higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N6

25 JULIE 2018

Hierdie nasienriglyn bestaan uit 17 bladsye.

LET WEL: NASIENRIGLYN = 200 / 2 = 100

VRAAG 1

1.1 $$z = \cos^2\left(x^2 y^2\right)$$

$$\frac{\partial z}{\partial x} = 2\cos\left(x^2 y^2\right)\left\{-\sin\left(x^2 y^2\right)\right\}2xy^2$$

(4)

1.2 $$z = \sin^{-1}\frac{y}{x}$$

$$\frac{\partial z}{\partial y} = \frac{\dfrac{1}{x}}{\sqrt{1-\left(\dfrac{y}{x}\right)^2}}$$

(2)

1.3 $$A = \frac{1}{2}xy$$

$$\Delta A = \frac{\partial A}{\partial x}\Delta x + \frac{\partial A}{\partial y}\Delta y$$

$$= \frac{1}{2}y\Delta x + \frac{1}{2}x\Delta y$$

$$= \frac{1}{2}(4)(0,2) + \frac{1}{2}(3)(-0,2)$$

$$= 0,1 \, units^2$$

(6)

[12]

VRAAG 2

2.1 $$y = \int x^2 \ln x^2 \, dx = \int x^2 \, 2\ln x \, dx$$

$$= 2\ln x \frac{x^3}{3} - \int \frac{2}{x}\frac{x^3}{3}\,dx$$

$$= 2\ln x \frac{x^3}{3} - \int \frac{2}{3}x^2 \, dx$$

$$= 2\ln x \frac{x^3}{3} - \frac{2}{3}\frac{x^3}{3} + C$$

$$= \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + C$$

(6)

Kopiereg voorbehou   Blaai om asseblief

2.2 $y = x \tan^2 x$

$\int x \tan^2 x \, dx$

$= \int x (\sec^2 x - 1) dx$ ✔

$= x(\tan x - x) - \int 1(\tan x - x) dx$ ✔ ✔ ✔

✔ ✔

$= x(\tan x - x) - \ln(\sec x) + \dfrac{x^2}{2} + C$

(6)

2.3 $x(x-1) + 1 = x^2 - x + 1$ ✔

✔ ✔

$= \left\{ x^2 - x + \left(\dfrac{1}{2}\right)^2 \right\} - \left(\dfrac{1}{2}\right)^2 + 1$

$= \left(x - \dfrac{1}{2} \right)^2 + \dfrac{3}{4}$ ✔

$= \int \dfrac{1}{\left(x - \dfrac{1}{2} \right)^2 + \dfrac{3}{4}} dx$ ✔

Of gebruik $\dfrac{4ac - b^2}{4a} + a\left(x + \dfrac{b}{2a} \right)^2$

✔ ✔ ✔

$= \dfrac{1}{\dfrac{\sqrt{3}}{2}} \tan^{-1} \dfrac{x - \dfrac{1}{2}}{\dfrac{\sqrt{3}}{2}} + C = \dfrac{2}{\sqrt{3}} \tan^{-1} \dfrac{2\left(x - \dfrac{1}{2} \right)}{\sqrt{3}} + C$

$= \dfrac{2}{\sqrt{3}} \tan^{-1} \dfrac{2x - 1}{\sqrt{3}} + C$

(8)

2.4 $\int \dfrac{1}{\tan^4 \dfrac{x}{4}} dx$

$= \int \cot^4 \dfrac{x}{4} dx$ ✔

$= \int \cot^2 \dfrac{x}{4} \cot^2 \dfrac{x}{4} dx$ ✔

$= \int \cot^2 \dfrac{x}{4} \left(\cos ec^2 \dfrac{x}{4} - 1 \right) dx$ ✔

$= \int \cot^2 \dfrac{x}{4} \cos ec^2 \dfrac{x}{4} dx - \int \cot^2 \dfrac{x}{4} dx$ ✔

$\overset{\checkmark}{} \quad \overset{\checkmark}{}$

$= -4 \left(\dfrac{1}{3} \right) \cot^3 \dfrac{x}{4} - \left\{ -4 \cot \dfrac{x}{4} - x \right\} + C$

$= -\dfrac{4}{3} \cot^3 \dfrac{x}{4} + 4 \cot \dfrac{x}{4} + x + C$ ✔✔

(8)

2.5 $\int \sin^5 3x \cos^3 3x \, dx$

$= \int \sin^5 3x \cos^2 3x \cos 3x \, dx$ ✔

$= \int \sin^5 3x (1 - \sin^2 3x) \cos 3x \, dx$ ✔

$\overset{\checkmark}{} \qquad \overset{\checkmark}{}$

$= \int \sin^5 3x \cos 3x \, dx - \int \sin^7 3x \cos 3x \, dx$

$\overset{\checkmark}{} \quad \overset{\checkmark}{} \quad \overset{\checkmark}{} \quad \overset{\checkmark}{}$

$= \dfrac{1}{3} \left(\dfrac{\sin^6 3x}{6} \right) - \dfrac{1}{3} \left(\dfrac{\sin^8 3x}{8} \right) + C$

(Oorweeg ook die u-substitusie.) (8)

Alternatief 2.5 (breek sin⁵3x op)

$\int \sin^5 3x \cos^3 3x dx$

$= \int \sin^4 3x \sin 3x \cos^3 3x dx$ ✓

$= \int \left(\sin^2 3x\right)^2 \sin 3x \cos^3 3x dx$ ✓

$= \int \left(1 - \cos^2 3x\right)^2 \sin 3x \cos^3 3x dx$ ✓

$u = \cos 3x$

$\dfrac{du}{dx} = -3 \sin 3x$

$dx = -\dfrac{du}{3 \sin 3x}$

$= \int \left(1 - u^2\right)^2 \sin 3x u^3 \left(-\dfrac{du}{3 \sin 3x}\right)$ ✓

$= -\dfrac{1}{3} \int \left(1 - 2u^2 + u^4\right) u^3 du$ ✓

$= -\dfrac{1}{3} \int \left(u^3 - 2u^5 + u^7\right) du$ ✓

$= -\dfrac{1}{3} \left[\dfrac{u^4}{4} - \dfrac{2u^6}{6} + \dfrac{u^8}{8}\right] + C$ ✓

$= -\dfrac{1}{3} \left[\dfrac{\cos^4 3x}{4} - \dfrac{\cos^6 3x}{3} + \dfrac{\cos^8 3x}{8}\right] +$ ✓

$= -\dfrac{\cos^4 3x}{12} + \dfrac{\cos^6 3x}{9} - \dfrac{\cos^8 3x}{24} + C$

[36]

VRAAG 3

3.1 $\int \dfrac{3x^2 - 3x + 7}{(3x+1)(x^2 - 2x + 2)} dx$

$\dfrac{3x^2 - 3x + 7}{(3x+1)(x^2 - 2x + 2)} = \dfrac{A}{3x+1} + \dfrac{Bx + C}{x^2 - 2x + 2}$ ✓

$3x^2 - 3x + 7 = A(x^2 - 2x + 2) + (Bx + C)(3x+1)$ ✓

$x = -\dfrac{1}{3} \qquad 3\left(-\dfrac{1}{3}\right)^2 - 3\left(-\dfrac{1}{3}\right) + 7 = A\left\{\left(-\dfrac{1}{3}\right)^2 - 2\left(-\dfrac{1}{3}\right) + 2\right\} \therefore A = 3$ ✓

$3x^2 - 3x + 7 = Ax^2 - 2Ax + 2A + 3Bx^2 + 3Cx + Bx + C$ ✓

$x^2 \qquad A + 3B = 3 \qquad \therefore B = 0$ ✓

$cons \tan ts \qquad 2A + C = 7 \qquad \therefore C = 1$ ✓

$\int \dfrac{3x^2 - 3x + 7}{(3x+1)(x^2 - 2x + 2)} dx = \int \dfrac{3}{3x+1} + \dfrac{1}{x^2 - 2x + 2} dx$ ✓

$\qquad\qquad = \int \dfrac{3}{3x+1} dx + \int \dfrac{1}{x^2 - 2x + 2} dx$ ✓

$\qquad\qquad = \int \dfrac{3}{3x+1} dx + \int \dfrac{1}{(x-1)^2 + 1} dx$ ✓ ✓

$\qquad\qquad = \ln(3x+1) + \tan^{-1}(x-1) + C$

$\qquad\qquad$ ✓ ✓

(12)

3.2 $\int \dfrac{x^2 - 7x + 15}{\left(x^3 - 6x^2 + 9x\right)} dx$

$\dfrac{x^2 - 7x + 15}{\left(x^3 - 6x^2 + 9x\right)} = \dfrac{x^2 - 7x + 15}{x\left(x^2 - 6x + 9\right)} = \dfrac{x^2 - 7x + 15}{x(x-3)^2} = \dfrac{A}{x} + \dfrac{B}{x-3} + \dfrac{C}{(x-3)^2}$ ✓

$x^2 - 7x + 15 = A(x-3)^2 + Bx(x-3) + C$ ✓

Let $x = 3$ $C = 1$ ✓

Let $x = 0$ $15 = 9A$ $\therefore A = \dfrac{5}{3}$ ✓

Let $x = 1$ $9 = 4A - 2B + C$ $\therefore B = -\dfrac{2}{3}$ ✓

$\int \dfrac{x^2 - 7x + 15}{\left(x^3 - 6x^2 + 9x\right)} dx = \int \dfrac{A}{x} dx + \int \dfrac{B}{x-3} dx + \int \dfrac{C}{(x-3)^2} dx$ ✓

$= \int \dfrac{\frac{5}{3}}{x} dx + \int \dfrac{-\frac{2}{3}}{x-3} dx + \int \dfrac{1}{(x-3)^2} dx$ ✓

$= \dfrac{5}{3} \ln x - \dfrac{2}{3} \ln(x-3) - \dfrac{1}{x-3} + C$

LET WEL: Krediteer die ontwikkelings- en die gelykstellingskoëffisiënt om die waardes van A, B en C te bepaal.   (12)

[24]

VRAAG 4

4.1

$$\ln x \frac{dy}{dx} + \frac{y}{x} - \cos x = 0$$

$$x \frac{dy}{dx} + \frac{y}{x \ln x} = \frac{\cos x}{\ln x} \qquad \checkmark$$

$$e^{\int \frac{1}{x \ln x} dx} = e^{\ln(\ln x)} = \ln x \qquad \checkmark \quad \checkmark \quad \checkmark$$

$$\int Q e^{\int pdx} dx = \int \frac{\cos x}{\ln x} (\ln x) dx = \int \cos x \, dx = \sin x \qquad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$y \ln x = \sin x + C \qquad \checkmark$$

$$2 \ln \frac{\pi}{3} = \sin \frac{\pi}{3} + C \quad \checkmark \quad C = -0,774 \qquad \checkmark$$

$$y \ln x = \sin x - 0,774 \quad \checkmark \tag{12}$$

4.2

$$3 \frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = x^2 - x + 5$$

$$3r^2 + r - 2 = 0 \qquad \checkmark$$

$$(r+1)(3r-2) = 0$$

$$r = -1 \quad r = \frac{2}{3} \qquad \checkmark$$

$$y_c = Ae^{-x} + Be^{\frac{2}{3}x} \qquad \checkmark$$

$$y = Cx^2 + Dx + E \qquad \checkmark$$

$$\frac{dy}{dx} = 2Cx + D \qquad \checkmark$$

$$\frac{d^2 y}{dx^2} = 2C \qquad \checkmark$$

$$\therefore 3(2C) + 2Cx + D - 2(Cx^2 + Dx + E) = x^2 - x + 5$$

$$6C + 2Cx + D - 2Cx^2 - 2Dx - 2E = x^2 - x + 5 \qquad \checkmark$$

$$-2C = -1 \qquad \therefore C = -\frac{1}{2}$$
$$\checkmark$$

$$2C - 2D = -1 \qquad D = 0$$

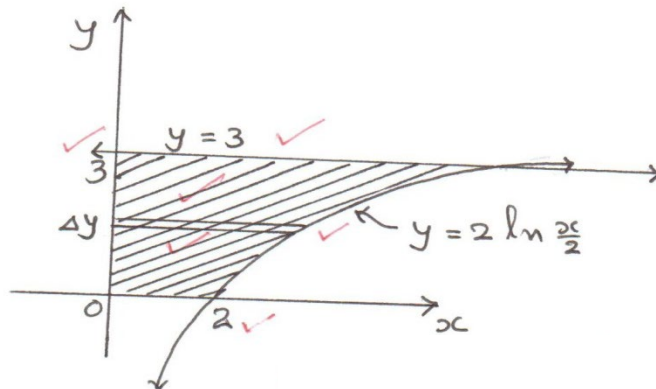$$6C + D - 2E = 5 \qquad E = -4 \quad \checkmark$$

$$y_p = -\frac{1}{2}x^2 - 4 \qquad \checkmark$$

$$y = Ae^{-x} + Be^{\frac{2}{3}x} - \frac{1}{2}x^2 - 4 \quad \checkmark \quad r \qquad y = Ae^{\frac{2}{3}x} + Be^{-x} + -\frac{1}{2}x^2 - 4$$

$$\tag{12}$$
$$\mathbf{[24]}$$

VRAAG 5

5.1   5.1.1

✓✓ x-afsnitte
✓✓ elke grafiek
✓ gebied ✓ strook   (6)

5.1.2

$$y = 2\ln\frac{x}{2} \qquad \therefore \frac{x}{2} = \overset{\frac{y}{\text{✓}}}{\quad} \qquad x = 2e^{\frac{y}{2}}$$

$$V_x = 2\pi\int_a^b y\left(x_1 - x_2\right)dy \qquad\qquad ✓$$

$$= 2\pi\int_0^3 y\left(2e^{\frac{y}{2}}\right)dy \quad ✓ \quad ✓ \quad ✓$$

$$= 4\pi\left[ye^{\frac{y}{2}}2 - \int e^{\frac{y}{2}}2 \right]_0^3 \quad ✓ \quad ✓$$

$$= 4\pi\left[ye^{\frac{y}{2}}2 - 4e^{\frac{y}{2}} \right]_0^3$$

$$= 4\pi\left[3e^{\frac{3}{2}}2 - 4e^{\frac{3}{2}} - \left(0 - 4\right) \right]$$

$$= 4\pi\left(2e^{\frac{3}{2}} + 4 \right) = 51{,}854\pi = 162{,}903 units \quad ✓ \qquad (8)$$

5.1.3

$$V_{m-y} = \int_a^b r\,dV$$

$$= \int_a^b \frac{x_1 + x_2}{2} 2\pi y \left(x_1 - x_2\right) dy \quad \checkmark$$

$$= \pi \int_a^b y \left(x_1^2 - x_2^2\right) dy \quad \checkmark$$

$$= \pi \int_0^3 y \left(2e^{\frac{y}{2}}\right)^2 dy \quad \checkmark$$

$$= \pi \int_0^3 y 4e^y dy \quad \checkmark$$

$$= 4\pi \left[ye^y - e^y \right]_0^3 \quad \checkmark \quad \checkmark$$

$$= 4\pi \left[3e^3 - e^3 - (0 - e^0) \right] \quad \checkmark$$

$$= 164{,}684\pi = 517{,}371 units^4 \quad \checkmark$$

$$\overset{\checkmark}{} \qquad \overset{\checkmark}{}$$

$$\bar{x} = \frac{164{,}684\pi}{51{,}854\pi} = 3{,}7159 = 3{,}716 units \qquad (10)$$

5.2   5.2.1

$$y = \frac{1}{4}x^2 \qquad y = \frac{1}{4}(x+2)$$

$$\frac{1}{4}x^2 = \frac{1}{4}(x+2)$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \qquad x = 2$$

$$y = \frac{1}{4} \qquad y = 1$$

$$\left(-1; \frac{1}{4}\right) \overset{\checkmark}{} \quad (2;1) \quad \checkmark$$

S.2.1

✓ elke grafiek
✓ gebied
✓ strook (6)

5.2.2
$$A = \int_a^b y_1 - y_2 \, dx \quad ✓$$

$$= \int_{-1}^{2} \frac{1}{4}(x+2) - \frac{1}{4}x^2 \, dx \quad ✓✓✓$$

$$= \frac{1}{4}\int_{-1}^{2} x + 2 - x^2 \, dx \quad ✓$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2} \quad ✓$$

$$= \frac{1}{4}\left[\frac{2^2}{2} + 2(2) - \frac{2^3}{3} - \left\{\frac{1}{2} - 2 + \frac{1}{3}\right\}\right] \quad ✓$$

$$= 1{,}125 \, units^2 \quad ✓ \tag{8}$$

5.2.3
$$A_{m-x} = \int_a^b r \, dA$$

$$= \int_{-1}^{2} \frac{1}{2}(y_1 + y_2)(y_1 - y_2) \, dx$$

$$= \frac{1}{2}\int_{-1}^{2}(y_1^2 - y_2^2) \, dx \quad ✓$$

$$= \frac{1}{2}\int_{-1}^{2}\left\{\frac{1}{4}(x+2)\right\}^2 - \left\{\frac{1}{4}x^2\right\}^2 \, dx \quad ✓✓✓$$

$$= \frac{1}{32}\int_{-1}^{2}(x+2)^2 - x^4 \, dx$$

$$= \frac{1}{32} \int_{-1}^{2} (x^2 + 4x + 4) - x^4 \, dx \quad \checkmark$$

Of $\frac{1}{32}\left[\frac{(x+2)^3}{3} - \frac{x^5}{5}\right]_{-1}^{2}$

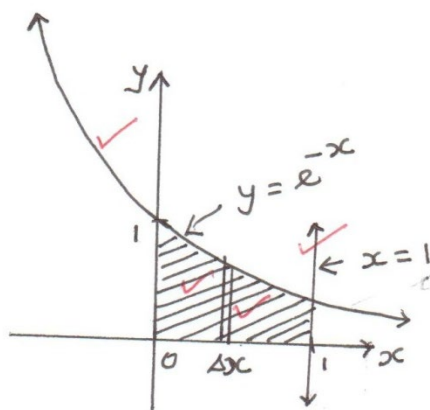$$= \frac{1}{32}\left[\frac{x^3}{3} + \frac{4x^2}{2} + 4x - \frac{x^5}{5}\right]_{-1}^{2} \quad \checkmark$$

$$\frac{1}{32}\left[\frac{2^3}{3} + 2^2 + 4(2) - \frac{2^5}{5} - \left\{\frac{(-1)^3}{3} + 2(-1)^2 + 4(-1) - \frac{(-1)^5}{5}\right\}\right] \quad \checkmark$$

$$= \frac{9}{20} = 0,45 \, units^3 \quad \checkmark$$

$$\bar{y} = \frac{1,125}{0,45}^{\checkmark} = 0,4 \, units \quad \checkmark$$

(10)

5.3   5.3.1

✓ elke grafiek
✓ gebied
✓ strook

(4)

5.3.2

$$I_x = \frac{1}{2}\pi\rho\int_{a}^{b} y^4 \, dx \quad \checkmark$$

$$= \frac{1}{2}\pi\rho\int_{0}^{1} \left(e^{-x}\right)^4 \, dx \quad \checkmark \quad \checkmark$$

$$= \frac{1}{2}\pi\rho\int_{0}^{1} e^{-4x} \, dx \quad \checkmark$$
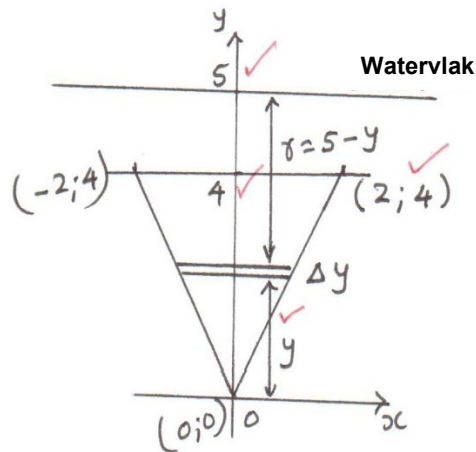
$$= \frac{1}{2}\pi\rho\left[\frac{e^{-4x}}{-4}\right]_{0}^{1} \quad \checkmark$$

$$= -\frac{1}{8}\pi\rho\left[e^{-4} - e^{0}\right] \quad \checkmark \quad \checkmark$$

$$= 0,1227\pi\rho = 0,386\rho \quad \checkmark$$

(8)

5.4   5.4.1

✓ vir die vorm   (4)

5.4.2 $m = \dfrac{4-0}{2-0} = 2$ ✓

$\therefore y = 2x$ ✓

Eerste moment

$= \displaystyle\int_a^b r\,dA$

$\boxed{\begin{array}{l} r = 5 - y \\[4pt] dA = x_1 - x_2 = 2x = y \end{array}}$

$= \displaystyle\int_0^4 (5-y)y\,dy$ ✓ ✓

$= \displaystyle\int_0^4 (5y - y^2)\,dy$ ✓

$= \left[\dfrac{5y^2}{2} - \dfrac{y^3}{3}\right]_0^4$ ✓

$= \left[\dfrac{5(4)^2}{2} - \dfrac{(4)^3}{3} - 0\right]$ ✓

$= \dfrac{56}{3} = 18,6667 m^3$ ✓

(8)

5.4.3

$$\int_a^b r^2 dA$$

$$= \int_0^4 (5-y)^2 \, y \, dy \qquad \checkmark$$

$$= \int_0^4 (25 - 10y + y^2) \, y \, dy \qquad \checkmark$$

$$= \int_0^4 (25y - 10y^2 + y^3) \, dy \qquad \checkmark$$

$$= \left[\frac{25y^2}{2} - \frac{10y^3}{3} + \frac{y^4}{4} \right]_0^4 \qquad \checkmark$$

$$= \left[\frac{25(4)^2}{2} - \frac{10(4)^3}{3} + \frac{(4)^4}{4} - 0 \right] \qquad \checkmark$$

$$= \frac{152}{3} = 50,6667 m^4 \qquad \checkmark \qquad \checkmark$$

$$\bar{y} = \left(\frac{152}{3} \right) \div \left(\frac{56}{3} \right) = \frac{56,6667 m^4}{18,6667 m^3} = 2,714 m \qquad \checkmark$$
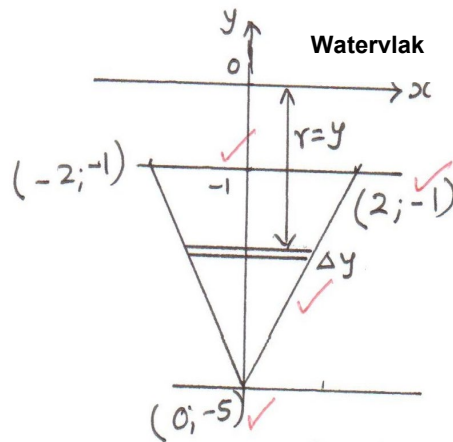
(8)

> Let wel:
>
> $$\int_0^4 (5-y)^2 \, y \, dy$$
>
> kan deel vir deel geïntegreer word.

Alternatief 5.4.1

✓ vir die vorm

Alternatief 5.4.2

$$m = \frac{-5+1}{0-2} = 2 \quad ✓$$
$$y + 5 = 2(x - 0)$$
$$\therefore 2x = y + 5 \quad ✓$$

Eerste moment

$$= \int_a^b rdA = \int_{-5}^{-1} y(y+5)dy \quad ✓ \quad ✓$$

$$= \int_{-5}^{-1} (y^2 + 5y)dy \quad ✓$$

$$= \left[\frac{y^3}{3} + \frac{5y^2}{2} \right]_{-5}^{-1} \quad ✓$$

$$= \frac{(-1)^3}{3} + \frac{5(-1)^2}{2} - \left\{ \frac{(-5)^3}{3} + \frac{5(-5)^2}{2} \right\} \quad ✓$$

$$= -18,6667m^3 \quad ✓$$

Plaas 'n regmerkie vir 0 by die waterhoogte en vir –5 aan die onderkant van die sluis.

5.4.3   Tweede moment =

$$= \int_a^b r^2 dA$$

$$= \int_{-5}^{-1} y^2(y+5)dy \quad \checkmark \quad \checkmark$$

$$= \int_{-5}^{-1} (y^3 + 5y^2)dy \quad \checkmark$$

$$= \left[\frac{y^4}{4} + \frac{5y^3}{3} \right]_{-5}^{-1} \quad \checkmark$$

$$= \left[\frac{(-1)^4}{4} + \frac{5(-1)^3}{3} - \frac{(-5)^4}{4} - \frac{5(-5)^3}{3} \right] \quad \checkmark$$

$$= \frac{152}{3} = 50,6667 m^4 \quad \checkmark$$

$$\bar{y} = \frac{50,6667}{-18,6667} = -2,714m \quad \checkmark$$

[80]

VRAAG 6

6.1

$$y = \frac{x^3}{6} + \frac{1}{2x}$$
$$\checkmark \quad \checkmark$$

$$\frac{dy}{dx} = \frac{x^2}{2} - \frac{1}{2x^2}$$

$$\left(\frac{dy}{dx} \right)^2 = \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4} \quad \checkmark$$

$$1 + \left(\frac{dy}{dx} \right)^2 = 1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4} \quad \checkmark$$

$$= \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}$$

$$= \left(\frac{x^2}{2} + \frac{1}{2x^2} \right)^2 \quad \checkmark$$

$$S = \int_1^2 \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2} \right)^2} dx \quad \checkmark \quad \checkmark$$

$$= \int_1^2 \frac{x^2}{2} + \frac{1}{2x^2} dx \quad \checkmark$$

$$= \left[\frac{x^3}{6} + \frac{x^{-1}}{-2} \right]_1^2 \quad \checkmark \quad \checkmark$$

$$= \left[\frac{x^3}{6} - \frac{1}{2x} \right]_1^2$$

$$= \frac{2^3}{6} - \frac{1}{2(2)} - \left\{ \frac{1}{6} - \frac{1}{2} \right\} \quad \checkmark$$

$$= 1,417 units \quad \checkmark$$

(12)

6.2 $y = \cos x$

$$\frac{dy}{dx} = -\sin x \quad \checkmark$$

$$\left(\frac{dy}{dx} \right)^2 = \sin^2 x \quad \checkmark$$

$$\qquad\qquad\quad \checkmark \quad \checkmark \quad \checkmark$$
$$A_x = 2\pi \int_0^{\frac{\pi}{2}} \cos x \sqrt{1 + \sin^2 x} \, dx$$

$$u = \sin x \qquad \frac{du}{dx} = \cos x \qquad dx = \frac{du}{\cos x}$$

$$x = 0 \quad u = \sin 0 = 0 \quad x = \frac{\pi}{2} \quad u = \sin \frac{\pi}{2} = 1$$

$$= 2\pi \int_0^1 \cos x \sqrt{1 + u^2} \cdot \frac{du}{\cos x} \qquad \checkmark \quad \checkmark \quad \checkmark$$

$$= 2\pi \int_0^1 \sqrt{1 + u^2} \, du$$

$$= 2\pi \left[\frac{u}{2} \sqrt{1 + u^2} + \frac{1}{2} \ln \left\{ u + \sqrt{1 + u^2} \right\} \right]_0^1 \quad \checkmark$$

$$= 2\pi \left[\frac{1}{2} \sqrt{2} + \frac{1}{2} \ln \left(1 + \sqrt{2} \right) - \left\{ 0 + \frac{1}{2} \ln \left(0 + \sqrt{1} \right) \right\} \right] \quad \checkmark$$

$$= 2\pi \left[\frac{1}{\sqrt{2}} + \frac{1}{2} \ln \left(1 + \sqrt{2} \right) - 0 \right] \quad \checkmark$$

$$= 7,212 units^2 \quad \checkmark$$

(12)

[24]

TOTAAL: 200