higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N6

29 JULIE 2019

Hierdie nasienriglyn bestaan uit 20 bladsye.

LET WEL: Hierdie vraestel word uit 200 nagesien en deur 2 gedeel om by 'n punt uit 100 uit te kom.

VRAAG 1

1.1

$$z = x^2 + 2xy + y^2$$

$$\frac{\partial z}{\partial x} = 2x + 2y \ \checkmark \qquad \frac{\partial z}{\partial y} = 2x + 2y \ \checkmark$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x(2x+2y) + y(2x+2y) \ \checkmark$$

$$= (2x+2y)(x+y) \ \checkmark$$

$$= 2(x+y)(x+y) \ \checkmark$$

$$= 2(x+y)^2$$

$$= 2(x^2 + 2xy + y^2) \ \checkmark$$

$$= 2z$$

Alternatief

$$z = x^2 + 2xy + y^2$$

$$= (x+y)^2 \ \checkmark \ \checkmark \ \checkmark \ \checkmark$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = x.2(x+y).1 + y.2(x+y).1$$

$$= 2(x+y)(x+y) \ \checkmark$$

$$= 2(x+y) \ \checkmark$$

$$= 2z$$

(6)

1.2

$$x = \sqrt{t} = t^{\frac{1}{2}} \quad \checkmark \qquad y = \frac{1}{\sqrt{t}} = t^{-\frac{1}{2}} \quad \checkmark$$

$$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}} \qquad \frac{dy}{dt} = -\frac{1}{2}t^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}t^{-\frac{3}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}} = -t^{-1} \quad \checkmark \ \checkmark$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = t^{-2}\frac{1}{\frac{1}{2}t^{-\frac{1}{2}}} = t^{-2}2t^{\frac{1}{2}} = 2t^{-\frac{3}{2}} \quad \checkmark \ \checkmark$$

$$x = \sqrt{t} \qquad y = \frac{1}{\sqrt{t}}$$

$$y = \frac{1}{x} \ \checkmark$$

$$\frac{dy}{dx} = -\frac{1}{x^2} = -x^{-2} \ \checkmark \ \checkmark$$

$$\frac{d^2y}{dx^2} = 2x^{-3} \ \checkmark$$

$$= 2\left(t^{\frac{1}{2}}\right)^{-3} = 2t^{-\frac{3}{2}} \ \checkmark \ \checkmark$$

(6)
[12]

VRAAG 2

2.1 $\int y\,dx = \int x^2 e^{3x}\,dx \qquad f(x) = x^2 \qquad g'(x) = e^{3x}$

$$= x^2\frac{e^{3x}}{3} - \int 2x\frac{e^{3x}}{3}dx$$

$$= x^2\frac{e^{3x}}{3} - \left[2x\frac{e^{3x}}{9} - \int 2\frac{e^{3x}}{9}dx\right] \qquad ✓✓✓✓✓✓$$

$$= x^2\frac{e^{3x}}{3} - 2x\frac{e^{3x}}{9} + \frac{2}{27}e^{3x} + c$$

Alternatief

$f(x)$	$g'(x)$	
x^2	e^{3x}	
$2x$	$\frac{e^{3x}}{3}$	✓
2	$\frac{e^{3x}}{9}$	✓
0	$\frac{e^{3x}}{27}$	✓

$$x^2\frac{e^{3x}}{3} - 2x\frac{e^{3x}}{9} + 2\frac{e^{3x}}{27} + c \qquad ✓✓✓$$

(6)

2.2 $\int \cos^5\frac{x}{5}dx = \int \cos^4\frac{x}{5}\cos\frac{x}{5}\,dx$ ✓

$$= \int\left(\cos^2\frac{x}{5}\right)^2 \cos\frac{x}{5}dx \quad ✓$$

$$= \int\left(1-\sin^2\frac{x}{5}\right)^2 \cos\frac{x}{5}dx \quad ✓ \qquad u = \sin\frac{x}{5} \qquad \frac{du}{dx} = \frac{1}{5}\cos\frac{x}{5}$$

$$= \int\left(1-u^2\right)^2 \cos\frac{x}{5}\frac{5du}{\cos\frac{x}{5}} \quad ✓ \qquad dx = \frac{5du}{\cos\frac{x}{5}}$$

$$= 5\int\left(1-u^2\right)^2 du \quad ✓$$

$$= 5\int\left(1-2u^2+u^4\right)du \quad ✓$$

$$= 5\left[u - 2\frac{u^3}{3} + \frac{u^5}{5}\right] + c \quad ✓$$

$$= 5\left[\sin\frac{x}{5} - 2\frac{\sin^3\frac{x}{5}}{3} + \frac{\sin^5\frac{x}{5}}{5}\right] + c \quad ✓$$

$$= 5\sin\frac{x}{5} - \frac{10}{3}\sin^3\frac{x}{5} + \sin^5\frac{x}{5} + c$$

(8)

2.3 $\int \tan^3 x \sec x dx$

$= \int \tan^2 x \sec x \tan x dx$ ✓

$= \int (\sec^2 x - 1) \sec x \tan x dx$ ✓

$= \int (u^2 - 1) du$

$= \frac{u^3}{3} - u + c$ ✓

$= \frac{\sec^3 x}{3} - \sec x + c$ ✓ ✓ ✓

$u = \sec x$	$\frac{du}{dx} = \sec x \tan x$ $dx = \frac{du}{\sec x \tan x}$	*kkk*

Alternatief

$\int \tan^3 x \sec x dx$ $\quad u = \tan x$

✓

$= \int u^3 \sec x \frac{du}{\sec^2 x}$ $\quad \frac{du}{dx} = \sec^2 x \quad dx = \frac{du}{\sec^2 x}$

$= \int u^3 \frac{du}{\sec x}$ ✓

$= \int u^3 \frac{du}{\sqrt{1+u^2}}$ $\quad v = 1 + u^2 \quad \Rightarrow \frac{dv}{du} = 2u \Rightarrow du = \frac{dv}{2u}$

$= \int \frac{u^3}{v^{\frac{1}{2}}} \frac{dv}{2u} = \frac{1}{2}\int u^2 v^{-\frac{1}{2}} dv$ ✓

$= \frac{1}{2}\int (v-1) v^{-\frac{1}{2}} dv \quad = \frac{1}{2}\int v^{\frac{1}{2}} - v^{-\frac{1}{2}} dv$ ✓

$= \frac{1}{2}\left[\frac{v^{\frac{3}{2}}}{\frac{3}{2}} - \frac{v^{\frac{1}{2}}}{\frac{1}{2}}\right] = \frac{1}{3} v^{\frac{3}{2}} - v^{\frac{1}{2}}$ ✓

$= \frac{1}{3}\sec^3 x - \sec x + c$ ✓

Alternatief

$$\int \tan^3 x \sec x dx$$

$$\int \frac{\sin^3 x}{\cos^3 x}\frac{1}{\cos x}dx$$

$$\int \frac{\sin^3 x}{\cos^4 x}dx \quad ✓$$

$$= \int \frac{\sin^2 x \sin x}{\cos^4 x}dx$$

$$= \int \frac{1-\cos^2 x \sin x}{\cos^4 x}dx \quad ✓ \qquad u = \cos x \qquad \frac{du}{dx} = -\sin x$$

$$= \int \frac{1-u^2 \sin x}{u^4}\frac{du}{-\sin x}$$

$$dx = \frac{du}{-\sin x}$$

$$= -\int \frac{1-u^2}{u^4}du \quad ✓$$

$$= -\int u^{-4} - u^{-2} du \quad ✓$$

$$= -\left[\frac{u^{-3}}{-3} - \frac{u^{-1}}{-1}\right] + c \quad ✓$$

$$= -\left[\frac{(\cos x)^{-3}}{-3} - \frac{(\cos x)^{-1}}{-1}\right] + c = \frac{\sec^3 x}{3} - \sec x + c \quad ✓$$

(6)

2.4

$$\int \frac{1}{9-4x-x^2}dx$$

$$= \int \frac{1}{13-(x+2)^2}dx \quad ✓$$

$$= \frac{1}{2\sqrt{13}}\ln\frac{\sqrt{13}+(x+2)}{\sqrt{13}-(x+2)} + c \quad ✓ ✓ ✓$$

$$9-4x-x^2$$

$$= -\left[x^2+4x+4-9-4\right] \quad ✓$$

$$= -\left[\left(x+2\right)^2 - 13\right] \quad ✓ ✓$$

$$= 13-\left(x+2\right)^2 \quad ✓$$

$$\text{of } \frac{1}{7,211} or 0,139 \ \ln\frac{\sqrt{13}+(x+2)}{\sqrt{13}-(x+2)} + c$$

$$\text{or } -\int \frac{1}{\left(x+2\right)^2 - 13}dx$$

$$= -\frac{1}{2\sqrt{13}}\ln\frac{\left(x+2\right)-\sqrt{13}}{\left(x+2\right)+\sqrt{13}} + c$$

$$ax^2+bx+c$$
$$= \frac{4ac-b^2}{4a} + a\left(x+\frac{b}{2a}\right)^2$$
$$9-4x-x^2$$
$$= \frac{4(-1)9-(4)^2}{4(-1)} - \left(x+\frac{-4}{2(-1)}\right)^2$$
$$= \frac{-36-16}{-4} - (x+2)^2$$
$$= 13-(x+2)^2$$

(8)

2.5 $\int \frac{1}{ab}\tan^{-1}\frac{bx}{a}dx \qquad f(x)=\tan^{-1}\frac{bx}{a}$ and $g'(x)=\frac{1}{ab}$ ✓ ✓

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{1}{ab}x\frac{\frac{b}{a}}{1+\frac{b^2x^2}{a^2}}dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{1}{ab}x\frac{ba}{a^2+b^2x^2}dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\int\frac{x}{a^2+b^2x^2}dx \quad ✓$$

$$=\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\frac{1}{2b^2}\int\frac{2b^2x}{a^2+b^2x^2}dx \quad ✓$$

$$==\frac{1}{ab}x\tan^{-1}\frac{bx}{a}-\frac{1}{2b^2}\ln\left(a^2+b^2x^2\right)+c \quad ✓ \quad ✓$$

$u=a^2+b^2x^2$ $\frac{du}{dx}=2b^2x$ $dx=\frac{du}{2b^2x}$

Alternatief

$$\int \frac{1}{ab}\tan^{-1}\frac{bx}{a}\,dx$$

$$u = \tan^{-1}\frac{bx}{a} \qquad \frac{bx}{a} = \tan u$$

$$\frac{b}{a} = \sec^2 u\frac{du}{dx} \qquad dx = \frac{a}{b}\sec^2 u\,du$$

$$\therefore \int \frac{1}{ab}\tan^{-1}\frac{bx}{a}\,dx$$

$$= \int \frac{1}{ab}u\frac{a}{b}\sec^2 u\,du$$

$$= \frac{1}{b^2}\int u\sec^2 u\,du \qquad f(u) = u \qquad g'(u) = \sec^2 u$$

$$= \frac{1}{b^2}\left[u\tan u - \int \tan u\,du\right]$$

$$\frac{1}{b^2}\left[u\tan u - \ln(\sec u)\right]$$

$$\frac{1}{b^2}\left[u\tan u - \ln(1+\tan^2 u)^{\frac{1}{2}}\right]$$

$$\frac{1}{b^2}\left[u\tan u - \frac{1}{2}\ln(1+\tan^2 u)\right]$$

$$\frac{1}{b^2}\left[\tan^{-1}\frac{bx}{a}\left(\frac{bx}{a}\right) - \frac{1}{2}\ln(1+\frac{b^2x^2}{a^2})\right] + c$$

$$= \frac{1}{b^2}\left[\left(\frac{bx}{a}\right)\tan^{-1}\frac{bx}{a} - \frac{1}{2}\ln\frac{a^2+b^2x^2}{a^2}\right] + c$$

$$= \frac{1}{b^2}\left[\left(\frac{bx}{a}\right)\tan^{-1}\frac{bx}{a} - \frac{1}{2}\ln\left(a^2+b^2x^2\right) + \frac{1}{2}\ln a^2\right] + c$$

$$= \frac{1}{ab}x\tan^{-1}\frac{bx}{a} - \frac{1}{2b^2}\ln\left(a^2+b^2x^2\right) + \frac{1}{2b^2}\ln a^2 + c$$

$$= \frac{1}{ab}x\tan^{-1}\frac{bx}{a} - \frac{1}{2b^2}\ln\left(a^2+b^2x^2\right) + K$$

where K is a constant $= \frac{1}{2b^2}\ln a^2 + c$

(8)

[36]

VRAAG 3

3.1 $\int \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} dx$

$2x^3 - 2x^2 + x - 1 = 2x^2(x-1) + (x-1)$ ✓

$= (x-1)(2x^2+1)$ ✓

$\frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} = \frac{8x^2 - 2x + 3}{(x-1)(2x^2+1)} = \frac{A}{x-1} + \frac{Bx + C}{2x^2+1}$ ✓ ✓

$8x^2 - 2x + 3 = A(2x^2+1) + (Bx + C)(x-1)$ ✓

$x = 1 \quad 8 - 2 + 3 = A(2+1) \quad \therefore A = 3$ ✓

$8x^2 - 2x + 3 = (2Ax^2 + A) + (Bx^2 + Cx - Bx - C)$ ✓

$2A + B = 8 \quad \therefore B = 2$ ✓

$C - B = -2 \quad \therefore C = 0$ ✓

Oorweeg dit om ander waardes van *x* te gebruik om B en C te bereken.

$\int \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} dx = \int \frac{3}{x-1} dx + \int \frac{2x}{2x^2+1} dx$ ✓

$= 3\ln(x-1) + \frac{1}{2}\ln(2x^2+1) + c$ ✓ ✓

Alternatief

$\int \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} dx$

$2x^3 - 2x^2 + x - 1 = 2x^2(x-1) + (x-1)$ ✓ and $8x^2 - 2x + 3 = 6x^2 + 3 + 2x^2 - 2x$ ✓

$= (x-1)(2x^2+1)$ ✓ $= 3\left(2x^2+1\right) + 2x(x-1)$ ✓

$\therefore \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} = \frac{3\left(2x^2+1\right) + 2x(x-1)}{(x-1)(2x^2+1)}$ ✓ ✓

$= \frac{3\left(2x^2+1\right)}{(x-1)(2x^2+1)} + \frac{2x(x-1)}{(x-1)(2x^2+1)}$ ✓

$= \frac{3}{x-1} + \frac{2x}{2x^2+1}$ ✓ ✓

$\therefore \int \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1} dx = \int \frac{3}{x-1} dx + \int \frac{2x}{2x^2+1} dx$ ✓

$= 3\ln(x-1) + \frac{1}{2}\ln(2x^2+1) + c$ ✓ ✓

(12)

3.2 $\frac{(3x+2)(2x-3)}{(3x+2)^2-(2x-3)^2} = \frac{6x^2-5x-6}{5x^2+24x-5}$ ✓✓

$= \frac{6}{5} + \frac{-\frac{169}{5}x}{(5x-1)(x+5)}$ *met behulp van die langdelingsmetode* ✓✓

$= 1{,}2 + \frac{-33{,}8x}{(5x-1)(x+5)}$ *waar*

$\frac{-\frac{169}{5}x}{(5x-1)(x+5)} = \frac{A}{(5x-1)} + \frac{B}{x+5}$ ✓

$\Rightarrow -\frac{169}{5}x = A(x+5) + B(5x-1)$ ✓

$x = -5 \Rightarrow B = -6{,}5 = -\frac{13}{2}$ ✓

$x = \frac{1}{5} \Rightarrow A = -1{,}3 = -\frac{13}{10}$ ✓

$\Rightarrow \int \frac{(3x+2)(2x-3)}{(3x+2)^2-(2x-3)^2}dx = \int \frac{6}{5}dx - \int \frac{13}{10(5x-1)}dx - \int \frac{13}{2(x+5)}dx$ ✓

$= \frac{6}{5}x - \frac{13}{50}\ln(5x-1) - \frac{13}{2}\ln(x+5) + C$ ✓✓✓ (12)

[24]

VRAAG 4

4.1
$$\frac{dy}{dx} = \tan x - y\cot x$$
$$\frac{dy}{dx} + y\cot x = \tan x \quad \checkmark$$
$$e^{\int pdx} = e^{\int \cot x dx} \quad \checkmark$$
$$= e^{\ln(\sin x)} \quad \checkmark$$
$$= \sin x \quad \checkmark$$
$$\int Qe^{\int pdx} = \int \tan x \sin x dx \quad \checkmark$$
$$= \int \frac{\sin x}{\cos x}\sin x dx$$
$$= \int \frac{\sin^2 x}{\cos x} dx \quad \checkmark$$
$$= \int \frac{1-\cos^2 x}{\cos x} dx \quad \checkmark$$
$$= \int \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} dx \quad \checkmark$$
$$= \int \sec x - \cos x dx \quad \checkmark$$
$$= \ln(\sec x + \tan x) - \sin x + c \quad \checkmark \quad \checkmark$$

$$\therefore y\sin x = \ln(\sec x + \tan x) - \sin x + c \quad \checkmark$$
(12)

4.2
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 2e^{2x}$$
$$r^2 - 2r + 2 = 0 \quad \checkmark$$
$$r = \frac{2 \pm \sqrt{4-4.1.2}}{2} \quad \checkmark$$
$$= \frac{2 \pm \sqrt{-4}}{2} \quad \checkmark$$
$$= \frac{2 \pm 2i}{2} = 1 \pm i \quad \checkmark$$
$$y_c = e^x\left[A\cos x + B\sin x\right] \quad \checkmark$$

$y = Ce^{2x}$ ✓

$\frac{dy}{dx} = 2Ce^{2x}$ ✓

$\frac{d^2y}{dx^2} = 4Ce^{2x}$ ✓

$\therefore 4Ce^{2x} - 2\left(2Ce^{2x}\right) + 2Ce^{2x} = 2e^{2x}$ ✓

$C = 1$ ✓

$y_p = e^{2x}$

$y = y_c + y_p$ ✓

$y = e^x\left[A\cos x + B\sin x\right] + e^{2x}$ ✓

OF

$y = 2Ce^{2x}$

$\frac{dy}{dx} = 4Ce^{2x}$

$\frac{d^2y}{dx^2} = 8Ce^{2x}$

$\therefore 8Ce^{2x} - 2\left(4Ce^{2x}\right) + 2(2Ce^{2x}) = 2e^{2x}$

$4Ce^{2x} = 2e^{2x} \quad \Rightarrow 4C = 2 \quad \therefore \; C = \frac{1}{2}$

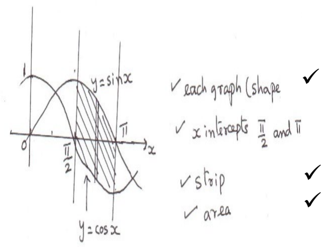
$y_p = 2Ce^{2x}$

$y_p = 2\left(\frac{1}{2}\right)e^{2x} = e^{2x}$

(12)
[24]

VRAAG 5

5.1 5.1.1



✓ vorm van elke grafiek (2 x 1)

(2 x 1)

✓ strook
✓ oppervlakte

(6)

5.1.2

$Area = \int_a^b y_1 - y_2 dx$ ✓

$= \int_{\frac{\pi}{2}}^{\pi} \sin x - \cos x dx$ ✓ ✓

$= \left[-\cos x - \sin x\right]_{\frac{\pi}{2}}^{\pi}$ ✓ ✓

$= \left[-\cos\pi - \sin\pi - \{-\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\}\right]$ ✓

or $-\left[\cos\pi + \sin\pi - \{\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\}\right]$

$= 2\text{units}^2$ ✓ ✓

(8)

5.1.3 $A_{m-y} = \int r dA$ ✓

$= \int_{\frac{\pi}{2}}^{\pi} x(\sin x - \cos x)dx$ ✓ ✓ ✓ $f(x) = x \quad g'(x) = \sin x - \cos x$

$= \left[x(-\cos x - \sin x) - \int (-\cos x - \sin x)dx \right]_{\frac{\pi}{2}}^{\pi}$ ✓ ✓

$= -\left[x(\cos x + \sin x) - \int (\cos x + \sin x)dx \right]_{\frac{\pi}{2}}^{\pi}$

$= -\left[x(\cos x + \sin x) - (\sin x - \cos x) \right]_{\frac{\pi}{2}}^{\pi}$ ✓

$= -\left[\pi(\cos\pi + \sin\pi) - (\sin\pi - \cos\pi) - \left\{ \frac{\pi}{2}(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}) - (\sin\frac{\pi}{2} - \cos\frac{\pi}{2} \right\} \right]$ ✓✓

$= -\left[\pi(-1+0) - (0 - -1) - \left\{ \frac{\pi}{2}(0+1) - (1-0) \right\} \right]$

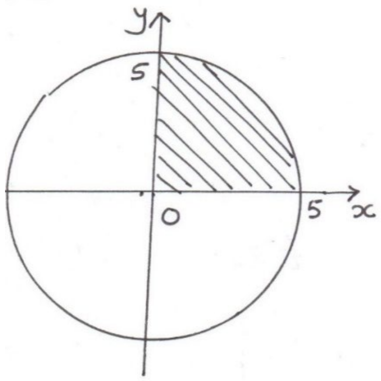
$= -\left[-\pi - 1 - \frac{\pi}{2} + 1 \right]$

$= \frac{3\pi}{2}$ or $4,712$ units^3 ✓

✓

$\bar{x} = \frac{A_{m-y}}{A} = \frac{4,712}{2} = 2,356$ *units* ✓ (12)

5.2 5.2.1



✓ strook (vertikaal of horisontaal in berekening gebruik)
✓ oppervlakte
✓ grafiek (vorm)
✓ afsnit (4)

5.2.2

$$V_x = \pi\int_a^b y_1^2 - y_2^2 dx \quad ✓$$

$$= \pi\int_0^5 25 - x^2 dx \quad ✓ \quad ✓$$

$$= \pi\left[25x - \frac{x^3}{3}\right]_0^5 \quad ✓$$

$$= \pi\left[25(5) - \frac{(5)^3}{3}\right] \quad ✓$$

$$= \frac{250}{3}\pi \quad \text{or} \quad 261{,}800 \text{ units}^3 \quad ✓$$

(6)

5.2.3

$$V_{m-y} = \pi\int_a^b x\left(y_1^2 - y_2^2\right)dx \quad ✓$$

$$= \pi\int_0^5 x\left(25 - x^2\right)dx \quad ✓ \quad ✓$$

$$= \pi\int_0^5 25x - x^3 dx \quad ✓$$

$$= \pi\left[\frac{25}{2}x^2 - \frac{x^4}{4}\right]_0^5 \quad ✓$$

$$= \frac{625}{4}\pi = 156{,}25\pi = 490{,}874\text{units}^4 \quad ✓$$

$$\bar{x} = \frac{261{,}800}{490{,}874} = \frac{15}{8} = 1{,}875 \text{ units} \quad ✓ \quad ✓$$

$$V_{m-y} = \int_a^b rdv$$

$$= \int_a^b \frac{x_1 + x_2}{2} 2\pi y\left(x_1 - x_2\right)dy$$

$$= \pi\int_a^b y\left(x_1^2 - x_2^2\right)dy \quad ✓$$

$$= \pi\int_0^5 y\left(25 - y^2\right)dy \quad ✓$$

$$= \pi\int_a^5 \left(25y - y^3\right)dy \quad ✓$$

$$= \pi\left[\frac{25y^2}{2} - \frac{y^4}{4}\right]_0^5 \quad ✓$$

$$= \pi\left[\frac{25(5)^2}{2} - \frac{y(5)^4}{4} - \{0\}\right]$$

$$= \frac{625}{4}\pi \quad \text{or} \quad 490{,}874 \text{ units}^4 \quad ✓$$

$$\bar{x} = \frac{261{,}800}{490{,}874} = \frac{15}{8} = 1{,}875 \text{ units} \quad ✓$$

Met behulp van die horisontale strook

$$V_x = 2\pi\int_a^b y\left(x_1 - x_2\right)dy \quad ✓$$

$$= 2\pi\int_0^5 y\sqrt{25 - y^2}dy \quad ✓✓$$

$$= -\pi\int_0^5 -2y\left(25 - y^2\right)^{\frac{1}{2}}dy$$

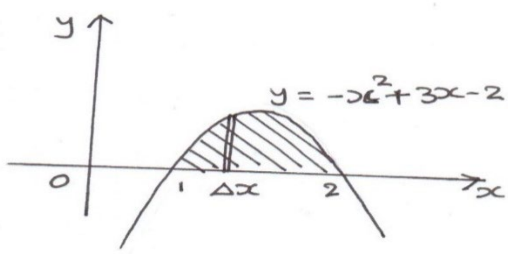
$$= -\frac{2}{3}\pi\left[\left(25 - y^2\right)^{\frac{3}{2}}\right]_0^5 \quad ✓✓$$

$$= -\frac{2}{3}\pi\left[0 - 25^{\frac{3}{2}}\right] = \frac{250}{3}\pi \quad ✓$$

$$= 83{,}333\pi = 261{,}800\text{units}^3$$

(8)

5.3 5.3.1



✓ grafiek
✓ *x*-afsnitte
✓ oppervlakte
✓ strook

(4)

5.3.2

$$A = \int_a^b y_1 - y_2 dx \quad ✓$$

$$= \int_1^2 -x^2 + 3x - 2\, dx \quad ✓$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2 \quad ✓ \quad ✓$$

$$= \left[-\frac{(2)^3}{3} + \frac{3(2)^2}{2} - 2(2) - \left\{ -\frac{1}{3} + \frac{3}{2} - 2 \right\} \right] \quad ✓$$

$$= \frac{1}{6} \text{ or } 0{,}167 \text{ units}^2 \quad ✓$$

(6)

5.3.3

$$I_y = \int_a^b r^2 dA \quad ✓$$

$$= \int_1^2 x^2(-x^2 + 3x - 2)\, dx \quad ✓$$

$$\int_1^2 (-x^4 + 3x^3 - 2x^2)\, dx \quad ✓$$

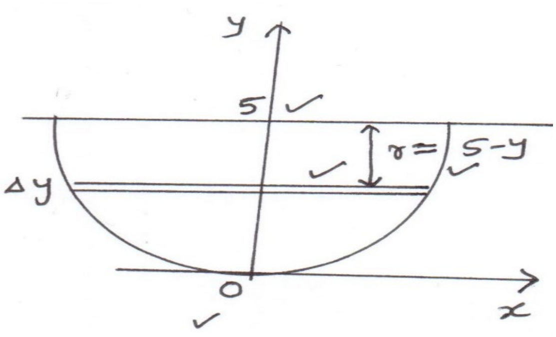
$$= \left[-\frac{x^5}{5} + \frac{3x^4}{4} - \frac{2x^3}{3} \right]_1^2 \quad ✓$$

$$\left[-\frac{(2)^5}{5} + \frac{3(2)^4}{4} - \frac{2(2)^3}{3} - \left\{ -\frac{1}{5} + \frac{3}{4} - \frac{2}{3} \right\} \right] \quad ✓$$

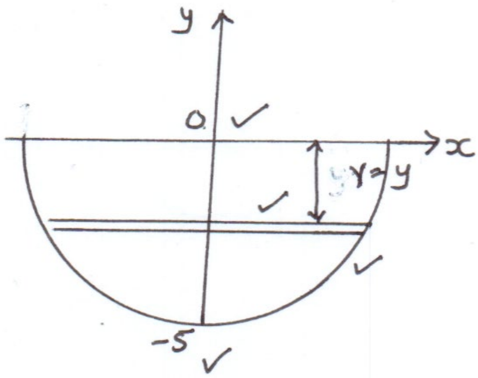
$$= \frac{23}{60} \quad or 0{,}383 \;✓ = \frac{0{,}383}{0{,}167} A = 2{,}295A \quad ✓ \quad ✓$$

(8)

5.4 5.4.1



Alternatief (met x-as op die watervlak)



(4)

5.4.2 $x^2 + (y-5)^2 = 25$

$x = \sqrt{25-(y-5)^2}$ ✓

first moment $= \int_a^b r\,dA$ ✓

$= \int_0^5 (5-y)2\sqrt{25-(y-5)^2}\,dy$ ✓ ✓ ✓ $u = 5 - y \quad dy = -du$

$= -2\int_5^0 u\sqrt{25-u^2}\,du$ ✓ $y = 0 \quad u = 5$

$= \frac{2}{3}\left[\left(25-u^2\right)^{\frac{3}{2}}\right]_5^0$ ✓ $y = 5 \quad u = 0$

$= \frac{2}{3}\left[25^{\frac{3}{2}} - 0\right]$ ✓ $= \frac{250}{3} = 83,333\ m^3$ ✓

$= \left[\frac{\left(25-u^2\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_5^0$

$= \frac{2}{3}\left[\left(25-u^2\right)^{\frac{3}{2}}\right]_5^0$

$= \frac{2}{3}\left[(25)^{\frac{3}{2}} - \{0\}\right]$

$= \frac{250}{3}$ or $83,333m^3$

5.4.3 $\bar{\bar{y}} = \frac{245,437}{83,333}$ ✓ $= 2,945m$ ✓

Alternatief

5.4.2 First moment of area $= \int_a^b r\,dA$ ✓

$= \int_a^b y2x\,dy$ ✓ ✓

But $x^2 + y^2 = 25 \Rightarrow x = \sqrt{25 - y^2}$

$= \int_{-5}^{0} y2\sqrt{25 - y^2}\,dy$ ✓ ✓

$= -\int_{-5}^{0} -y2\left(25 - y^2\right)^{\frac{1}{2}}\,dy$ ✓ ✓

$= -\left[\dfrac{\left(25 - y^2\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-5}^{0}$ ✓

$= -\dfrac{2}{3}\left[\left(25 - y^2\right)^{\frac{3}{2}}\right]_{-5}^{0}$

$= -\dfrac{2}{3}\left[(25)^{\frac{3}{2}} - \left\{25 - (-5)^2\right\}^{\frac{3}{2}}\right]$ ✓ ✓

$= -\dfrac{2}{3}(25)^{\frac{3}{2}} = -83\dfrac{1}{3} \quad \text{or} -\dfrac{250}{3}m^3$ ✓ (12)

5.4.3 $\overline{y} = \dfrac{245,437m^4}{-83,333m^3} = -2,945m$ ✓ ✓ (2)

[80]

VRAAG 6

6.1 $2y = x^2$

$\frac{dy}{dx} = x$ ✓

$\left[\frac{dy}{dx}\right]^2 = x^2$ ✓

$1+\left[\frac{dy}{dx}\right]^2 = 1+x^2$ ✓

$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dy$ ✓

$= \int_2^4 \sqrt{1+x^2}\,dy$ ✓ ✓

$= \left[\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right)\right]_2^4$ ✓ ✓

$\left[\frac{4}{2}\sqrt{1+4^2} + \frac{1}{2}\ln\left(4+\sqrt{1+4^2}\right) - \left\{\frac{2}{2}\sqrt{1+2^2} + \frac{1}{2}\ln\left(2+\sqrt{1+2^2}\right)\right\}\right]$ ✓ ✓

$= \left[2\sqrt{17} + \frac{1}{2}\ln\left(4+\sqrt{17}\right) - \left\{\sqrt{5} + \frac{1}{2}\ln\left(2+\sqrt{5}\right)\right\}\right]$

$= 6{,}336$ units ✓ ✓

Alternatief

$$2y = x^2$$

$$x = \sqrt{2}y^{\frac{1}{2}} \qquad or \sqrt{2}\sqrt{y}$$

$$\frac{dx}{dy} = \sqrt{2}\frac{1}{2}y^{-\frac{1}{2}} \quad ✓$$

$$\left[\frac{dx}{dy}\right]^2 = \frac{1}{2y} \quad ✓$$

$$1+\left[\frac{dx}{dy}\right]^2 = 1+\frac{1}{2y} \quad ✓$$

$$= \frac{2y+1}{2y}$$

$$S = \int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \quad ✓$$

$$= \int_2^8 \sqrt{\frac{2y+1}{2y}}\,dy$$

$$\int_2^8 \frac{\sqrt{2y+1}}{\sqrt{2y}}dy \quad ✓$$

$$= \int_2^4 \frac{\sqrt{1+u^2}}{u}udu \quad ✓$$

$$= \int_2^4 \sqrt{1+u^2}\,du \quad ✓$$

$$= \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u+\sqrt{1+u^2}\right)\right]_2^4 \quad ✓ \quad ✓$$

$$\left[\frac{4}{2}\sqrt{1+4^2} + \frac{1}{2}\ln\left(4+\sqrt{1+4^2}\right) - \left\{\frac{2}{2}\sqrt{1+2^2} + \frac{1}{2}\ln\left(2+\sqrt{1+2^2}\right)\right\}\right] \quad ✓$$

$$= \left[2\sqrt{17} + \frac{1}{2}\ln\left(4+\sqrt{17}\right) - \left\{\sqrt{5} + \frac{1}{2}\ln\left(2+\sqrt{5}\right)\right\}\right]$$

$$= 6{,}336 \text{ units} \quad ✓ \quad ✓$$

(12)

6.2

$$x = \frac{1}{9}y^2$$

$$\frac{dx}{dy} = \frac{2}{9}y \quad ✓$$

$$\left[\frac{dx}{dy}\right]^2 = \left(\frac{2}{9}y\right)^2 \quad ✓$$

$$1+\left[\frac{dx}{dy}\right]^2 = 1+\left(\frac{2}{9}y\right)^2 = 1+\frac{4y^2}{81} = \frac{81+4y^2}{81} \quad ✓ \; ✓$$

$$A_x = 2\pi\int_0^6 y\frac{\sqrt{81+4y^2}}{9}dy \quad ✓ \; ✓$$

$$= \frac{2}{9}\pi\int_{81}^{225} y u^{\frac{1}{2}}\frac{du}{8y} \quad ✓$$

$$= \frac{2}{9}\frac{1}{8}\pi\int_{81}^{225} u^{\frac{1}{2}}du$$

$$= \frac{1}{36}\pi\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{81}^{225} \quad ✓ \; ✓$$

$$= \frac{1}{36}\frac{2}{3}\pi\left[u^{\frac{3}{2}}\right]_{81}^{225}$$

$$= \frac{1}{54}\pi\left[225^{\frac{3}{2}} - 81^{\frac{3}{2}}\right] \quad ✓$$

$$= 49\pi \text{ or } 153{,}938 \text{ units} \quad ✓ \; ✓$$

Alternatief

$$x = \frac{1}{9}y^2$$

$$y^2 = 9x \checkmark \Rightarrow y = 3\sqrt{x}$$

$$2y\frac{dy}{dx} = 9$$

$$\frac{dy}{dx} = \frac{9}{2y} = \frac{9}{2\left(3\sqrt{x}\right)} = \frac{9}{6\sqrt{x}} = \frac{3}{2\sqrt{x}} \quad \checkmark \quad \checkmark$$

$$\left[\frac{dy}{dx}\right]^2 = \left(\frac{3}{2\sqrt{x}}\right)^2 \quad \checkmark$$

$$1 + \left[\frac{dy}{dx}\right]^2 = 1 + \left(\frac{3}{2\sqrt{x}}\right)^2 = 1 + \frac{9}{4x} = \frac{4x+9}{4x} \quad \checkmark$$

$$A_x = 2\pi\int_0^4 y\sqrt{1 + \left[\frac{dy}{dx}\right]^2}\,dx \quad \checkmark \quad y = 0 \quad x = 0$$

$$= 2\pi\int_0^4 y\sqrt{\frac{4x+9}{4x}}dx \quad \checkmark \qquad y = 6 \quad x = \frac{1}{9}y^2 = \frac{1}{9}(6)^2 = 4 =$$

$$2\pi\int_0^4 3\sqrt{x}\,\frac{\sqrt{4x+9}}{2\sqrt{x}}dx \quad \checkmark$$

$$= 3\pi\int_0^4 \sqrt{4x+9}\,dx$$

$$= 3\pi\frac{1}{4}\left[\frac{(4x+9)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 \quad \checkmark$$

$$\frac{1}{2}\pi\left[(4x+9)^{\frac{3}{2}}\right]_0^4$$

$$= \frac{1}{2}\pi\left[\left(4(4)+9\right)^{\frac{3}{2}} - (0+9)^{\frac{3}{2}}\right] \quad \checkmark$$

$$\frac{1}{2}\pi\left[(25)^{\frac{3}{2}} - (9)^{\frac{3}{2}}\right]$$

$$= 49\pi \quad \text{or} \quad 153{,}938\text{units}^2 \quad \checkmark \quad \checkmark$$

(12)

[24]

TOTAAL = 200 ÷ 2: 100