higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N6

27 Julie 2021

Hierdie nasienriglyn bestaan uit 16 bladsye.

VRAAG 1

1.1

$$z=\sqrt{x^2-y^2}$$
$$=\left(x^2-y^2\right)^{\frac{1}{2}}$$
$$\frac{\partial z}{\partial x}=\frac{1}{2}\left(x^2-y^2\right)^{-\frac{1}{2}}.2x=\frac{x}{\sqrt{x^2-y^2}} \quad ✓ ✓$$
$$\frac{\partial z}{\partial y}=\frac{1}{2}\left(x^2-y^2\right)^{-\frac{1}{2}}.-2y=\frac{-y}{\sqrt{x^2-y^2}} \quad ✓ ✓$$
$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{x^2}{\sqrt{x^2-y^2}}+\frac{-y^2}{\sqrt{x^2-y^2}} \quad ✓$$
$$=\frac{x^2-y^2}{\sqrt{x^2-y^2}}$$
$$=\sqrt{x^2-y^2} \quad ✓$$

$$Z=\sqrt{x^2-y^2} \Rightarrow Z^2=x^2-y^2$$
$$\Rightarrow 2Z\frac{\partial Z}{\partial x}=2x \quad ✓$$
$$\Rightarrow \frac{\partial Z}{\partial x}=\frac{x}{Z}=\frac{x}{\sqrt{x^2-y^2}} \quad ✓$$
$$2Z\frac{\partial Z}{\partial y}=-2y \quad ✓$$
$$\Rightarrow \frac{\partial Z}{\partial y}=-\frac{y}{Z}=\frac{-y}{\sqrt{x^2-y^2}} \quad ✓$$
$$\therefore x\frac{\partial Z}{\partial x}+y\frac{\partial Z}{\partial y}=\frac{x^2}{\sqrt{x^2-y^2}}-\frac{y^2}{\sqrt{x^2-y^2}} \quad ✓$$
$$=\frac{x^2-y^2}{\sqrt{x^2-y^2}} \quad ✓$$
$$=\sqrt{x^2-y^2}$$

(3)

1.2 1.2.1

$x=6\cos\theta$ $\qquad y=6\sin\theta$

$$\frac{dx}{d\theta}=-6\sin\theta \quad ✓ \qquad \frac{dy}{d\theta}=6\cos\theta \quad ✓$$
$$\frac{dy}{dx}=\frac{dy}{d\theta}\frac{d\theta}{dx}=\frac{6\cos\theta}{-6\sin\theta} \; ✓ =-\cot\theta \quad ✓$$

$$x^2+y^2=36cos^2\theta+36sin^2\theta=36 \; ✓$$
$$\Rightarrow 2x+2y\frac{dy}{dx}=0 \; ✓$$
$$\Rightarrow \frac{dy}{dx}=-\frac{x}{y}=-\frac{\cos\theta}{\sin\theta}=-\cot\theta \; ✓✓$$

(2)

1.2.2

$$\frac{d^2y}{dx^2}=\frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}=\cos ec^2\theta\frac{1}{-6\sin\theta} \quad ✓$$
$$=-\frac{1}{6}\cos ec^3\theta \text{ or } =\frac{1}{-6\sin^3\theta} \quad ✓$$

$$\frac{d^2y}{dx^2}=\frac{-y+x\frac{dy}{dx}}{y^2} \; ✓$$
$$=\frac{-6\sin\theta+6\cos\theta(-\cot\theta)}{36sin^2\theta}$$
$$=-6\left(\sin\theta+\frac{cos^2\theta}{\sin\theta}\right)\div 36sin^2\theta$$
$$=\frac{1}{6sin^3\theta} \; ✓$$

(1)
[6]

VRAAG 2

2.1 $\int y\,dx = \int (1+\tan^2 3x)(\sec^2 3x - 1)dx$

$= \int \sec^2 3x \tan^2 3x dx$ ✓ ✓

Of u = $\tan 3x$

$= \frac{1}{3}\int 3\sec^2 3x \tan^2 3x dx$ ✓

$= \frac{1}{3}\frac{\tan^3 3x}{3} + C$ ✓

Alternative

$\int y\,dx \quad = \int \left(1+\tan^2 3x\right)\left(\sec^2 3x - 1\right)dx$

$= \int \left(\sec^2 3x + \tan^2 3x \sec^2 3x - 1 - \tan^2 3x\right)dx$ ✓

$= \frac{1}{3}\tan 3x + \frac{1}{3}\frac{\tan^3 3x}{3} - x - \left(\frac{1}{3}\tan 3x - x\right)$ ✓ ✓

$= \frac{1}{3}\frac{\tan^3 3x}{3} + C$ ✓

(2)

2.2 $\int y\,dx = \int \sin^{-1} bx dx$

$f(x) = \sin^{-1} bx \quad g'(x) = 1$ ✓

$f'(x) = \frac{b}{\sqrt{1-b^2x^2}} \qquad g(x) = x$ ✓

✓ ✓

$= x\sin^{-1} bx - \int \frac{b}{\sqrt{1-b^2x^2}}x\,dx$

$u = 1 - b^2x^2 \;\; dx = \frac{du}{-2b^2x}$ ✓

$= x\sin^{-1} bx - \int \frac{b}{\sqrt{u}}x\,\frac{du}{-2b^2x}$ ✓ ✓

$= x\sin^{-1} bx + \frac{1}{2b}\int u^{-\frac{1}{2}}du$ ✓

$= x\sin^{-1} bx + \frac{1}{2b}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$ ✓

$= x\sin^{-1} bx + \frac{1}{b}\sqrt{1-b^2x^2} + C$ ✓

(5)

2.3 $6x - x^2 = -\left[x^2 - 6x\right]$ ✓

$= -\left[x^2 - 6x + 9 - 9\right]$ ✓

$= -\left[(x-3)^2 - 9\right]$ ✓ $= 9 - (x-3)^2$ ✓

$\int y\,dx = \int \sqrt{9 - (x-3)^2}\,dx$ ✓

$= \dfrac{a^2}{2b}\sin^{-1}\dfrac{bx}{a} + \dfrac{x}{2}\sqrt{a^2 - b^2x^2} + C$ (formula used) ✓ ✓ ✓

$= \dfrac{9}{2}\sin^{-1}\dfrac{x-3}{3} + \dfrac{x-3}{2}\sqrt{9-(x-3)^2} + C$ or $\dfrac{9}{2}\sin^{-1}\dfrac{x-3}{3} + \dfrac{x-3}{2}\sqrt{6x-x^2} + C$

Alternative: $\dfrac{4ac - b^2}{4a} + a\left(x + \dfrac{b}{2a}\right)^2 = \dfrac{0-36}{-4} - \left(x + \dfrac{6}{-2}\right)^2 = 9 - (x-3)^2$

(4)

2.4 $\int y\,dx = \int \cos^4\left(\dfrac{3}{2}x\right)dx \quad = \int \left[\cos^2\left(\dfrac{3}{2}x\right)\right]^2 dx$ ✓

$= \int \left[\dfrac{1}{2} + \dfrac{1}{2}\cos 3x\right]^2 dx$ or $\int \dfrac{1}{4}(1 + \cos 3x)^2\,dx$ ✓

$= \int \dfrac{1}{4} + \dfrac{1}{2}\cos 3x + \dfrac{1}{4}\cos^2 3x\,dx$ ✓ ✓

$= \dfrac{1}{4}x + \dfrac{1}{2}.\dfrac{1}{3}\sin 3x + \dfrac{1}{4}\left\{\dfrac{x}{2} + \dfrac{\sin 6x}{12}\right\}$ ✓ ✓ ✓

$= \dfrac{1}{4}x + \dfrac{1}{6}\sin 3x + \dfrac{x}{8} + \dfrac{\sin 6x}{48} + C$ ✓ or $= \dfrac{3}{8}x + \dfrac{1}{6}\sin 3x + \dfrac{\sin 6x}{48} + C$

(4)

2.5 $\int y\,dx = \int x\tan x + \ln\sec x\;dx$

Use integration by parts for the first integral only

$= \int x\tan x\,dx + \int \ln\sec x\,dx$ ✓ using $f(x) = x$ and $g'(x) = \tan x$

$f'(x) = 1$ ✓ $g(x) = \ln\sec x$ ✓

$= \left\{x\ln\sec x - \int \ln\sec x\,dx\right\} + \int \ln\sec x\,dx$ ✓ ✓

$= x\ln\sec x + C$ ✓

(3)

[18]

VRAAG 3

3.1 $2x^3 - x^2 - 6x = x\left(2x^2 - x - 6\right)$ ✓

$= x\left(2x+3\right)\left(x-2\right)$ ✓

$\dfrac{\left(x+3\right)\left(x-4\right)}{2x^3 - x^2 - 6x} = \dfrac{A}{x} + \dfrac{B}{2x+3} + \dfrac{C}{x-2}$ ✓ ✓

$\left(x+3\right)\left(x-4\right) = A\left(2x+3\right)\left(x-2\right) + Bx\left(x-2\right) + Cx\left(2x+3\right)$ ✓

when $x = -\dfrac{3}{2}$ $\left(-\dfrac{3}{2}+3\right)\left(-\dfrac{3}{2}-4\right) = B.-\dfrac{3}{2}\left(-\dfrac{3}{2}-2\right)$

$-\dfrac{33}{4} = \dfrac{21}{4}B \quad \therefore B = -\dfrac{33}{21}$ ✓ $= -\dfrac{11}{7}$

when $x = 2 \quad 5.-2 = C.2.(4+3) \quad -10 = 14C \quad \therefore C = -\dfrac{5}{7}$ ✓

when $x = 0 \quad -12 = A.3(-2) \quad \therefore A = 2$ ✓

$\int \dfrac{\left(x+3\right)\left(x-4\right)}{2x^3 - x^2 - 6x}dx = \int \dfrac{2}{x}dx + \int \dfrac{-\frac{11}{7}}{2x+3}dx + \int \dfrac{-\frac{5}{7}}{x-2}dx$ ✓

$= 2\ln x - \dfrac{11}{7}.\dfrac{1}{2}\ln(2x+3) - \dfrac{5}{7}\ln(x-2) + C$ ✓ ✓ ✓

$= 2\ln x - \dfrac{11}{14}\ln(2x+3) - \dfrac{5}{7}\ln(x-2) + C$ (6)

3.2 $\dfrac{3x^2+16x+26}{(x-2)(x^2+3x+4)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+3x+4}$ ✓ ✓

$3x^2+16x+26 = A(x^2+3x+4) + (Bx+C)(x-2)$ ✓

when $x = 2 \quad 12+32+26 = A(4+6+4) \quad 70 = 14A \quad \therefore A = 5$ ✓

when $x = 0 \quad 26 = A(4) + C(-2)$

$26 = 20 - 2C \quad \therefore C = -3$ ✓

when $x = 1 \quad 3+16+26 = A(1+3+4) + (B+C)(-1)$ ✓

$45 = 8A - B - C$

$45 = 40 - B + 3 \quad \therefore B = -2$ ✓

$\int \dfrac{3x^2+16x+26}{(x-2)(x^2+3x+4)}dx = \int \dfrac{5}{x-2}dx + \int \dfrac{-2x-3}{x^2+3x+4}dx$ ✓ ✓

$= \int \dfrac{5}{x-2}dx - \int \dfrac{2x+3}{x^2+3x+4}dx$ ✓

$= 5\ln(x-2) - \ln(x^2+3x+4) + C$ ✓ ✓

Alternatief

$$\frac{3x^2+16x+26}{(x-2)(x^2+3x+4)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+3x+4} \quad ✓ \quad ✓$$

$3x^2+16x+26=A(x^2+3x+4)+(Bx+C)(x-2)$ ✓

when $x=2$ $\quad 12+32+26=A(4+6+4) \quad 70=14A \quad \therefore A=5$ ✓

Expanding $\quad 3x^2+16x+26=Ax^2+3Ax+4A+Bx^2+Cx-2Bx-2C$ ✓

Equating x^2 $\quad A+B=3 \quad \Rightarrow 5+B=3 \quad \therefore B=-2$ ✓

Equating x $\quad 3A+C-2B=16 \quad \Rightarrow 3(5)+C-2(-2)=16 \quad C=-3$ ✓

$$\int\frac{3x^2+16x+26}{(x-2)(x^2+3x+4)}dx=\int\frac{5}{x-2}dx+\int\frac{-2x-3}{x^2+3x+4}dx \quad ✓ \quad ✓$$

$$=\int\frac{5}{x-2}dx-\int\frac{2x+3}{x^2+3x+4}dx \quad ✓$$

$$=5\ln(x-2)-\ln(x^2+3x+4)+C \quad ✓ \quad ✓$$

(6)

[12]

VRAAG 4

4.1

$e^{\int pdx}=e^{-\int\frac{1}{x\ln x}dx}$ ✓

$=e^{-\int\frac{\frac{1}{x}}{\ln x}dx}$ ✓ or using $u=\ln x$

$=e^{-\ln(\ln x)}$ ✓

$=(\ln x)^{-1}$ ✓

$=\frac{1}{\ln x}$ ✓

$\int Qe^{\int pdx}dx=\int\frac{1}{x}\frac{1}{\ln x}dx$ ✓ ✓

$=\int\frac{\frac{1}{x}}{\ln x}dx$ or using $u=\ln x$ as before

$=\ln(\ln x)$ ✓

$\therefore y\frac{1}{\ln x}=\ln(\ln x)+C$ ✓

$x=2 \quad y=0 \quad 0=\ln(\ln 2)+C$ ✓ $\quad \therefore C=0{,}367$ ✓

$\therefore y\frac{1}{\ln x}=\ln(\ln x)+0{,}367$ ✓

(6)

4.2 $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=2e^{-3x}$

$r^2+r-6=0$ ✓

$(r+3)(r-2)=0$ ✓

$r=-3 \quad r=2$ ✓

$y_c=Ae^{-3x}+Be^{2x}$ ✓

For y_p $\quad y=Cxe^{-3x}$...........(1) ✓

If used $y=2Cxe^{-3x}$ follow up to get $C=-\frac{1}{5}$

$\frac{dy}{dx}=-3Cxe^{-3x}+Ce^{-3x}$ ✓

$\frac{d^2y}{dx^2}=-3.-3Cxe^{-3x}-3Ce^{-3x}-3Ce^{-3x}$ ✓

$=9Cxe^{-3x}-6Ce^{-3x}$ ✓

Substituting $\frac{d^2y}{dx^2},\frac{dy}{dx}$ and y in (1)

$9Cxe^{-3x}-6Ce^{-3x}+(-3Cxe^{-3x}+Ce^{-3x})-6Cxe^{-3x}=2e^{-3x}$ ✓

$-5Ce^{-3x}=2e^{-3x} \qquad \therefore C=-\frac{2}{5}$ ✓

$\therefore y_p=-\frac{2}{5}xe^{-3x}$ ✓

$y=y_c+y_p$

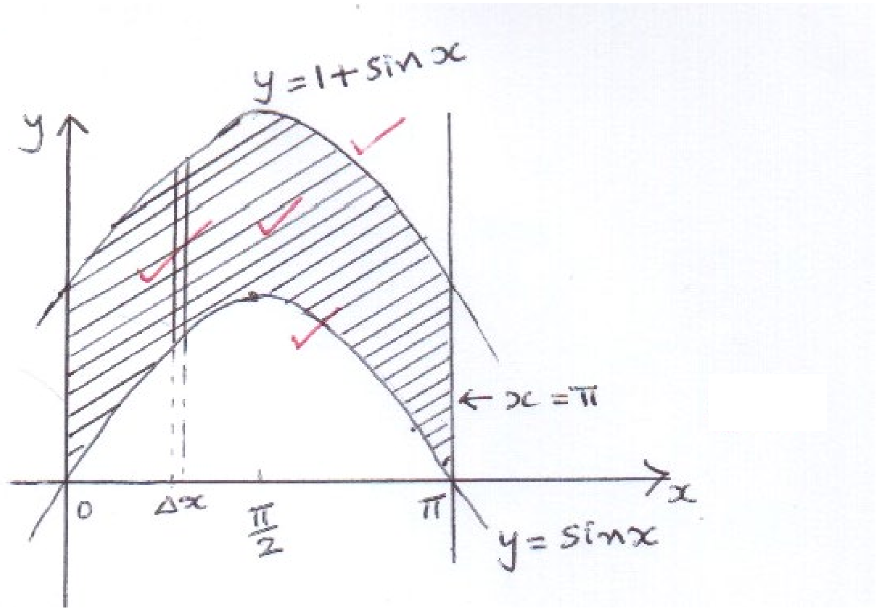
$y=Ae^{-3x}+Be^{2x}-\frac{2}{5}xe^{-3x}$ ✓

(6)

[12]

VRAAG 5

5.1 5.1.1



(2)

5.1.2

$$V_x = \pi\int_a^b (y_1^2 - y_2^2)dx \quad ✓$$

$$= \pi\int_0^\pi \left\{(1+\sin x)^2 - \sin^2 x\right\} dx \quad ✓ \; ✓ \; ✓$$

$$= \pi\int_0^\pi \left\{(1+2\sin x+\sin^2 x) - \sin^2 x\right\} dx \quad ✓$$

$$= \pi\int_0^\pi \left\{1+2\sin x\right\} dx \quad ✓$$

$$= \pi\left[x - 2\cos x\right]_0^\pi \quad ✓ \; ✓$$

$$= \pi\left[\pi - 2\cos\pi - (0 - 2\cos 0)\right] \quad ✓ \qquad = \pi\left[\pi - 2(-1) + 2\right] = \pi\left[\pi + 4\right]$$

$$= 22,4360\text{units}^3 \quad ✓$$

(5)

5.1.3

$$V_{m-y} = \int_a^b r dV \quad ✓$$

$$= \int_a^b \frac{y_1 + y_2}{2} 2\pi x\left(y_1 - y_2\right) dx$$

$$= \pi\int_a^b x(y_1^2 - y_2^2)dx \quad ✓$$

$$= \pi\int_0^\pi x\left\{(1+\sin x)^2 - \sin^2 x\right\} dx \quad ✓$$

$$\pi\int_0^\pi x\left\{(1+2\sin x+\sin^2 x) - \sin^2 x\right\} dx \quad ✓$$

$$= \pi\int_0^\pi x\left\{1+2\sin x\right\} dx \quad ✓$$

using integration by parts: $f(x) = x \quad g'(x) = 1 + 2\sin x$

$f'(x) = 1 \quad g(x) = x - 2\cos x$

$$= \pi\left[x(x - 2\cos x) - \int 1.(x - 2\cos x)dx\right]_0^\pi \quad ✓ \; ✓$$

$$= \pi\left[(x^2 - 2x\cos x) - \left(\frac{x^2}{2} - 2\sin x\right)\right]_0^\pi \quad ✓$$

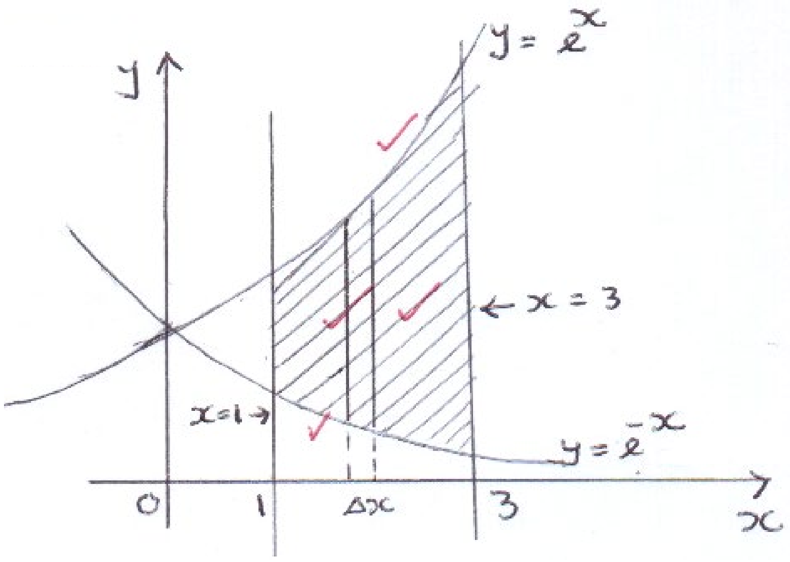
$$= \pi\left[(\pi^2 - 2\pi\cos\pi) - \left(\frac{\pi^2}{2} - 2\sin\pi\right) - (0)\right] \quad ✓ \qquad = \pi\left[\frac{\pi^2}{2} + 2\pi\right]$$

$$= 35,2423\text{units}^4 \quad ✓$$

$$\overline{x} = \frac{V_{m-y}}{V_x} = \frac{35,2423\text{units}^4}{22,4360\text{units}^3} \overset{✓}{=} 1,571\text{units} \quad ✓$$

(6)

5.2 5.2.1



(2)

5.2.2

$$A = \int_a^b (y_1 - y_2)dx$$

$$= \int_1^3 e^x - e^{-x} dx \quad ✓$$

$$= \left[e^x + e^{-x}\right]_1^3 \quad ✓$$

$$= \left[e^3 + e^{-3} - \left(e^1 + e^{-1}\right)\right] \quad ✓$$

$$= 17{,}0492 \text{units}^2 \quad ✓$$

(2)

5.2.3

$$A_{m-y} = \int_a^b r\,dA \quad \checkmark$$

$$= \int_a^b x(y_1 - y_2)dx \quad \checkmark$$

$$= \int_1^3 x(e^x - e^{-x})dx \quad \checkmark$$

$\checkmark$ ing integration by parts: $f(x) = x \qquad g'(x) = e^x - e^{-x}$

$f'(x) = 1 \qquad g(x) = e^x + e^{-x}$

$$= \left[x\left(e^x + e^{-x}\right) - \int (e^x + e^{-x})dx \right]_1^3 \quad \checkmark \; \checkmark$$

$$= \left[x\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right) \right]_1^3 = \left[x\left(e^x + e^{-x}\right) - e^x + e^{-x} \right]_1^3 \quad \checkmark$$

$$= \left[3e^3 + 3e^{-3} - e^3 + e^{-3} - \left\{ e^1 + e^{-1} - e^1 + e^{-1} \right\} \right] = \left[2e^3 + 4e^{-3} - 2e^{-1} \right] \quad \checkmark$$

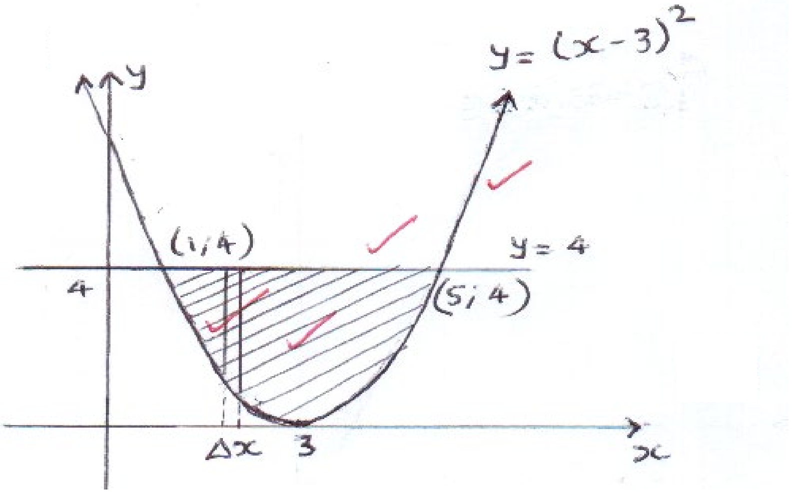
$$= 39{,}6345 \quad \checkmark \; \checkmark$$

$$\overline{x} = \frac{A_{m-y}}{A_x}$$

$$= \frac{39{,}6345}{17{,}0492} = 2{,}325 \text{units} \quad \checkmark \; \checkmark$$

(6)

5.3 5.3.1



(2)

5.3.2

$$A=\int_1^5 4-(x-3)^2\,dx \quad ✓ \quad ✓$$

$$=\int_1^5 4-x^2+6x-9dx$$

$$=\int_1^5 -5-x^2+6x\,dx \quad ✓$$

$$=\left[-5x-\frac{x^3}{3}+3x^2\right]_1^5 \quad ✓$$

$$=\left[-5(5)-\frac{5^3}{3}+3(5^2)-\left\{-5-\frac{1}{3}+3\right\}\right] \quad ✓$$

$$=10,667\text{units}^2 \quad ✓$$

Alternatief

$$A=\int_1^5 4-(x-3)^2\,dx \quad ✓ \quad ✓$$

$$=\left[4x-\frac{(x-3)^3}{3}\right]_1^5 \quad ✓ \quad ✓$$

$$=\left[20-\frac{8}{3}-\left\{4-\frac{-8}{3}\right\}\right] \quad ✓$$

$$=10,667 \quad ✓$$

(3)

5.3.3

$$I_y=\int_a^b r^2dA \quad ✓$$

$$=\int_1^5 x^2\left\{4-(x-3)^2\right\}dx \quad ✓ \quad ✓$$

$$\int_1^5 x^2\left\{-5-x^2+6x\right\}dx \quad ✓$$

$$=\int_1^5\left\{-5x^2-x^4+6x^3\right\}dx \quad ✓$$

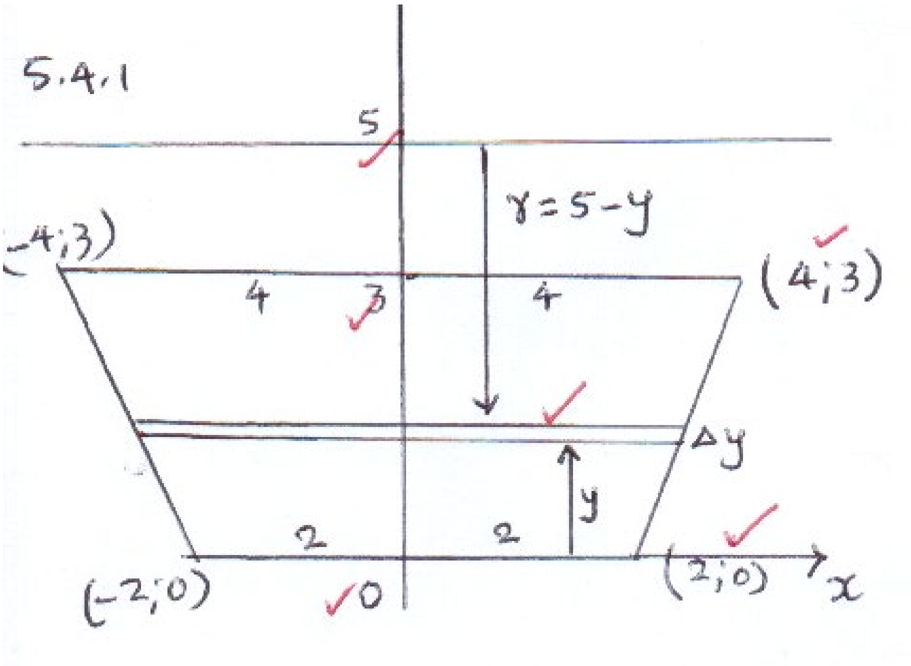
$$=\left[-\frac{5}{3}x^3-\frac{x^5}{5}+\frac{6}{4}x^4\right]_1^5 \quad ✓$$

$$=\left[-\frac{5}{3}.5^3-\frac{5^5}{5}+\frac{3}{2}5^4-\left\{-\frac{5}{3}-\frac{1}{5}+\frac{3}{2}\right\}\right] \quad ✓$$

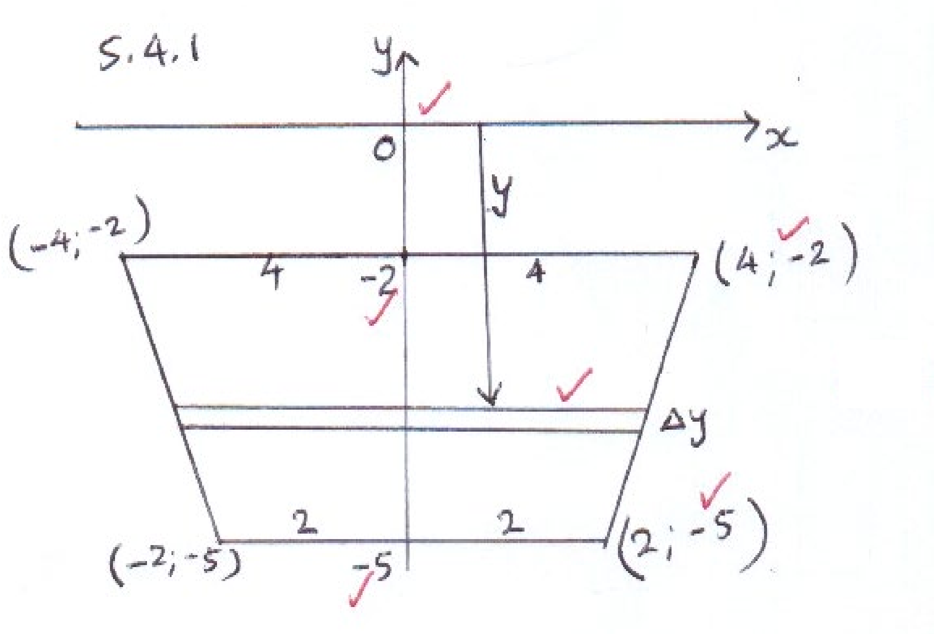
$$=104,533\text{units}^4 \quad ✓$$

(4)

5.4 5.4.1



Alternatief (met x-as op die watervlak)



(3)

5.4.2 slope of the right edge $m=\dfrac{3-0}{4-2}=\dfrac{3}{2}$ ✓

$y-y_1=m\left(x-x_1\right)$ using(2;0)

$y=\dfrac{3}{2}\left(x-2\right)$

$y=\dfrac{3}{2}x-3$

$x=\dfrac{2}{3}\left(y+3\right)$ ✓ $\therefore$ length of strip $=2x=\dfrac{4}{3}\left(y+3\right)$

first moment of area $=\int_a^b r\,dA$

$=\int_0^3 (5-y)\dfrac{4}{3}(y+3)\,dy$ ✓ ✓

$=\dfrac{4}{3}\int_0^3 2y-y^2+15\,dy$ ✓

$=\dfrac{4}{3}\left[y^2-\dfrac{y^3}{3}+15y\right]_0^3$ ✓ $=\dfrac{4}{3}\left[3^2-\dfrac{3^3}{3}+15.3-\{0\}\right]$ ✓

$=60\text{units}^3$ ✓

$\bar{y}=\dfrac{213}{60}=3,55\text{units}$ ✓ ✓

Alternatief:

slope of the right edge $m=\dfrac{-2+5}{4-2}=\dfrac{3}{2}$ ✓

$y-y_1=m\left(x-x_1\right)$ using(2;-5)

$y+5=\dfrac{3}{2}\left(x-2\right)$

$y+5=\dfrac{3}{2}x-3$

$x=\dfrac{2}{3}\left(y+8\right)$ ✓

length of strip $=2x=\dfrac{4}{3}\left(y+8\right)$

(4)

first moment of area $= \int_a^b r dA$

$= \int_{-5}^{-2} y \frac{4}{3}(y+8)dy$ ✓ ✓

$= \frac{4}{3}\int_{-5}^{-2} y^2 + 8y\, dy$ ✓

$= \frac{4}{3}\left[\frac{y^3}{3} + 4y^2\right]_{-5}^{-2}$ ✓

$= \frac{4}{3}\left[\frac{(-2)^3}{3} + 4(-2)^2 - \left\{\frac{(-5)^3}{3} + 4(-5)^2\right\}\right]$ ✓

$= -60\text{units}^3$ ✓

$\overline{\overline{y}} = \frac{213}{-60}$ ✓ $= -3,55\text{units}$

5.4.3 $Y = \frac{213}{-60} = -3,55 units$ ✓✓ (1)

[40]

VRAAG 6

6.1

$$x = \frac{y^2}{4}$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx \quad \checkmark$$

$$\frac{dx}{dy} = \frac{2y}{4} = \frac{1}{2}y \quad \checkmark$$

$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{1}{2}y\right)^2 = \frac{y^2}{4} \quad \checkmark$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{4} = \frac{4 + y^2}{4} \quad \checkmark \checkmark$$

$$\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx = \int_0^4 \frac{\sqrt{y^2 + 4}}{2} dx \quad \checkmark \checkmark$$

$$= \frac{1}{2}\left[\frac{y}{2}\sqrt{y^2 + 4} + \frac{4}{2}\ln\left\{y + \sqrt{y^2 + 4}\right\}\right]_0^4 \quad \checkmark \checkmark$$

$$= \frac{1}{2}\left[\frac{4}{2}\sqrt{4^2 + 4} + 2\ln\left\{4 + \sqrt{4^2 + 4}\right\} - \left(0 + 2\ln\left\{0 + \sqrt{0 + 4}\right\}\right)\right] \quad \checkmark \checkmark$$

$$= \frac{1}{2}\left[2\sqrt{20} + 2\ln\left\{4 + \sqrt{20}\right\} - 2\ln 2\right]$$

$$= 5{,}916 \text{units} \quad \checkmark$$

(6)

6.2 $y = 2\sin x$

$$A_x = 2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = 2\cos x \quad ✓$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+4\cos^2 x \quad ✓ \quad ✓$$

$$A_x = 2\pi\int_{\frac{\pi}{2}}^{\pi} 2\sin x\sqrt{1+4\cos^2 x}\,dx \quad ✓ \qquad u = 2\cos x \qquad \frac{du}{dx} = -2\sin x \quad ✓$$

$$dx = \frac{du}{-2\sin x}$$ follow up if used $u = \cos x$

$$= 2\pi\int_{\frac{\pi}{2}}^{\pi} 2\sin x\sqrt{1+4\cos^2 x}\,\frac{du}{-2\sin x} \quad ✓ \quad x = \frac{\pi}{2} \quad u = 2\cos\frac{\pi}{2} = 0$$

$$x = \pi \quad u = 2\cos\pi = -2$$

$$= -2\pi\int_0^{-2}\sqrt{1+u^2}\,du \quad ✓ \quad ✓$$

$$= -2\pi\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left\{u+\sqrt{1+u^2}\right\}\right]_0^{-2} \quad ✓ \quad ✓$$

$$= -2\pi\left[\frac{-2}{2}\sqrt{1+(-2)^2} + \frac{1}{2}\ln\left\{-2+\sqrt{1+(-2)^2}\right\} - \left(0+\frac{1}{2}\ln 1\right)\right] \quad ✓$$

$$= -2\pi\left[-\sqrt{5} + \frac{1}{2}\ln\left\{-2+\sqrt{5}\right\}\right]$$

$$= 18,585\text{units}^2 \quad ✓$$

(6)
[12]

200÷ 2 100

TOTAAL: 100