = 2 \quad \therefore D = 1 \quad \checkmark$

Put $D = 1$ in (2) $C = -3 \quad \checkmark$

When $x = 0$ $-1 = -B + D \quad \therefore B = 2 \quad \checkmark$

Equating x^3 $A + C = 0 \Rightarrow A - 3 = 0 \quad \therefore A = 3 \quad \checkmark$

**Let daarop
dat die
waardes van
C en D
verander het.**

$$\int \frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} dx = \int \frac{3x + 2}{x^2 + 1} dx + \int \frac{-2}{x + 1} dx + \int \frac{-1}{x - 1} dx$$

$$\int \frac{3x^2 - 6x - 1}{(x^2 + 1)(x + 1)(x - 1)} dx = \int \frac{3x + 2}{x^2 + 1} dx + \int \frac{-3x + 1}{x^2 - 1} dx \quad \checkmark$$

$$= \int \frac{3x}{x^2 + 1} dx + \int \frac{2}{x^2 + 1} dx + \int \frac{-3x}{x^2 - 1} dx + \int \frac{1}{x^2 - 1} dx \quad \checkmark$$

$$= \frac{3}{2} \ln(x^2 + 1) + 2 \tan^{-1} x - \frac{3}{2} \ln(x^2 - 1) + \frac{1}{2} \ln \frac{x - 1}{x + 1} + C \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= \frac{3}{2} \ln(x^2 + 1) + 2 \tan^{-1} x - \frac{3}{2} \ln(x + 1) - \frac{3}{2} \ln(x - 1) + \frac{1}{2} \ln(x - 1) - \frac{1}{2} \ln(x + 1) + C$$

$$= \frac{3}{2} \ln(x^2 + 1) + 2 \tan^{-1} x - 2 \ln(x + 1) - \ln(x - 1) + C$$

(6)

3.2 $\int \frac{x^3 - 10x - 15}{x^2 - 9} dx$ $\quad x^2 - 9)x^3 - 10x - 15(x$

$$x^3 - 9x$$

$$-x - 15 \quad \checkmark$$

$$\frac{x^3 - 10x - 15}{x^2 - 9} = x + \frac{-x - 15}{x^2 - 9} \quad \checkmark$$

$$\frac{-x - 15}{x^2 - 9} = \frac{A}{x + 3} + \frac{B}{x - 3} \quad \checkmark$$

$$-x - 15 = A(x - 3) + B(x + 3) \quad \checkmark$$

when $x = 3$ $-18 = B \cdot 6 \quad \therefore B = -3$

when $x = -3$ $3 - 15 = A(-6) \quad \therefore A = 2$

$$\int \frac{x^3 - 10x - 15}{x^2 - 9} dx = \int x + \frac{2}{x + 3} + \frac{-3}{x - 3} dx \quad \checkmark$$

$$= \frac{x^2}{2} + 2 \ln(x + 3) - 3 \ln(x - 3) + C \quad \checkmark$$

Alternatief

$$\frac{x^3 - 10x - 15}{x^2 - 9} = x + A + \frac{B}{x+3} + \frac{C}{x-3} \quad \checkmark$$

$$x^3 - 10x - 15 = (x + A)(x + 3)(x - 3) + B(x - 3) + C(x + 3) \quad \checkmark$$

$$x = 3 \quad 27 - 30 - 15 = C(3 + 3) \quad C = -3$$

$$x = -3 \quad -27 + 30 - 15 = B(-3 - 3) \quad B = 2 \quad \checkmark$$

$$x = 0 \quad -15 = A(3)(-3) - 3B + 3C$$

$$-15 = -9A - 3(2) + 3(-3) \quad \therefore A = 0 \quad \checkmark$$

$$\int \frac{x^3 - 10x - 15}{x^2 - 9} dx = \int x + 0 + \frac{2}{x+3} + \frac{-3}{x-3} dx \quad \checkmark$$

$$= \frac{x^2}{2} + 2\ln(x+3) - 3\ln(x-3) + C \quad \checkmark$$

(6)
[12]

VRAAG 4

4.1 $\frac{dy}{dx} + y \sec x = x \cos x$

$$p = \sec x \quad Q = x \cos x$$

$$e^{\int p dx} = e^{\int \sec x dx}$$

$$= e^{\ln(\sec x + \tan x)} \quad \checkmark$$

$$= \sec x + \tan x$$

$$\int Q e^{\int p dx} dx = \int x \cos x (\sec x + \tan x) dx \quad \checkmark$$

$$\int x \cos x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right) dx$$

$$= \int x + x \sin x dx$$

$$= \int x dx + \int x \sin x dx \quad \text{using by parts for the second integral}$$

$$= \frac{x^2}{2} + \left\{ x(-\cos x) - \int 1 \cdot -\cos x dx \right\} \quad \checkmark$$

$$= \frac{x^2}{2} + \left\{ -x \cos x + \int \cos x dx \right\} \quad \checkmark$$

$$= \frac{x^2}{2} - x \cos x + \sin x \quad \checkmark$$

$$y(\sec x + \tan x) = \frac{x^2}{2} - x \cos x + \sin x + C \quad \checkmark$$

(6)

$$4.2 \quad r^2 - 6r + 12 = 0$$

$$r = \frac{6 \pm \sqrt{36 - 48}}{2}$$

$$= \frac{6 \pm \sqrt{-12}}{2} = \frac{6 \pm 2\sqrt{3}i}{2}$$

$$= 3 + \sqrt{3}i \quad \checkmark$$

$$\therefore y_c = e^{3x} [A \cos \sqrt{3}x + B \sin \sqrt{3}x] \quad \checkmark$$

$$y = y_c + y_p \quad \text{But } y_p = 0$$

$$y = e^{3x} [A \cos \sqrt{3}x + B \sin \sqrt{3}x]$$

$$x = 0 ; y = 2 \quad 2 = e^0 [A \cos 0 + B \sin 0] \quad \checkmark$$

$$\therefore A = 2$$

$$\frac{dy}{dx} = e^{3x} [-\sqrt{3}A \sin \sqrt{3}x + \sqrt{3}B \cos \sqrt{3}x] + 3e^{3x} [A \cos \sqrt{3}x + B \sin \sqrt{3}x] \quad \checkmark$$

$$x = 0; \quad \frac{dy}{dx} = 6 + 4\sqrt{3} \quad 6 + 4\sqrt{3} = e^0 [-\sqrt{3}A \sin 0 + \sqrt{3}B \cos 0] + 3e^0 [A \cos 0 + B \sin 0]$$

$$6 + 4\sqrt{3} = \sqrt{3}B + 3A$$

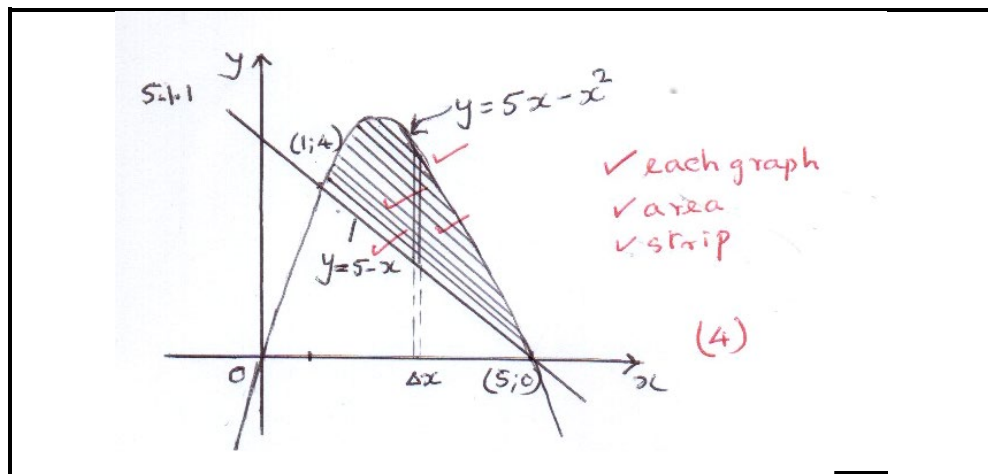
$$\text{put } A = 2 \quad 6 + 4\sqrt{3} = \sqrt{3}B + 6 \quad \therefore B = 4 \quad \checkmark$$

$$\therefore y = e^{3x} [2 \cos \sqrt{3}x + 4 \sin \sqrt{3}x] \quad \checkmark$$

(6)
[12]

VRAAG 5

5.1 5.1.1



(4)

$$\begin{aligned}
 5x - x^2 &= 5 - x && \checkmark \\
 x^2 - 6x + 5 &= 0 \\
 (x - 5)(x - 1) &= 0 && \checkmark \\
 x = 5 & \quad x = 1 \\
 y = 0 & \quad y = 4 \\
 (5; 0) & \quad (1; 4) && \checkmark
 \end{aligned}$$

OF

$$\begin{aligned}
 5x - x^2 &= 5 - x \\
 x(5 - x) &= 5 - x \\
 x(5 - x) - (5 - x) &= 0 \\
 (5 - x)(x - 1) &= 0 \\
 x = 5 & \quad x = 1 \\
 y = 0 & \quad y = 4
 \end{aligned}$$

(3)

5.1.2

$$\begin{aligned}
 A &= \int_1^5 5x - x^2 - (5 - x) dx && \checkmark \\
 &= \int_1^5 (6x - x^2 - 5) dx \\
 &= \left[\frac{6x^2}{2} - \frac{x^3}{3} - 5x \right]_1^5 && \checkmark \\
 &= \left[3(5)^2 - \frac{(5)^3}{3} - 5(5) - \left\{ 3(1)^2 - \frac{(1)^3}{3} - 5(1) \right\} \right] \\
 &= 10\frac{2}{3} \text{ or } \frac{32}{3} \text{ or } 10,667 \text{ units}^2 && \checkmark
 \end{aligned}$$

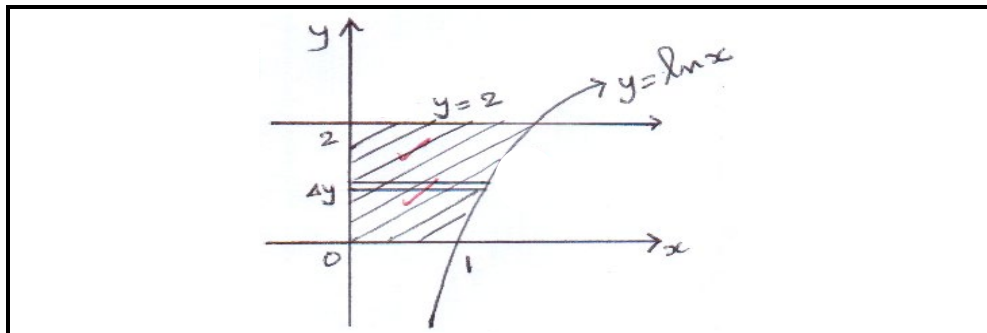
(3)

5.1.3

$$\begin{aligned}
 V_y &= 2\pi \int_a^b x(y_1 - y_2) dx \quad \checkmark \\
 &= 2\pi \int_1^5 x(6x - x^2 - 5) dx \\
 &= 2\pi \int_1^5 (6x^2 - x^3 - 5x) dx \\
 &= 2\pi \left[6\frac{x^3}{3} - \frac{x^4}{4} - 5\frac{x^2}{2} \right]_1^5 \quad \checkmark \\
 &= 2\pi \left[6\frac{5^3}{3} - \frac{5^4}{4} - 5\frac{5^2}{2} - \left\{ 6\frac{1^3}{3} - \frac{1^4}{4} - 5\frac{1^2}{2} \right\} \right] \quad \checkmark \\
 &= 64\pi = 201,062 \text{units}^3 \quad \checkmark
 \end{aligned}$$

(4)

5.2 5.2.1



(2)

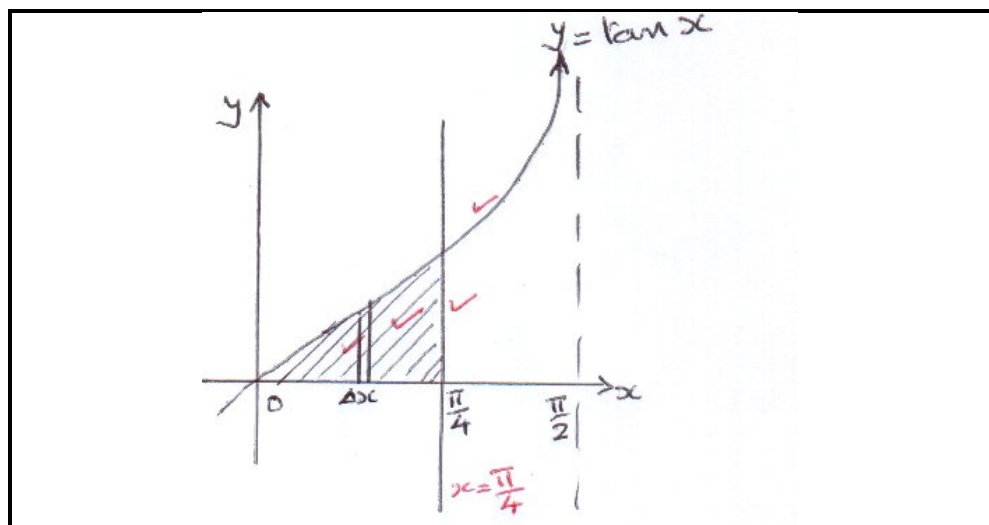
5.2.2

$$\begin{aligned}
 A &= \int_a^b (x_1 - x_2) dy \quad \checkmark \quad y = \ln x \Rightarrow x = e^y \quad \checkmark \\
 &= \int_0^2 e^y dy \quad \checkmark \\
 &= [e^y]_0^2 \quad \checkmark \\
 &= [e^2 - e^0] \quad \checkmark \\
 &= 6,389 \text{units}^2 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 I_x &= \int_a^b r^2 dA \\
 &= \int_{-4}^{-2} y^2 \frac{3}{2} (y+4)^{\frac{1}{2}} dy \quad u = y+4 \quad dy = du \quad y = u-4 \\
 &\quad y = -4 \quad u = 0 \quad \text{and} \quad y = -2 \quad u = 2 \\
 &= \frac{3}{2} \int_0^2 (u-4)^2 u^{\frac{1}{2}} du \quad \checkmark \\
 &= \frac{3}{2} \int_0^2 (u^2 - 8u + 16) u^{\frac{1}{2}} du \quad \checkmark \\
 &= \frac{3}{2} \int_0^2 (u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}}) du \quad \checkmark \\
 &= \frac{3}{2} \left[\frac{2}{7} u^{\frac{7}{2}} - 8 \cdot \frac{2}{5} u^{\frac{5}{2}} + 16 \cdot \frac{2}{3} u^{\frac{3}{2}} \right]_0^2 \quad \checkmark \\
 &= \frac{3}{2} \left[\frac{2}{7} 2^{\frac{7}{2}} - 8 \cdot \frac{2}{5} 2^{\frac{5}{2}} + 16 \cdot \frac{2}{3} 2^{\frac{3}{2}} - (0) \right] \quad \checkmark \\
 &= 22,95 \text{ units}^4 \quad \checkmark
 \end{aligned}$$

(6)

5.3 5.3.1



(2)

5.3.2

$$I_x = \frac{1}{2} \pi \rho \int_a^b y^4 dx$$

$$\frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^4 x dx$$

$$\frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^2 x \tan^2 x dx \quad \checkmark$$

$$\frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^2 x (\sec^2 x - 1) dx$$

$$\frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} (\tan^2 x \sec^2 x - \tan^2 x) dx \quad \checkmark$$

$$\frac{1}{2} \pi \rho \left[\frac{\tan^3 x}{3} - (\tan x - x) \right]_0^{\frac{\pi}{4}}$$

$$\frac{1}{2} \pi \rho \left[\frac{\tan^3 \frac{\pi}{4}}{3} - \tan \frac{\pi}{4} + \frac{\pi}{4} - (0) \right] \quad \checkmark$$

$$\frac{1}{2} \pi \rho \left[\frac{1}{3} - 1 + \frac{\pi}{4} \right]$$

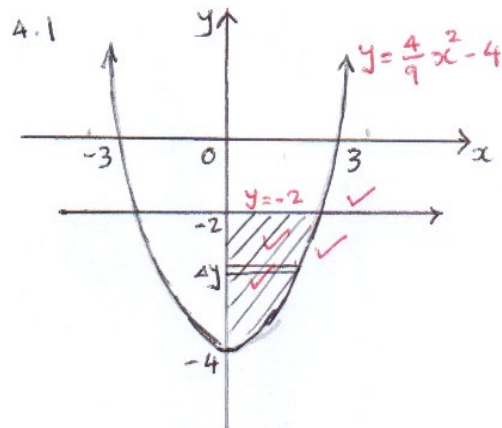
$$= 0,187 \rho \quad \checkmark$$

For x intercepts $\frac{4}{9}x^2 - 4 = 0 \quad x^2 = 9 \quad \therefore x = -3 \quad x = 3 \quad \checkmark$

y intercept $y = -4 \quad \checkmark$

(6)

5.4 5.4.1



$$y = \frac{4}{9}x^2 - 4$$

$$\frac{4}{9}x^2 = y + 4$$

$$x^2 = \frac{9}{4}(y + 4)$$

$$x = \frac{3}{2}(y + 4)^{\frac{1}{2}} \quad \checkmark$$

(4)

5.4.2

$$I_x = \int_a^b r^2 dA$$

$$= \int_{-4}^{-2} y^2 \frac{3}{2}(y+4)^{\frac{1}{2}} dy \quad u = y+4 \quad dy = du \quad y = u-4$$

$$y = -4 \quad u = 0 \quad \text{and} \quad y = -2 \quad u = 2$$

$$= \frac{3}{2} \int_0^2 (u-4)^2 u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{3}{2} \int_0^2 (u^2 - 8u + 16)u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{3}{2} \int_0^2 (u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}}) du \quad \checkmark$$

$$= \frac{3}{2} \left[\frac{2}{7}u^{\frac{7}{2}} - 8 \cdot \frac{2}{5}u^{\frac{5}{2}} + 16 \cdot \frac{2}{3}u^{\frac{3}{2}} \right]_0^2 \quad \checkmark$$

$$= \frac{3}{2} \left[\frac{2}{7}2^{\frac{7}{2}} - 8 \cdot \frac{2}{5}2^{\frac{5}{2}} + 16 \cdot \frac{2}{3}2^{\frac{3}{2}} - (0) \right] \quad \checkmark$$

$$= 22,951 \text{ units}^4 \quad \checkmark$$

(6)
[40]

VRAAG 6

6.1

$$y = \left(3 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2} \left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot -\frac{2}{3} x^{-\frac{1}{3}} = -\left(3 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} x^{-\frac{1}{3}}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(3 - x^{\frac{2}{3}}\right) x^{-\frac{2}{3}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{\left(3 - x^{\frac{2}{3}}\right)}{x^{\frac{2}{3}}} = \frac{x^{\frac{2}{3}} + \left(3 - x^{\frac{2}{3}}\right)}{x^{\frac{2}{3}}} = \frac{3}{x^{\frac{2}{3}}} \quad \checkmark$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{\sqrt{3}}{x^{\frac{1}{3}}} = \sqrt{3} x^{-\frac{1}{3}} \quad \checkmark$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_0^2 \sqrt{3} x^{-\frac{1}{3}} dx \quad \checkmark \quad \text{Vir limiete}$$

$$= \sqrt{3} \left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}} \right]_0^2 \quad \checkmark \quad \text{Vir integrasie}$$

$$= \sqrt{3} \frac{3}{2} \left[2^{\frac{2}{3}} - 0 \right] \quad \checkmark \quad \text{Vir substitusie van limiete}$$

$$= 4,124 \text{ units} \quad \checkmark$$

Alternatief

$$y = (3 - x^{2/3})^{3/2}$$

$$y^{2/3} = 3 - x^{2/3}$$

$$x = (3 - y^{2/3})^{3/2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{3}{2} (3 - y^{2/3})^{1/2} \times -\frac{2}{3} y^{-1/3}$$

$$\Rightarrow \left(\frac{dx}{dy}\right)^2 = (3 - y^{2/3}) \times y^{-2/3}$$

$$\Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{(3 - y^{2/3})}{y^{2/3}} = \frac{y^{2/3} + (3 - y^{2/3})}{y^{2/3}} = \frac{3}{y^{2/3}} \quad \checkmark$$

$$\Rightarrow S = \int_{1.679}^{5.196} \sqrt{\frac{3}{y^{2/3}}} dy \quad \checkmark$$

$$= \sqrt{3} \int_{1.679}^{5.196} y^{-1/3} dy \quad \checkmark$$

$$= \sqrt{3} \left[\frac{y^{2/3}}{2/3} \right]_{1.679}^{5.196} \quad \checkmark$$

$$= \frac{3\sqrt{3}}{2} \left[(5.196)^{2/3} - (1.679)^{2/3} \right] \quad \checkmark$$

$$= 4.124 \quad \checkmark$$

$$x = 0 \Rightarrow y = 5,196$$

$$x = 2 \Rightarrow y = 1,679$$

(6)

$$6.2 \quad x = \sqrt{r^2 - y^2}$$

$$\Rightarrow y = \sqrt{r^2 - x^2} = (r^2 - x^2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \times -2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}$$

$$\begin{aligned} A_y &= \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ &= \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} 2\pi x \sqrt{\frac{r^2}{r^2 - x^2}} dx \quad \checkmark \\ &= 2\pi r \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark \\ &= -\pi r \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} -2x(r^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark \\ &= -\pi r \left[\frac{(r^2 - x^2)^{\frac{1}{2}}}{1/2} \right]_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} \quad \checkmark \\ &= -2\pi r [a - (a + h)] \quad \checkmark \\ &= 2\pi r h \quad \checkmark \end{aligned}$$

Alternatief 1

$$x = \sqrt{r^2 - y^2}$$

$$\Rightarrow x^2 = r^2 - y^2$$

$$\Rightarrow 2x \frac{dx}{dy} = -2y$$

$$\Rightarrow \frac{dx}{dy} = -\frac{y}{x}$$

$$\Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{y^2}{x^2}$$

$$\Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{x^2} = \frac{x^2 + y^2}{x^2} = \frac{r^2}{x^2}$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \checkmark$$

$$= \int_a^{a+h} 2\pi x \sqrt{\frac{r^2}{x^2}} dy \checkmark$$

$$= 2\pi r \int_a^{a+h} dy \checkmark$$

$$= 2\pi r [y]_a^{a+h} \checkmark$$

$$= 2\pi r [a + h - a] \checkmark$$

$$= 2\pi r h \checkmark$$

Alternatief 2

$$x = \sqrt{r^2 - y^2}$$

$$\Rightarrow y = \sqrt{r^2 - x^2} = (r^2 - x^2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \times -2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{r^2 - x^2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{r^2 - x^2} = \frac{r^2}{r^2 - x^2}$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} 2\pi x \sqrt{\frac{r^2}{r^2 - x^2}} dx \checkmark$$

$$= 2\pi r \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} x(r^2 - x^2)^{-\frac{1}{2}} dx \checkmark$$

$$= -\pi r \int_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} -2x(r^2 - x^2)^{-\frac{1}{2}} dx \checkmark$$

$$= -\pi r \left[\frac{(r^2 - x^2)^{\frac{1}{2}}}{1/2} \right]_{\sqrt{r^2 - (a+h)^2}}^{\sqrt{r^2 - a^2}} \checkmark$$

$$= -2\pi r [a - (a + h)] \checkmark$$

$$= 2\pi r h \checkmark$$

$$y = a \Rightarrow x = \sqrt{r^2 - a^2}$$

$$y = a + h \Rightarrow x = \sqrt{r^2 - (a + h)^2}$$

(6)
[12]**TOTAAL: 100**